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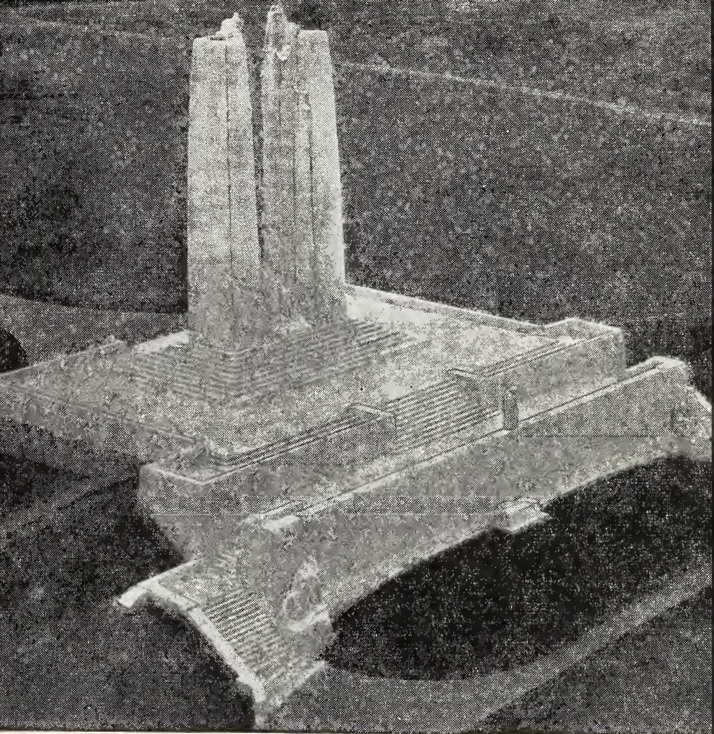
MATHEMATICS FOR CANADIANS



(*National Film Boa*

The Canadian War Memorial at Vimy exemplifies the use of simpl geometric forms in art.

MATHEMATICS FOR CANADIANS

Book I

BY

HENRY BOWERS
Principal of the Normal School, Stratford

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Professor of Mathematics in Queen's University
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AND

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Authorized by the Catholic Committee of the Council
of Public Instruction, Quebec

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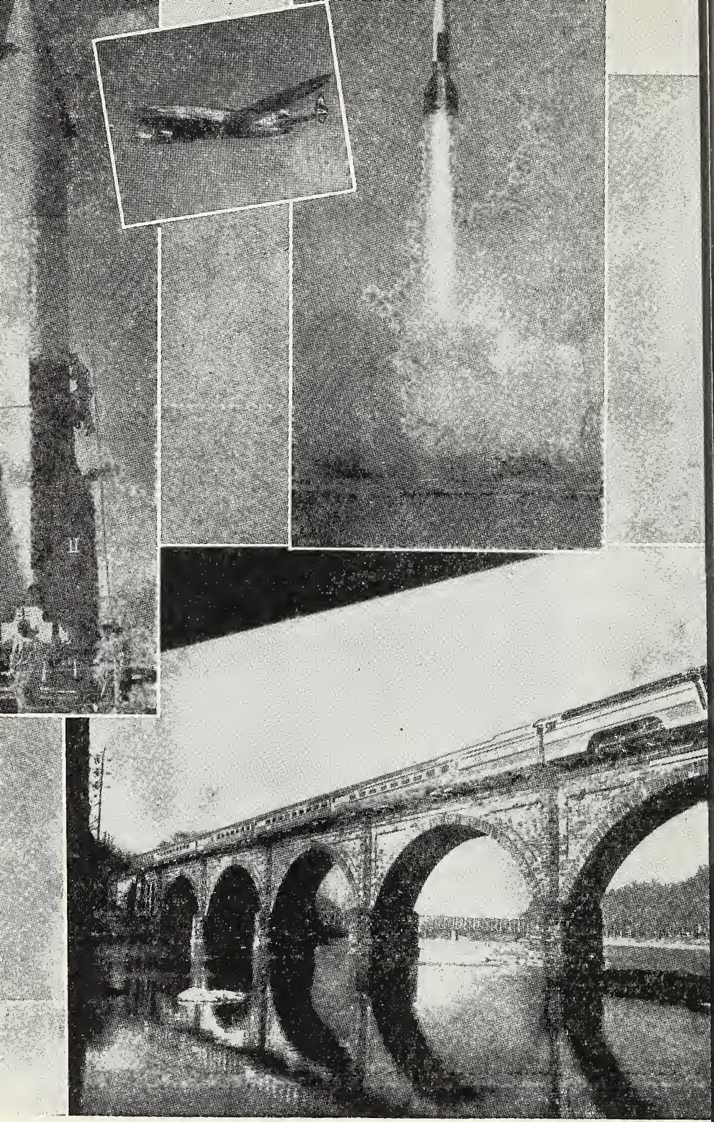


(Courtesy "Science Illustrat

Rockets, stream-lined trains, strato-liners--these are some of the products of those sciences which rest on the foundation of mathematics.

INTRODUCTION

WHY STUDY MATHEMATICS?

You are probably wondering what this book holds in store or you. What is it all about? Will it be interesting? Are the roblems difficult? Does it involve hard work? Perhaps you ave an even more important question: *Why should I study athematics?*

This is a fair question. If an effort is required of you, you ave a right to demand worth-while results. Of course, your eading of books, magazines, and newspapers may already have iven you a partial answer to your question. Even those who read ut little are soon aware of the fact that mathematics is very ecessary to scientists, engineers, and airmen. Suppose, however, you plan to be none of these—is mathematics still of value o you? The following pages are written in order to help you ee the place of mathematics in the education of a Canadian itizen.

Mathematics and Science

It is hardly necessary to remind young people that we are iving in the Age of Science. The horse and buggy, the sailing essel, the pony express, and the coal-oil lamp are relics of a anishing past. We live in an age of streamliners, strato-liners, ockets, radios, and atomic energy. These are some of the products of those sciences which rest on the foundation of mathematics.

Once you have grasped the basic role of mathematics in scientific achievement, you will probably agree that the citizen of today should have some acquaintance with the methods of mathematics. When one is faced at every turn with the present wonders and future possibilities of the sciences, he must have some appreciation of their background in order to make intelligent decisions. Look at the plate on the next page. It is composed of material taken from the handbook of a company

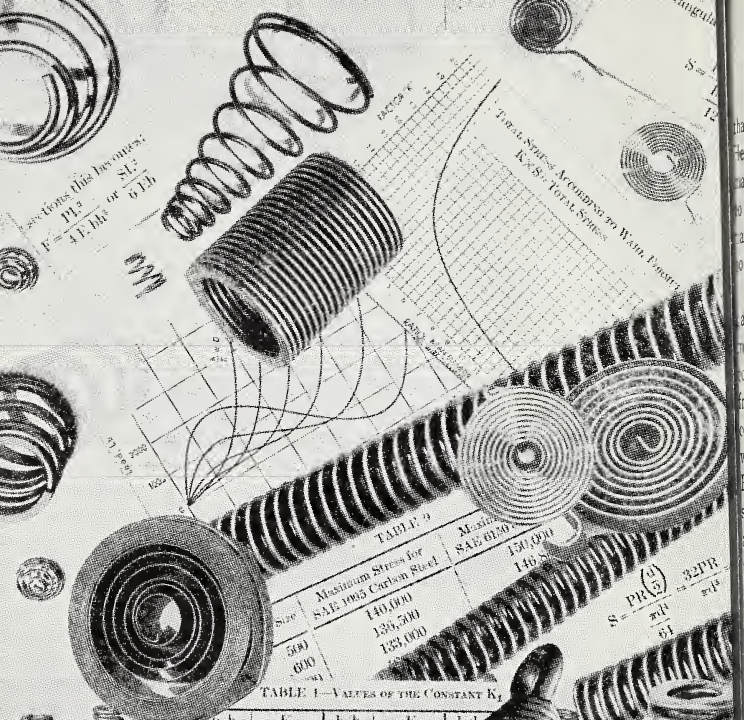


TABLE 3
Maximum Stress for SAE 1095 Carbon Steel
SAE 6150
140,000
136,500
133,000
150,000
146,5
TABLE 1—Values of the Constant K_1
b/h
K_1
1.0
.1795
2.2
.3027
1.1
.1970
2.3
.3076
1.2
.2131
2.4
.3120
.2269
2.5
.3170
2.6
.3201
Stress
Again referring to the fundamental formulas for beams
$S = \frac{Mc}{I}$
$F = \frac{PL^3}{4Ebh^3}$ or $\frac{SL^2}{6Eh}$
$S = \frac{PR\left(\frac{d}{2}\right)}{\frac{\pi d^4}{64}} = \frac{32PR}{\pi d^3}$
Total Stress According to Wahl Formula
K × S = Total Stress

(Courtesy Wallace Barnes Co. Ltd.,

Mathematics is essential in many industries. For example, the formula, graph, and the table of values play an important part in the making of sprin hat makes springs—springs for a great variety of machines. Here you see formulas, graphs, tables of values—basic tools of mathematics for solving scientific problems. It would be easy o fill this entire book with such examples of the important part played by mathematics in all sorts of machines from springs o strato-liners.

It may be that you plan to be an engineer, scientist, navi-ator, aviator, architect, astronomer, or banker. If so, mathematics is a necessity for you. Many of these fields are interesting o girls as well as boys. The day has come when girls may choose from a wide selection of professions, some of which require a considerable knowledge of mathematics. But what of those who plan to make their living in work remote from these fields? Experience suggests that, as dwellers on a planet ordered by cience, they should know something of mathematics if their bilities permit. The problems of peace demand it no less than the problems of war, which have all too recently required thousands of our citizens to acquire technical training in the interests of national defence.

Mathematics in Everyday Life

The study of mathematics provides you with a number of acts which are very useful in ordinary day-to-day life. Does the all can contain more than the shorter, stubbier one? How does the size of an egg depend on its length? In buying round cookies, pies, etc., how does the volume change with the diameter? In what way does a thick skin alter the value of an orange? How does one change a recipe in order to get fewer or more servings? Such questions are a natural and proper field for the use of mathematics. Other uses are not hard to find.

You will soon learn, for example, that the facts of mathematics are used in your other school subjects. In physics, chemistry, and shopwork, mathematics is helpful and often necessary. Business subjects also require mathematics. Nowadays many people buy household articles on the instalment plan. They pay extra for the privilege of getting their refrigerators early. Is this wise?

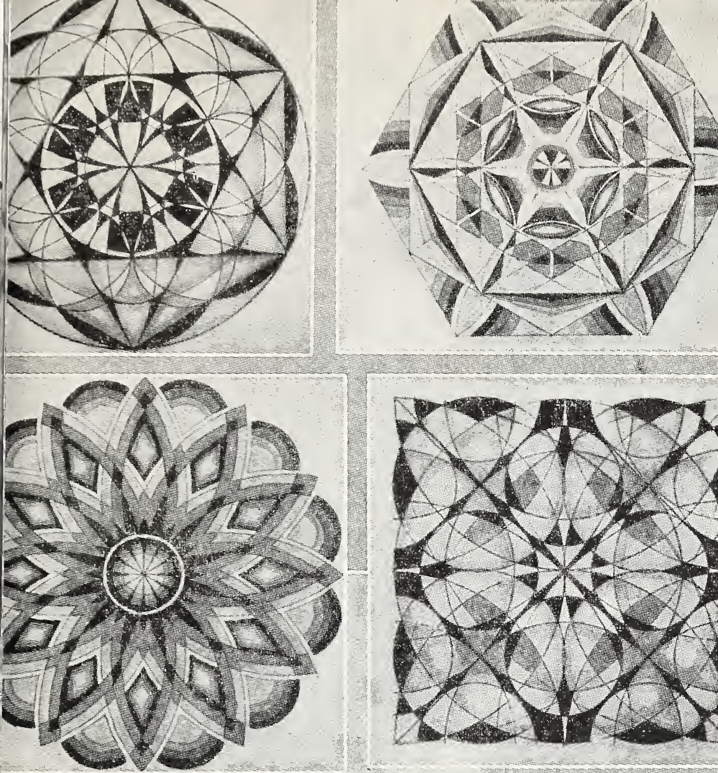
It is difficult to make an intelligent decision without some know edge of the mathematics involved.

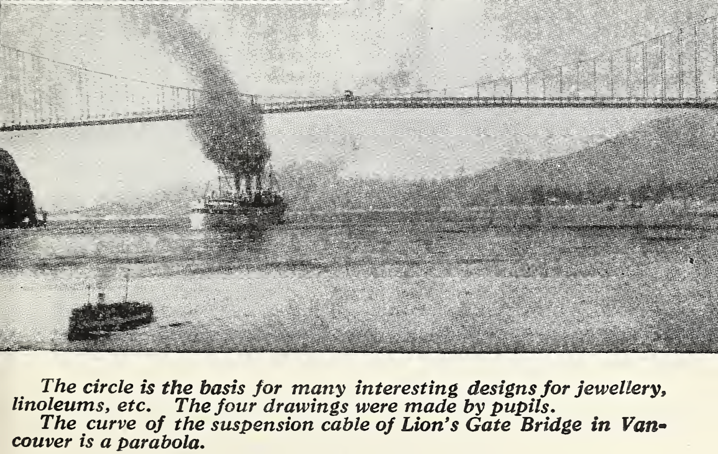
Your studies in this book will give you some informati about geometry. Some of the facts known to men for thousands years are within your reach. They are needed just as mu today as during the time of the Pharaohs and the Greek arch tects! Such ancient masterpieces as temples, pyramids, an irrigation projects both required and stimulated the growth geometry. Modern design uses geometrical ideas at every tur Consider for example our buildings, monuments (see frontispiece landscapes, furniture, baseball fields, race tracks, gymnasium tennis courts, highways, jewellery, linoleum, and so on. We liv in a world that makes lavish use of mathematics.

Mathematics and Clear Thinking

Never before in history have people been called upon to thin about so many different things as today. It seems as if everybody wants to sell something to everybody else. It may b insurance or a new vacuum cleaner or a subscription to fiv magazines at a special rate. Some of the decisions required the citizen are trivial, but others are of national importanc Bombarded by propaganda from every side on all important issues, John Citizen must learn to do some thinking for himself The ability to study the facts of a problem, draw logical conclusions, and make correct judgments is one of the most difficul and most important skills to acquire.

Mathematics is a way of thinking. It provides you with many opportunities to face problems and think them through A mathematical proof has no holes in it; the conclusions follow inevitably from the basic assumptions and the given data. In the pages that follow, you will be given practice in clear thinking both in mathematical and other situations. The drawing of correct conclusions from given data and the making of reasonably safe general statements from studies of special cases are abilities that every citizen in a democracy needs.





The circle is the basis for many interesting designs for jewellery, linoleums, etc. The four drawings were made by pupils.

The curve of the suspension cable of Lion's Gate Bridge in Vancouver is a parabola.

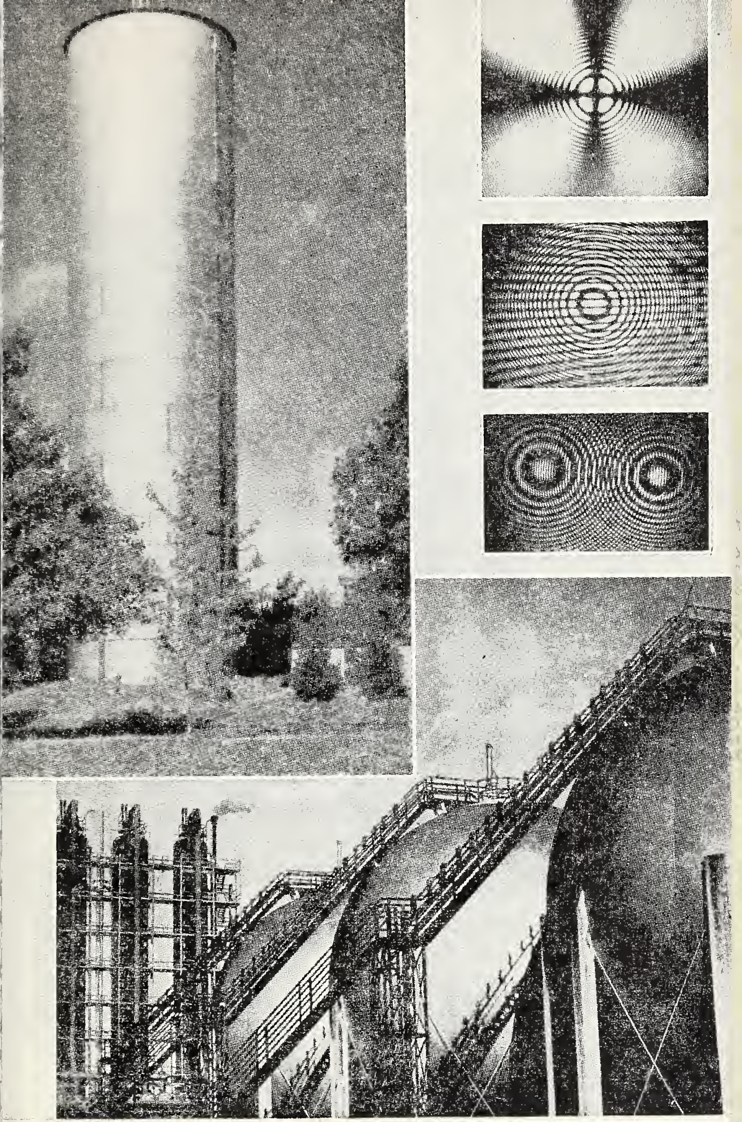
A little thought will convince you that in this world of ou one thing usually depends on another. The gasoline used by car depends on the speed of travel, the postage on a parcel depen on its weight, the volume of a ball depends on its radius, and on. The whole problem of dependence is basic in mathemati as well as in everyday life. You should therefore not fail take full advantage of the effective methods of mathematics f dealing with problems in relational thinking. The symbols algebra, the formula, tables of values, and the graph provi tools of great usefulness. The engineer using his formula and th nurse studying a temperature chart are both using a mathemat cal method of dealing with relationships.

> "*It is evident that the understanding of relations is a major concern of all men and women . . . mathematics is sometimes defined to be the science of relations.*"
>
> C. J. Keyser

Mathematics and Culture

Most of the foregoing underlines the usefulness of mathematic in helping you make a living and fulfil your obligations as citizen in a free country. While the aim of direct usefulness i valid and important, other objectives may in the long run giv you more satisfaction. Thousands of years ago, the great geometer Euclid gave a withering retort to a pupil who asked, "What' the use of all this stuff?" Summoning a slave, Euclid ordered "Give this fellow three pennies, for he must make a profit o everything he does."

Euclid's reply showed his contempt for those who can find n pleasure in study for its own sake. A parallel can be drawn fo our own times. The machine may soon teach us how to do th world's work in four hours a day. What will we do with ou leisure hours? Many writers have urged the necessity of a well-



(Photos courtesy National Film Board and Ja

metry---man-made and in nature: the familiar cylindrical water tank; circl patterns in atomic photography; the great horton storage spheres at the Polymer Corporation, Sarnia.

stored mind as a good basis for a happy life. One faces emp days, indeed, if none of the great events swirling about him c be viewed in the light of a rich stock of facts and experiences.

Consider atomic energy. Surely its possibilities for good a evil are better appreciated by those who know some history a have at least a nodding acquaintance with molecules and atom Popular magazines point out that Dr. Einstein gave the bas formula $E = mc^2$ many years ago. What a simple formula t lead to so fantastic a result! Even slight contact with math matics and the sciences helps to provide an inner satisfacti and deeper appreciation of the wonders within the atom.

Mathematics has its roots deep in the soil of human need As man's needs became more complex, as his horizons widene and his civilization developed, his mathematics grew in sco and utility. Indeed, mathematics has been called "the mirror civilization".* Primitive man had simple needs; a few number met all his mathematical requirements. Five thousand years ag the Babylonians and the Greeks had expanded mathemati until they were able to construct temples, palaces and oth great structures of unequalled beauty and usefulness. Today we are living in a maze of scientific development which stem from an ever-increasing body of mathematics. Without som knowledge of it, you will be unable to appreciate the world i which you live.

This appreciation, once gained, will give you a fuller an richer life. To observe geometric forms in nature, in architectur and in machinery adds to the beauty of the many forms tha surround us. To know something of the mathematical basis o the sciences, of industry, and of business makes for a bette understanding of the problems of the citizen. You may even b numbered among the many who find great beauty in the symbol and ideas of mathematics unadorned by the garnish of practica usefulness. You may, indeed, be numbered among the thousand who get fun and satisfaction from the working of mathematica problems and puzzles.

---

*Hogben: *Mathematics for the Million.*

Summary

Of what use is mathematics? It is a story with many chapters, subject of many parts. To some, mathematics provides the asic information and methods with which they work; these are e many engineers, scientists, and technologists of all kinds who ake our age possible. To all, it may provide many facts useful meeting the problems of everyday life, many opportunities to evelop the faculty of clear thinking, and the basis for a rich ppreciation of many aspects of our civilization.

> "*Mathematics, rightly viewed, possesses not only truth but supreme beauty—a beauty cold and austere, . . . without the gorgeous trappings of painting and music . . . , capable of a stern perfection such as only the greatest art can show.*"
>
> Bertrand Russell

CHAPTER I

THE DRAWING OF ACCURATE FIGURES

How Geometry Began

Even in the good old days there were taxes. To take an xample—the farmers in Ancient Egypt enjoyed the privilege of aying taxes on their land. There was a difficulty—in addition o the usual one of finding the cash. Every year, the Nile flooded s valley and washed away the landmarks. You can see what hat led to. As soon as the flood subsided, each farmer had a quabble with his neighbour and the Department of Inland Revenue had words with both. A deplorable situation! Somehing had to be done.

The priests, who were the most learned men in the country, hought over the problem and devised methods of re-establishing he boundary lines. In their primitive land surveying, geometry egan. Its origin is indicated by the derivation of the word from wo Greek words, *geos* (the earth) and *metron* (a measure).

For another quite different reason, geometry was studied by he priests of Egypt. They were very particular about the direction in which their temples faced. To determine directions, t a time when the magnetic compass was unknown, it was necessary to make observations on the stars and to know somehing about the properties of lines and angles.

These priests were interested in geometry mainly because it was useful in their work. However, the Greeks who settled in Egypt did not confine themselves to the purely practical side of the subject. They looked on it as an intellectual pastime, playing with it as people today play chess. It became a delightful puzzle-game. If a new geometrical discovery had no apparent practical use, that did not lessen the fun of making it. These Greeks liked climbing intellectual peaks, not for the sake of finding three pennies at the top, but for the sheer joy of pitting their wits against difficulties. In trying to solve geometric pro lems—just as people nowadays try to solve murder problems detective stories—they added greatly to the known properties

The Greeks did not climb intellectual peaks for the sake of money.

angles, lines, planes and solids. Also, they gave proofs for thei discoveries and arranged these proofs in a fairly logical chain But that is another story.

Striking up an Acquaintance with a Blue-Print

On the next page (Fig. 1) we see the plan of the ground floor of a summer cottage. This copy of a blue-print is worth a little study.

Notice the dimensions. The plan is 3 inches long, but this length is marked 24 feet. Since a length of 24 feet is represented by a length of 3 inches, the ***scale*** is 8 feet to the inch. Measure the width of the drawing and compare it with the width of the cottage. Does the same scale hold good?

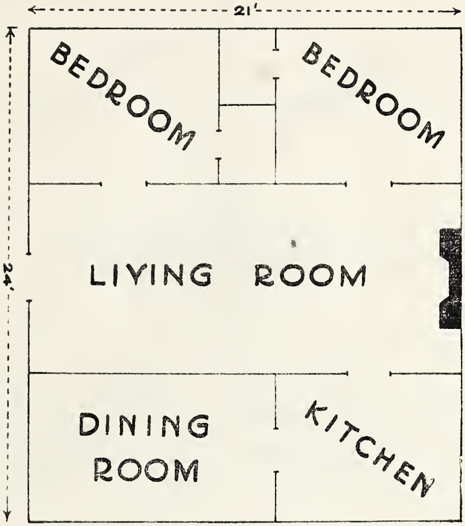


FIGURE 1

Observe how fine the lines are. They must be fine because hey are merely boundaries. If these lines were one-twelfth of n inch thick, on the scale they would represent a width of 8 nches. On the same scale, a line only one hundredth of an inch hick would indicate a width of ——— ?

* * *

The ideal line has no thickness. Unfortunately, if ideal ines—drawn we may suppose by an ideal architect using an deal pencil—were used, the plan would look like Figure 2 on he next page.

* * *

What do you think the builder would say?

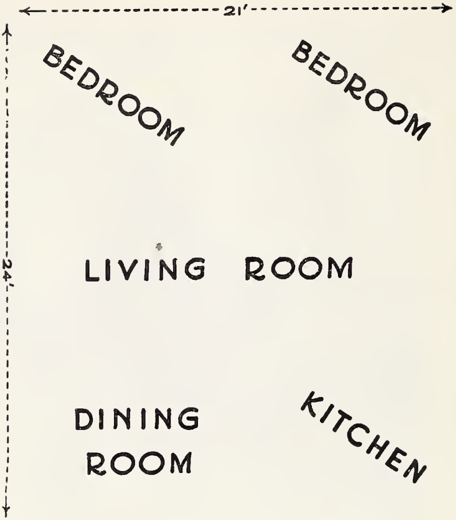
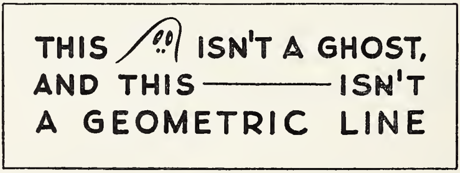
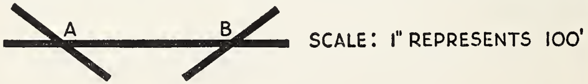

21′
24′
BEDROOM
BEDROOM
LIVING ROOM
DINING ROOM
KITCHEN

Figure 2

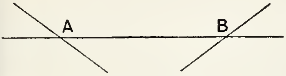
Since it is impossible to draw geometric lines, we have to b content to represent them by the usual thin marks.


THIS ISN'T A GHOST,
AND THIS ISN'T
A GEOMETRIC LINE

A student made the following drawing from informatio given in a problem. He wanted to obtain the distance betwee


A
B
SCALE: 1" REPRESENTS 100'

e intersections at A and B. Would his answer be as accurate if he had made this drawing?

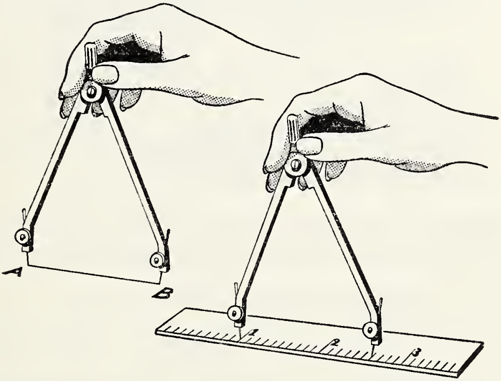


SCALE: 1" REPRESENTS 100'

Take this hint:

When you are trying to make an accurate figure, use a well-sharpened pencil and draw very fine lines.

Measuring a straight line. A pair of dividers may be used measuring a straight line. The instrument is adjusted until point rests on each end. Then it is transferred to the ruler d the length obtained. Note that we have taken for granted e accuracy of the graduations.

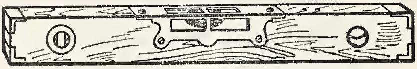


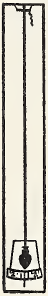
In this course, it will be sufficiently accurate most of the time measure lines to the nearest tenth, twelfth or sixteenth of an ch, depending upon how your ruler is graduated.

EXERCISES

1. In your room, find straight and curved lines.

2. Obtain a spirit level. Use it to find out if the wind ledge is level or ***horizontal.*** If the surface is perfectly level, th every line on it is horizontal. Find examples of horizontal lin in the room.





3. Use a plumb-line to find out if the edge of t door is perfectly upright or ***vertical.*** Find vertical lin in the room.

4. A line that is neither horizontal nor vertical called ***oblique.*** Point out examples of oblique lines.

5. See how well you can *estimate* the length of line Draw a straight line. Estimate its length in inche Then measure the line carefully. Record both estima and measurement. Do this for about ten lines. Are yo estimates gaining in accuracy?

6. Estimate the lengths of much longer lines, in the roo in the building, outside. Check the accuracy of your estimate

7. Give examples of the use of scale drawings.

8. What do you understand by a scale of 2 inches to the mile

9. What are some of the scales used in the drawing of maps

10. When making scale drawings, why is it desirable to us as large a scale as possible?

11. Copy the following table and fill in the missing values.

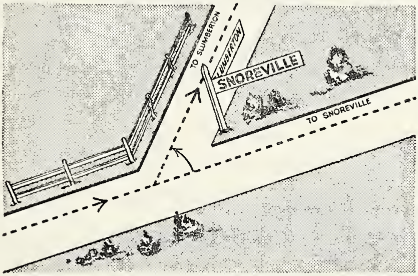
ctual length in feet	1	4	12	30	43	50			
cale length in inches (Scale ½ in. to 1 ft.) . .							27	30	31.25

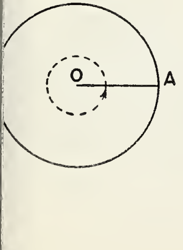
12. Copy the following table and complete it.

ctual length in feet	1	2	5	27	98	100			
cale length in inches . . .	0.1	0.2					11.3	16.9	20.5

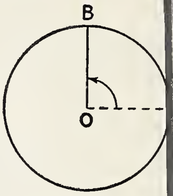
Angles

Suppose that you are going by bicycle to the village of umberton. When you come to the intersection of the Slumber-n and Snoreville roads you must change direction. The ***nount of turning*** made by your handlebars is an ***angle.***



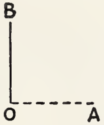


The complete revolution. The illustration shows one door of a set of swinging doors. Let us suppose that it was swung completely around once. In that one complete revolution, the door turned through an angle of three hundred and sixty degrees (360°).



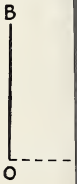
A quarter of a complete revolution. If the door mad quarter of a revolution, so that it came to rest in the p tion now shown, the angle through which it turned was a quarter of 360° or 90°. This square angle is called a ***right angle.*** Notice a few of the right angles in the room in which you are sitting.

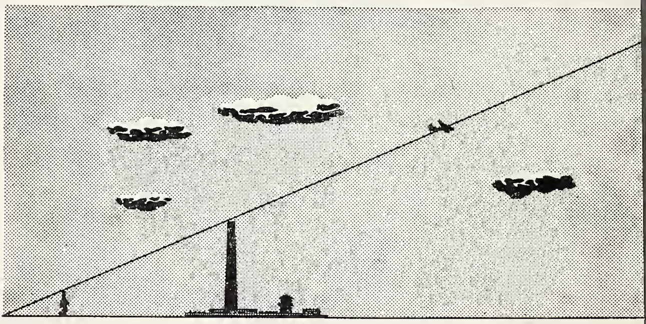
The door turned from the position *OA* to the position *OB*. These two lines (*OA* and *OB*) are called the ***arms*** of the angle. You should know that the length of the arms has nothing to with the size of the angle. Rotate one leg o small pair of dividers through a quarter revo tion. Also, rotate one leg of a pair of blac board compasses through a quarter revoluti The two angles are equal.



Furthermore, the two arms of the angle need not be of the same length. You can imagine that the line *OA* rotated until it lay along *OB*.

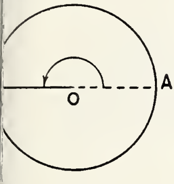
These two facts, namely that increasing or diminishing the arms has no effect on the angle and that the arms may be unequal, are true of all angles.





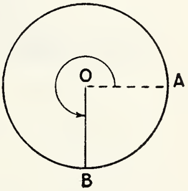
The angle has the same size irrespective of whether the arm reaches to the man, the chimney, the aircraft or the star.

The straight angle. This time we shall turn the door until s exactly in line with its original position. The angle formed is often called a ***straight angle.*** Obviously, to complete the revolution, it would be necessary to turn the door through another straight angle.

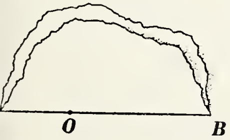


Two straight angles are equal to 360°. Therefore, one straight angle is 180°. How many right angles are there in one straight angle?

The angle of 270°. When the door has de three-quarters of a complete revolu-n, it is in the position shown. The angle rned through is three-quarters of 360°, at is to say, 270°. How many right angles there in three-quarters of a complete volution?

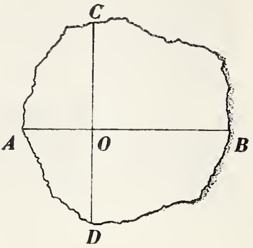


Creasing paper to form angles. Fold a piece of paper as shown in the illustration. Then fold it again so that *OA* lies along *OB*. Open the paper and examine the four angles made by the creases. What is the sum of these four angles? How do they compare in size? (Give the reason for your answer.) What is the size of each angle?

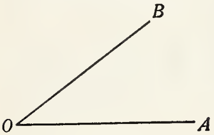


When two lines, for example *OC* and *OB*, are arms of a ht angle we say that each is ***perpendicular*** to the other. hat line is perpendicular to *AO*?

Fold the same piece of paper so as to ***bisect*** (or divide into o equal parts) one of the right angles. Try to fix in mind this angle of 45°. How many such angles are there in a co plete revolution?

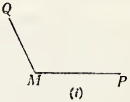


Naming angles. To refer conveniently to an angle, we lett the arms. In the illustration, O is t point at which the lines AO and intersect. The angle is called AOB BOA. Note that in both cases, the lett at the turning-point is in the middle.

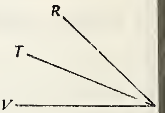


For practice, name the angle formed by:

(*a*) MP and MQ
(*b*) SR and ST
(*c*) ST and SV
(*d*) SR and SV

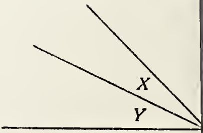


(i)

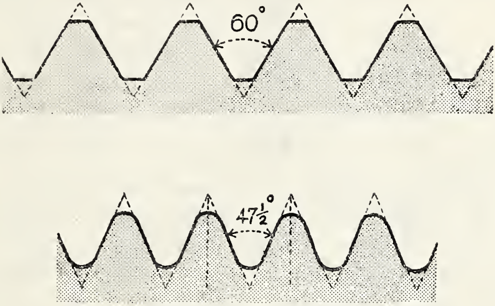


(ii)

Frequently, we use a single letter to denote an angle. Th in (i) above, we may call the angle either PMQ or M. In the accompanying figure two angles are designated by the single letters X and Y. In the figure above, why would it be objectionable to refer to the angle TSV as the angle S?



Instead of writing the word *angle*, we frequently use the nbol $\angle$. Thus, instead of writing *angle* VST, we may write re briefly $\angle VST$.



The angles of gear teeth.

EXERCISES

1. Through what angle do you turn in making (*a*) a left turn,) a right incline, (*c*) an about turn?

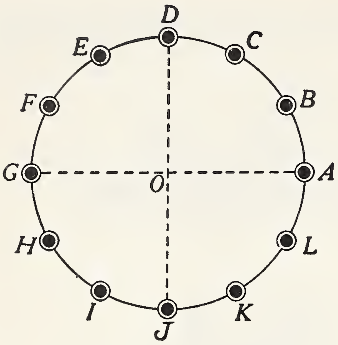
2. State the length of time required

(*a*) for the minute hand of a clock to turn through 90°;
(*b*) for the hour hand of a clock to turn through 90°.

3. State the number of degrees through which a spoke of a eel would rotate when making each of the following turns:

(*a*) two revolutions;
(*b*) one sixth of a revolution;
(*c*) one eighth of a revolution;
(*d*) one twelfth of a revolution;
(*e*) one three hundred and sixtieth of a revolution.

4. The earth rotates on its axis once each day. Through w many degrees does it rotate per hour?



5. A, B, C, etc. are ri evenly placed in a circu metal patch. Calculate number of degrees in eac the following angles:

(*a*) AOB; (*e*) AOF;
(*b*) AOG; (*f*) BOC;
(*c*) AOE; (*g*) COE;
(*d*) AOC; (*h*) BOF.

6. What fraction of a re lution is each of the follow angles?

(*a*) 120°; (*d*) 45°; (*g*) 108°; (*j*) 240°;
(*b*) 15°; (*e*) 60°; (*h*) 135°; (*k*) 270°;
(*c*) $22\frac{1}{2}$°; (*f*) 72°; (*i*) 225°; (*l*) 300°.

7. State the number of degrees turned through from 3 p. to 4 p.m. on the same day by (*a*) the hour hand of a clock, (*b*) t minute hand.

The magnetic compass. Examine a magnetic compass. Y should be familiar with the following directions:

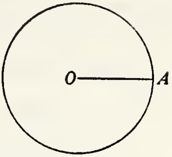
(*a*) North, South, East and West.
(*b*) North-East (NE); NW; SE and SW.

8. State the angle between the directions:

(*a*) N and E; (*d*) S and SE; (*g*) NE and SE;
(*b*) E and W; (*e*) W and SW; (*h*) NE and SW;
(*c*) N and NE; (*f*) W and NW; (*i*) N and SW.

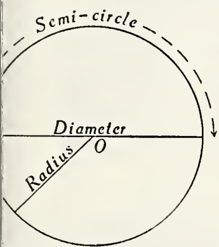
A familiar figure. Take any point on a sheet of pape Imagine that a second point must obey the following rules: (*a*) must be on the paper; (*b*) it must be neither more nor less th two inches from the first point. Mark a number of possib positions of the second point. Along what path must it lie?

* * *



you have seen, the path traced out by a nt moving on a level or ***plane*** surface so t it is always at a fixed distance from a d point is a circle. The plane surface need be horizontal, but may be tilted at any le to the horizontal.

The fixed point O is called the ***centre*** of the circle. Any line drawn from the centre to the circle is called a ***radius*** (plural, ***radii***). A straight line which passes through the centre and which has its ends on the circle is called a ***diameter.*** How does a diameter compare in length with a radius?



The end points of any diameter divide the circle into two equal parts each of which is a ***semicircle.***

EXERCISE

A pupil who was asked to define a circle said, "A circle is the h of a movable point which is always a fixed distance from a d point." Criticize his definition.

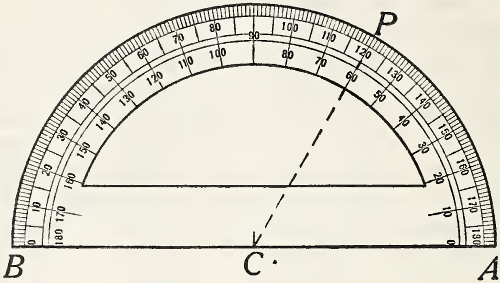
How to measure an angle. In your box of instruments, you find a protractor similar to the one shown in the illustration the next page. Examine it closely.

How would you describe the curved line from A to B?

At the middle of the straight portion is a mark. The point C, ere this mark meets the lower edge, is the centre of the circle vhich APB is half.

We have drawn CP one of the radii of the circle. Imagine t CP is movable and hinged at C. If CP set out from the position *CA* and turned into its present position *CP*, then angle formed (*ACP*) is, as you can see, 60°.

Perhaps you are wondering about the 120° mark at the s place.

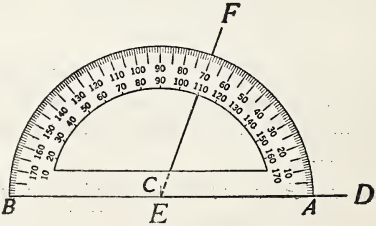


If *CP* had set out from *CB* and had turned in the *clock* sense—that is, the sense in which the hands of a clock move would have made ∠*BCP*, which is 120°.

Clearly, ∠*ACP* is less than 90°. Use your common se and you won't have any trouble in deciding which reading take.

* * *

To measure ∠*DEF*, place the protractor so that *C* is exac at *E* and so that *CA* is placed accurately along *ED*.



Now note where EF meets the semicircle. Which set of
duations must be used? How many degrees are in ∠DEF?

The accuracy with which an angle is measured depends to a large extent on the care with which the protractor is placed.

Making an angle of a given size. Draw a line OA. From
raw a line OB so that ∠AOB is 65°. Do this easy exercise
hout assistance. Then turn to page 405 and find out if you
e carefully followed the procedure described there.

EXERCISES

1. Construct the angle which the hands of a clock make at
2 o'clock, (*b*) 3 o'clock, (*c*) 6 o'clock.

2. Construct the following angles: (*a*) 185°; (*b*) 225°;
270°; (*d*) 315°.

3. Draw any angle ABC. On the side of BA remote from
, draw a line BD so that the angle ABD is equal to the angle
C. (Use your protractor.)

Hint: Draw a rough freehand sketch. Then plan your
cedure.

Measure the angle DBC. How does it compare in size with
angle ABC? *Note:* BA is (or should be!) the bisector of
angle DBC.

Exercises 4 - 9 refer to the figure on p. 16

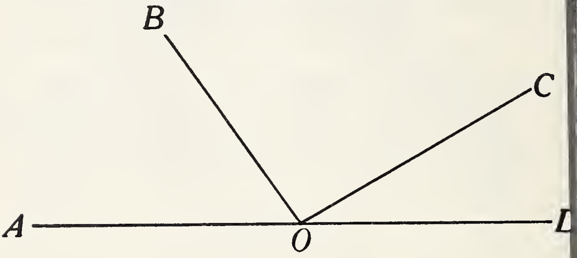
4. Measure the following angles and record your measure-
nts: (*a*) DOC; (*b*) COB; (*c*) BOA; (*d*) DOA; (*e*) DOB; (*f*) COA.

5. What two angles make up ∠DOB?

6. (*a*) Add your measurements of ∠DOC and ∠COB. Com-
e the sum with your measurement of ∠DOB.

(*b*) If you find that the sum of the measurements of $\angle$ and $\angle COB$ is equal to the measurement of $\angle DOB$, hav proved conclusively that your work is accurate? Give reasons.

(*c*) If you find that the two results are not equal, you proved that your work is inaccurate? Why?



7. Compare the sum of the measurements of $\angle COB$ $\angle BOA$ with the measurement of $\angle COA$.

8. Compare the sum of the measurements of $\angle$sDOC, and BOA with the measurement of $\angle DOA$.

9. Write the following statements in your work books fill in the blanks:

(*a*) $\angle BOD - \angle BOC = \angle \quad$;
(*b*) $\angle AOC - \angle \quad = \angle AOB$;
(*c*) $\angle AOD - \angle \quad = \angle AOB + \angle COD$.

10. Floraville is 3 miles west of Sunniton. Honiburg i miles north of Sunniton.

(*a*) How far is it from Floraville to Honiburg as the c flies?

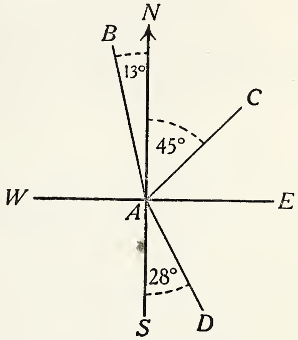
(*b*) What is the bearing of Honiburg from Floravi (Don't be uneasy. This will be explained in a moment.)

(*c*) What is the bearing of Floraville from Honiburg?

(*d*) What is the bearing of Sunniton from Floraville?

Hints: First make a rough sketch. Mark the distances on e sketch. Choose a suitable scale.

The bearing of a place *B* from another place *A* is the point the compass that you would e if you stood at *A* and ked towards *B*. In the ac- mpanying illustration, the aring of *B* as seen from *A* is °W of N (or N 13°W). The aring of *C* from *A* is NE. The aring of *D* from *A* is 28°E of or S 28°E). There are other thods of expressing bearing.



11. My dog, Postulate, who, hough not brilliant, fancies nself in mathematics, made following entry in his "Reserve Bone Book":

"Five bounds from tree stump in direction S 75° W. Then ee bounds on bearing S 55° E. Dig."

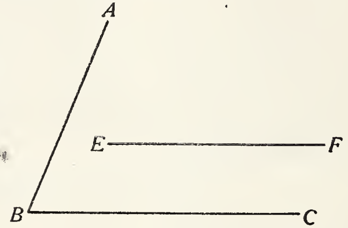
A fence prevented Postulate from making a direct obser- ion from the stump to the place of burial. If the fence had been there, what simpler entry could Postulate have made the "Bone Book"?

Estimation of the size of an angle. The ability to make a se estimate of the size of an angle is sometimes useful. You l now be given practice in estimating the number of degrees ngles drawn on paper. Later, you will have an opportunity stimating angles out of doors.

Draw an angle and estimate its size. Then measure it care- y. Record your estimate, the actual size, and the error of imate in a table similar to the following:

Estimated size af angle	*Actual size of angle*	*Error of estimate*
20°	13°	+7°
35°	45°	−10°
60°	55°	+5°

Repeat estimation and measurement until you become re onably expert in the former.



Copying an angle. A man who was laying out walks in garden wished to obtain a line DE so that $\angle DEF$ is equal $\angle ABC$.

We shall try to solve this little problem on paper. Then shall see how it could be solved in the garden. Draw the line your work book. Then, using your protractor, obtain the l DE.

* * *

We have mentioned that the Greeks regarded geometry a game. One rule of the game was that the use of a protrac was forbidden. Another forbade the use of a ruler *with gra ations on it.* It was permitted to draw straight lines with the of an ungraduated ruler or straight edge. There was noth unsportsmanlike in using the compasses.

You have copied the angle by using your protractor. Y could also have copied it by using tissue paper. (This would h given Euclid a fit.) Now play under the old rules: *Using strai edge and compasses only, make an angle DEF equal to the an ABC.*

A SUGGESTION

Some run for help whenever they meet a difficulty. Others, more independent, scorn to seek assistance before they exert to the utmost their own powers. The effort you make in trying to solve a problem is often worth more than the obtaining of the answer.

The construction of the required angle is given on page 405. *Compare your method with the one described. If your construction is different from the one described, bring it to your teacher's notice. Correct solutions differing from those in the textbook interest the teacher.*

Those set squares. Place one of the set squares on your er. Draw lines along the edges. Remove the set square produce the lines to meet. Measure the three angles med. Similarly, measure the three angles of the other set are.

One of the set squares provides you with a 'ready made' angle 5°. The other gives you the angles 30° and 60°. Both set ares are useful for the construction of right angles.

EXERCISES

1. Give a description of an angle.

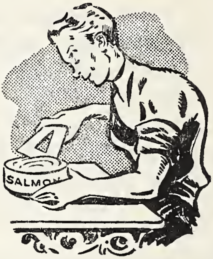
2. How many degrees are there in one revolution?

3. What is (*a*) a right angle, (*b*) a straight angle?

4. How would you crease a piece of paper so as to form a t angle?

HINT

Keep all your instruments from injury. A pair of compasses with a blunt point is useless. Bent metal set squares are merely scrap.



5. (*a*) When are two straight lines said to be perpendicu to each other?

(*b*) In your room find examples of (i) two perpendicu straight lines, (ii) three lines each perpendicular to the othe (iii) three lines each perpendicular to another line.

6. Draw two straight lines making an angle of 270°.

7. Draw two angles, one less than 90°, the other greater th 90° but less than 180°. Measure the angles.

8. What do you understand by the bisector of an angle?

9. Draw an angle. Measure it with your protractor. Usi compasses and straight edge, construct an angle double its si Check the accuracy of your result by careful measurement.

10. How would you make an angle equal to a given angle) using the protractor, (*b*) using compasses and straight edge ly? Make drawings to illustrate your answers.

11. Draw a straight line AB. Take any point Q in AB and y point P not in AB. Join PQ. Using straight edge and mpasses, draw a line PR so that $\angle RPQ = \angle PQB$.

12. Give a description of each of the following: (*a*) a circle,) a radius, (*c*) a diameter.

13. Explain the following statement: "The bearing of Mudrg from Mireville is S 10°E".

14. A town B bears NW from a town A and is 10 miles distnt from it. A man travelled from A to B. From B, he went niles due west to C. At C, he turned south and travelled to D, distance of 7 miles. Make a drawing to scale of the journey. om the drawing, find the distance from D to A, and the bearing A from D.

Symmetry

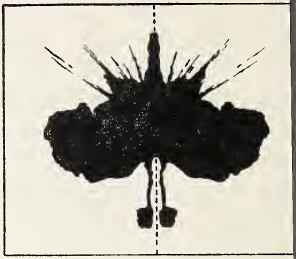
Put a drop of ink on the inner side of the crease formed by lding a sheet of white paper. Fold the paper again and press it. Open the paper and examine the ink figure. Observe s formation with respect to the crease. Again fold the paper ong the crease and hold it before a window or lamp. Does the udge on the right of the crease exactly cover that on the left?

On another piece of paper, put an ink spot a little to the right left of a crease. Examine the figures formed by refolding the per. It is impossible not to observe the balance in this as well in the previous figure.

Tear the paper along the crease. Compare the figure on one rt with the mirror image of the other.

Compare the mirror image of the palm of your left hand with e palm of your right hand.

Figures such as those produced by the ink spots are called ***mmetrical.*** The crease is the ***axis of symmetry.*** You

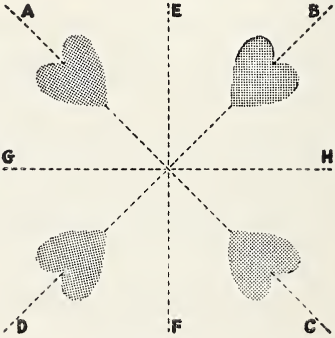


have noticed that each part is the mirror image of the other.

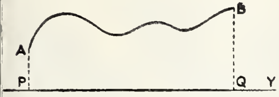
The palms of your hands are more or less symmetrical if held in such a way that each is a mirror image of the other. Then their axis of symmetry is a line running between them.

The type of symmetry we have described is called line sy metry. There are other forms of symmetry to which we sh not refer here.

The pattern shown in the accompanying illustration w obtained by cutting a hole in a folded paper square. How ma examples of line symmetry can you discover? In each case gi the axis of symmetry.



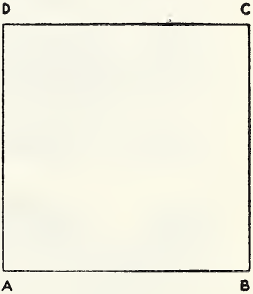
EXERCISES



1. Draw a straight line XY. Then, freehand, draw any curve such as AB. Using your set square, draw AP and BQ perpendicular to XY. With the aid of a set square and a pair of dividers, obtain a figure symmetrical with $ABQP$ about the is XY. (The curve may be drawn freehand through the points tained.)

2. The following exercise should be performed mentally. he accuracy of your answer may be checked with the aid of a irror, or by means of a drawing.)

In the accompanying illustration the order of the letters A, B, C, D ckwise or counter-clockwise? In e mirror image of the illustration e the letters in clockwise or unter-clockwise order?



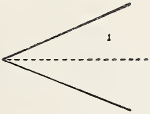
3. The following exercise should so be done mentally.

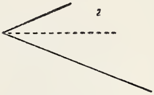
The two gloves of a pair are laid ose together on a table so that the lms are uppermost and the fingers rected away from you. Does the pair form a symmetrical patrn? If the right glove is turned inside out, can the pair form symmetrical pattern? Give the reason for your answer.

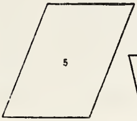
4. Examine each of the drawings on the next page and swer these questions:

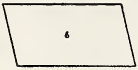
(*a*) Is the figure symmetrical about a line (or lines)?

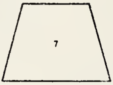
(*b*) If the figure is symmetrical, how many axes of symetry does it possess? (Make a sketch showing the axis or axes symmetry.)

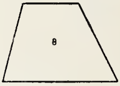


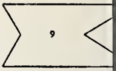


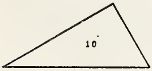


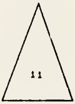


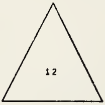


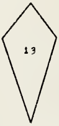


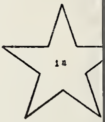


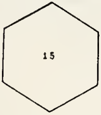


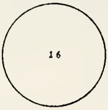


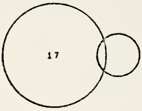


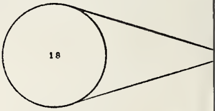


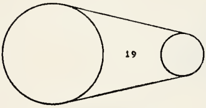


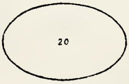


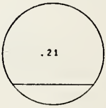


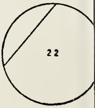












A

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B

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E

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F

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Z

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Y

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N

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M

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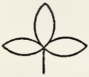
S



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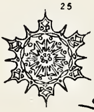

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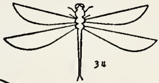

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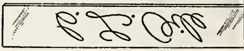

C. L. Oile

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OTTO

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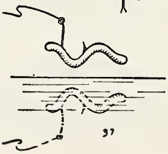


37

AH!HA

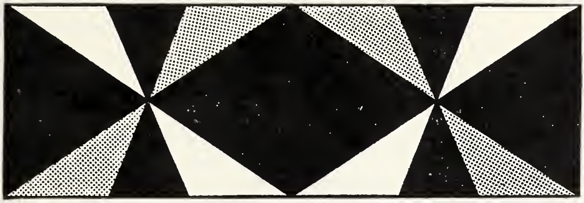
38

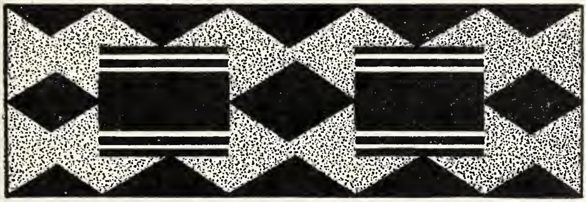
AH!AH

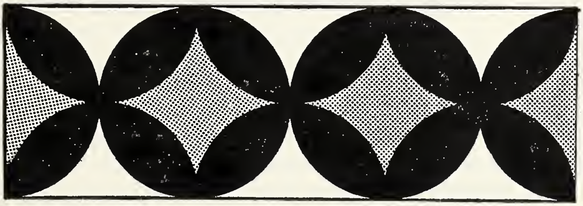
39

5. What is the mirror image of b p q d when the mirror is eld (*a*) at the right of the letters, (*b*) below the letters?

6. Lay a ruler along an axis of symmetry in each of the de-gns shown in the accompanying illustration.



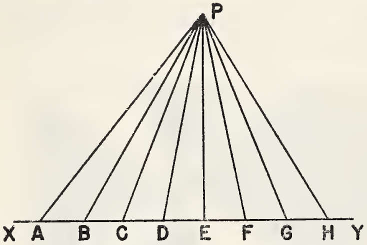


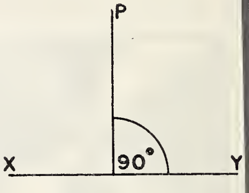


Learning More About the Use of Those Instruments

The distance from a point to a line. Draw a straight line XY. Take a point P outside the line and draw a number of straight ines PA, PB, PC. . . . . PH to XY. Measure the lengths of

PA, PB, PC, etc. and the angles PAY, PBY, . . . PHY. En the results in a table.





Length of line		*Size of angle*	
PA		PAY	
PB		PBY	
PC		PCY	
Etc.		Etc.	

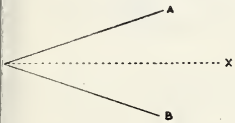
What do you suspect about the size of the angle when th line is shortest? Test this suspicion by drawing a perpendicul from P to XY and measuring it. If you are unable to draw perpendicular from P to XY with the aid of a set square, s page 406. Measure other lines close to the perpendicular.

In geometry, when we speak of the distance from a point to a line, we refer to the length of the perpendicular from the point to the line.

How to bisect an angle. You will recall that the bisector of angle is the line that divides the angle into two equal parts. ou can bisect an angle by dividing the number of degrees by 2 d then using your protractor. However, in the Euclidean me, this is a breach of the rules. Try to bisect an angle using ly compasses and straight edge.

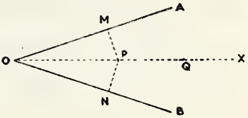
* * *

If you fail to discover the method, draw an angle AOB. ith the aid of your protractor, draw the bisector OX. Now we shall try to find out how to obtain OX with compasses and straight edge.

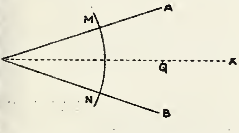


Take *any* point P on OX. How do you think the distance from P to OA (PM) would compare with the dis-nce from P to OB (PN)? Test the truth of your supposition.

How do you think the line OM compares in length with ON? est your answer.



How could you obtain two points and N equally distant from O? btain these two equidistant points another figure. (If you were per-itted to use a set square now, how uld you find P?) Look back again at the second figure. If u took any other point Q on OX, how do you think MQ would compare in length with NQ? Measure the two distances.



Turn now to the third figure. Using your compasses, find a point Q equidistant from M and N. Join OQ. Measure the angles AOQ and BOQ.

State the successive steps taken when bisecting an angle.

EXERCISES

1. When you place your compass point at M or N (page 2
why must the radius be greater than half the distance MN?

2. (*a*) Using your protractor, draw angles of 60°, 90°, 16
170°, 179°, and 180°.

(*b*) With the aid of compasses and straight edge, bis
each of these angles. Test the accuracy of your work by mea
of the protractor.

3. When you bisect a straight angle, what is the relation
the bisector to the arms of the straight angle?

4. At the point A in a straight line AB, draw a perpendicul
without using protractor or set square.

5. A perpendicular to a line on a tennis court at a point
the line was required. A boy obtained the perpendicular
using a rope and a few pegs. How did he do it? Compare yo
solution with that given on p. 406.

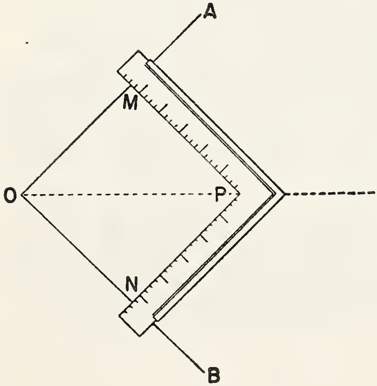
6. With straight edge and compasses, construct an angle o
and a half times as large as a given angle.

7. When one bisects an angle and measures the two halv
a small error is oft
found. What are the po
sible sources of the erro

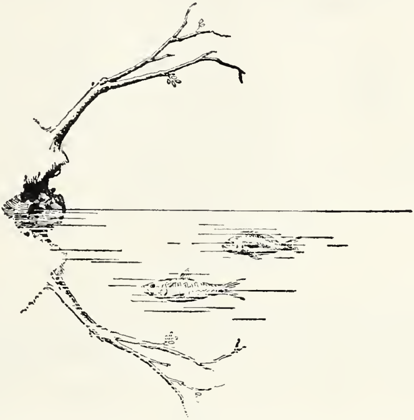
8. Using compasses an
straight edge only, co
struct (*a*) an angle of 45
(*b*) an angle of 135°, (
an angle of $67\frac{1}{2}$°.

9. Divide an angle int
four equal parts.

10. When a carpent
wishes to bisect an ang
AOB, he places his squa

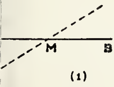


that $OM = ON$ and $PM = PN$. Then OP is the bisector. ake a carpenter's square out of cardboard and try the method. the angle MPN were not a right angle, could this method be ed?

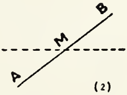


Find other examples in nature of the bisection of an angle.

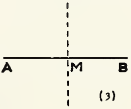
The bisector of a straight line. In each of the four figures, is the middle point of AB and the dotted line is a bisector of

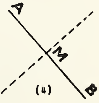


(1)



(2)



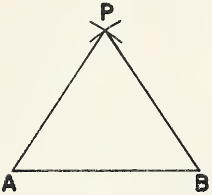


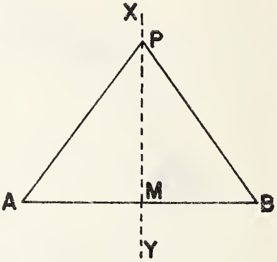
B. In (3) and (4), the two lines are perpendicular to each ther and the dotted line is called the ***right bisector*** of AB.

How to obtain the right bisector of a straight line. You are hallenged to construct the right bisector of a straight line using ompasses and straight edge only.

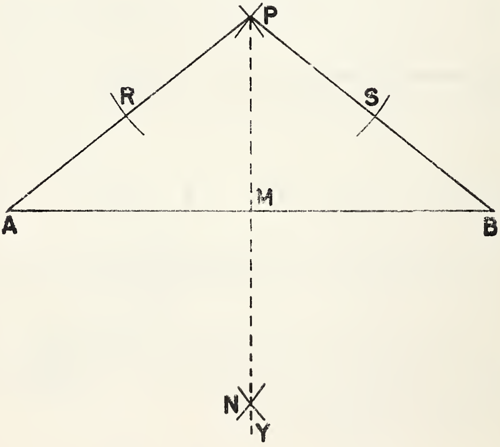
* * *

If you cannot do this exercise, draw a straight line AB. M sure the line. Divide the length by 2 and find the middle po M. With the aid of a set square, draw XY perpendicu to AB at the point M. Is the figure symmetrical? Which l is the axis of symmetry?





If you take any point P on XY, how do you think t length of PA would compare with that of PB? Measure PA a PB.



Draw another line AB and find a point P such that $AP =$ P. Which instrument did you use?

Now examine the first drawing. What relation may exist tween the angle APB and the line PY? Measure the angles PY and BPY. Is your suspicion confirmed?

Turn again to the second figure. How can you obtain the e PY? Check the accuracy of your work by using your di- ders, and by measuring either of the angles PMA or PMB. Vhy is it unnecessary to measure both?)

In following these suggestions, you placed the point of your mpasses at A, B, P, R and S. Try to obtain the right bisec- r by placing the points at A and B only.

EXERCISES

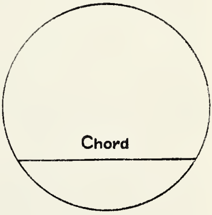
1. When bisecting a line, can you make the radius of your rcs as great as you please? Is there a limit beyond which you nnot reduce its length?

2. Without measuring a line, draw a line equal to three- alves of it.

3. Using straight edge and compasses, divide a straight line to four equal parts. Check the accuracy of your work by eans of your dividers.

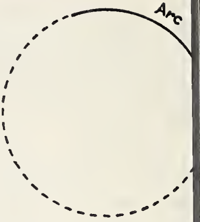
4. Without making measurements, draw a line three-eighths he length of a given line.

5. Draw a circle and in it place a hord. Bisect the chord at right angles. s the resulting figure symmetrical? Vhat is the axis of symmetry? Through hat important point would you expect he axis of symmetry to pass? Why?



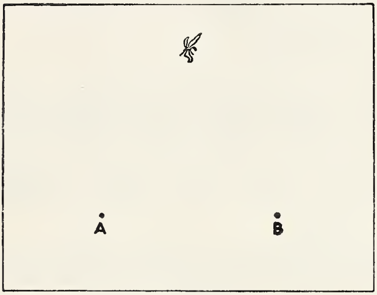
6. Using straight edge and com- asses, how would you find the centre of circle?

7. If you were given an arc of a circle, how would you construct the rest of the circle?



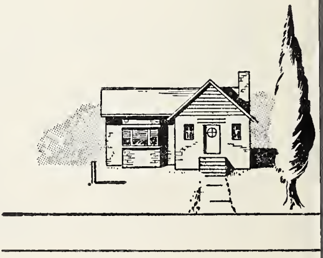
8. How would you draw a circle to pass through three points not in a straight line?

9. On a lawn are three oak trees not in the same straight line. A man wishes to place a fountain so that it will be equidistant from the tre
How may he determine the spot at which to erect the fountai



10. Mira, one of t
junior members of
troupe of performi
insects, was trained
jump so that, whenev
she alighted, she was
close to one fixed poi
A as to another,
Along what path w
Mira at liberty to ma
her six-point landings

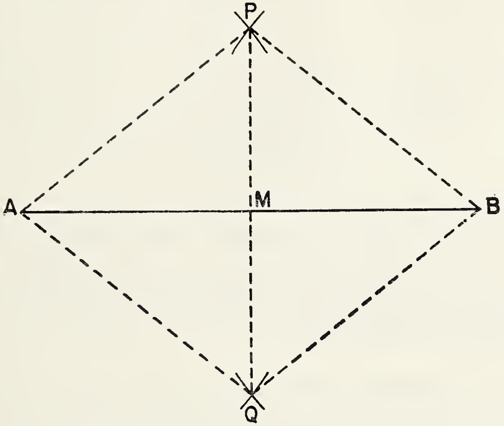
Finding the distance from a point to a straight line. A m
wished to find the distance from a post near his house to
straight road. His knowledge of geometry had become hazy, but, after a few attempts, he remembered how to solve the problem on paper with the aid of a set square. (Page 406.) Then, using some pegs and a rope, he laid out the line from the post to the road.



Try to construct the perpendicular with compasses and aight edge.

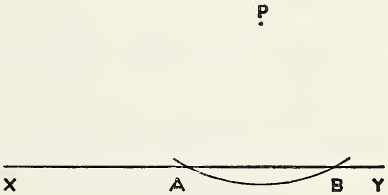
* * *

The following suggestions are given to assist those who are ving difficulty, and for the sake of comparison by those who ieve they have found a solution.



Examine the figure obtained when you bisected a straight e AB at right angles. ere we have a perndicular drawn to the e AB from a point P tside it. You will obve again that PA = B.

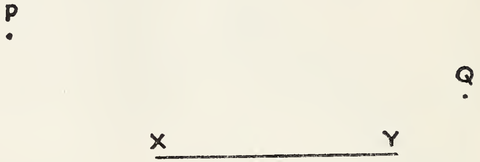
Then let us find two ints A and B on XY uidistant from P. You will also recall that AQ and BQ are ual. Then let us find a point Q equidistant from A and B. w shall we finish the construction?



Now, how would you solve this problem out of doors with aid of a rope and some pegs? (Page 407.)

EXERCISES

1. Draw a figure like the one shown. (*a*) Using a set squ



draw a perpendicular from P to XY. (*b*) Using straight e and compasses, draw a perpendicular from Q to XY.

2. Draw a circle and in it place a chord AB. From centre draw a perpendicular to the chord meeting it at the p N. Measure AN and NB.

3. Draw two lines OA and OB at right angles to each ot Bisect the angle AOB and on the bisector take any point From P draw PQ perpendicular to OA and PR perpendicula OB. Measure the angle QPR and the lines QP, PR, RO OQ. What name would you give the figure $OQPR$?

When are two straight lines parallel? Let us listen to a b dialogue.

Stranger: "Would you please tell me where Baker Street is?"
Native: "With pleasure! Turn to your right at the next int section, walk one block, and you will find Baker Str running parallel to this street."
Stranger: "Many thanks."

What meaning would you take from the word ***parallel***? (Y are being asked for a description, not for a formal definition.

Give five examples of parallel straight lines in the room. ust parallel straight lines be horizontal? If two lines are par- el, can they meet however far they are extended (or, as we say, oduced) in both directions?

Now be on your guard! You agree that if two lines are rallel, they cannot meet however far they are produced.

Do you agree that, if two lines do not meet however far they e produced, they are parallel?

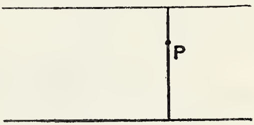
Represent lines by means of two pencils and show that it is ite possible for two lines not to meet however far they are oduced in both directions and yet not be parallel.

Now you are ready for the definition.

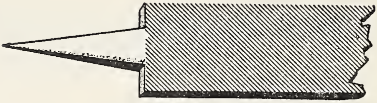
If two straight lines are in the same plane and do not meet however far they are produced, they are parallel.

otice that the definition is reversible. We may say, "If two aight lines in the same plane are parallel, they do not meet wever far they are produced in both directions."

Select two lines on a ruled sheet. Preferably, the lines should three or four spaces apart. From a point P between the two es draw a perpendicular to one of em. Produce the perpendicular meet the other line. By means your protractor or set square, d out whether the perpendicular the one line is also a perpendi- lar to the other.

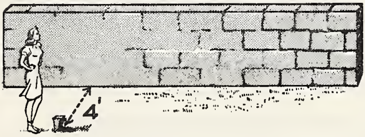


Draw a number of perpendiculars to the two parallel lines. easure the parts intercepted between the parallel lines. Are ey equal?



Parallel lines on a file.

How to draw through a given point a line parallel to a gi line. A girl was laying out a perennial border along a strai



wall. She took a poin feet from the wall and t wished to run a line par to the wall through point. How could she tain the line?

* * *

Try to solve a similar problem on paper. Through a poin not on the straight line AB, draw a parallel to AB. Do thi two ways: first, with a set square; then with compasses.

* * *

If you fail to construct the parallel, see page 407.

Learning to construct a few well-known figures. Us straight edge and compasses, construct a square. Then perfo the two constructions described below. If your method of c structing a square is different from the two, bring it to the att tion of the teacher.

1. Draw two equal straight lines AD and AB at right an to each other. Through B draw BX parallel to AD. Throu D draw DY parallel to AB. Let BX and DY intersect at Compare the lengths of BC and CD with that of AB or A Measure the angles B, C and D.

2. Draw a straight line AB. At A erect AX perpendicu to AB. At B erect BY perpendicular to AB and on the sa

e of AB as AX. From AX cut off AD equal to AB, and m BY cut off BC equal to AB. Join DC. Compare the gths of DC and AB. Measure the angles C and D.

Let us make a few changes in the foregoing construction.

3. Draw AB 3 inches in length. Draw the perpendiculars and BY as before. This time, however, cut off AD and each 1 inch long. Join DC as before. Compare the lengths DC and AB.

State wherein the figure obtained from the third construction resembles a square, (b) differs from a square.

Both figures are called ***rectangles.*** It is clear that ***the uare is a special kind of rectangle.***

Now you will become acquainted with the construction of other kind of figure. Using compasses and straight edge, ry out the following directions:

4. Draw two straight lines AX and AY at any angle less n 180° but not equal to 90°. From AX and AY cut off AD d AB, each $2\frac{1}{2}$ inches in length. Through B draw BP parallel AD. Through D draw DQ parallel to AB. Let BP and DQ ersect at C.

Examine the figure $ABCD$. Measure the sides BC and AD d the four angles.

Wherein does the figure $ABCD$ resemble a square? What tinguishes it from a square?

This figure, which looks like a square pushed out of shape, alled a ***rhombus. A square is a special kind of rhombus.***

Now for the fourth figure.

5. Repeat the construction outlined in (4) above, but make and AB respectively 2 inches and 3 inches long.

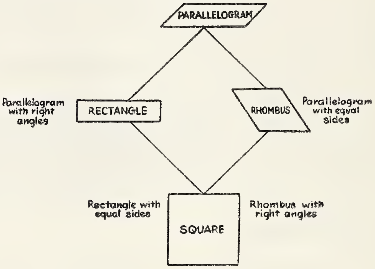
Examine the figure $ABCD$. Measure the sides BC and AD d the four angles.

State wherein the figure (i) resembles a rhombus, (ii) differs m a rhombus.

State wherein the figure (i) resembles a rectangle, (ii) differs m a rectangle.

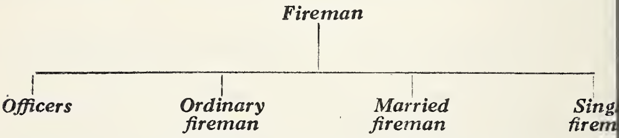
This figure is called a ***parallelogram.*** It makes one th of a rectangle pushed out of shape. ***Rectangles and squ are special kinds of parallelograms.***

The following arrangement brings out clearly the relat between the four figures. It will be noted that the most spe type of parallelogram is the square.


PARALLELOGRAM
Parallelogram with right angles
RECTANGLE
RHOMBUS
Parallelogram with equal sides
Rectangle with equal sides
SQUARE
Rhombus with right angles

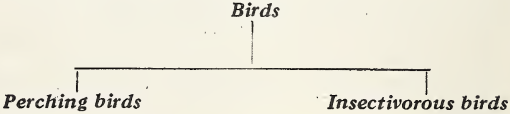
EXERCISES

1. Criticize the following classification:


Fireman
Officers
Ordinary fireman
Married fireman
Sing
firem

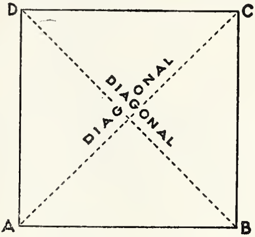
Hints: 1. Is it possible for a married fireman to be an officer?
2. Is it possible for an ordinary fireman to be single?
Make a better classification of firemen.

2. Criticize the following classification:


Birds
Perching birds
Insectivorous birds

Hints: 1. Are all birds included under the two headings 'perching' 'insectivorous'?
2. Are there any perching birds which live on insects?

3. Using any instruments you think fit, construct a square. ıw the diagonals. Do the diagonals bisect the angles of square? Compare your results h those of the other pupils.



A warning about generalizations. raveller in Ruritania was fortun- in that every time he met a ive of the country he was treated h kindness. As a result of these eriences, he made the following eral statement (or ***generaliza-n***) on his return home: *All Ruritanians are courteous.*

The traveller had not met every Ruritanian in existence. If had remained five minutes longer in the country, he might e met a boorish citizen. This contrary instance would have n enough to contradict the generalization that all Ruritanians courteous.

You drew a square and its diagonals. By measurement you ıd that the diagonals bisected certain angles. This does *not* ve the generalization: *In all squares, the diagonals bisect the les through which they pass.*

Then you compared your results with those of other members he class. The fact that the results are in agreement is not ugh to establish completely the truth of the generalization. ıe squares were of different sizes, the *probability* of the truth he generalization is higher than it was. However, no matter many measurements supported the generalization, it would contradicted if even *one* careful measurement were in dis-ement.

It is a failing of many people to generalize rashly, that is to to arrive at a rule after considering a few cases. We shall rn to this matter again.

EXERCISES

Which of the following generalizations are rash and which true or probably true?

(*a*) All cone-bearing trees are evergreen.

(*b*) All numbers the sum of whose digits is exactly divis by 9 are themselves exactly divisible by 9.

(*c*) All republics are democracies.

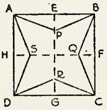
(d) All animals which have cloven hoofs chew the cud.

(*e*) "The police are very strict about parking in Punkinv I hadn't left my car more than twenty minutes in front of hydrant when I received a ticket."

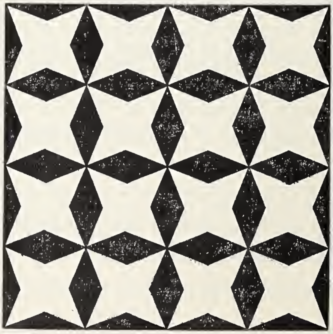
(f) "Those Neanderthal automobiles are no good."
"What makes you say that of the Ne's?"
"The last two I owned cost me a small fortune in repai

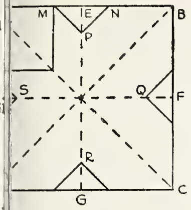
(g) "They have a wonderfully good system of educatio Balconia. An official of the Ministry of Education brought to four schools and they were far above the average of ours.

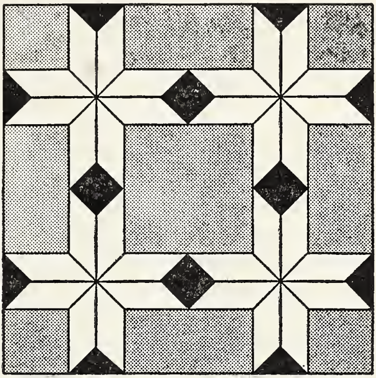
Simple geometric designs. Geometric designs are foun tiled, parquet and mosaic floors and in linoleums. Three sin designs are given below. You will probably receive more in esting practice in the use of your instruments if you devi design of your own than if you copy those in the textbook.



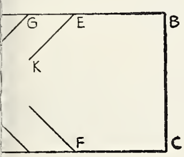
ABCD is a square. *E*, *F*, *G* and *H* are the middle points of *AB*, *BC*, *CD* and *DA* respectively.
$EP = FQ = GR = HS$.

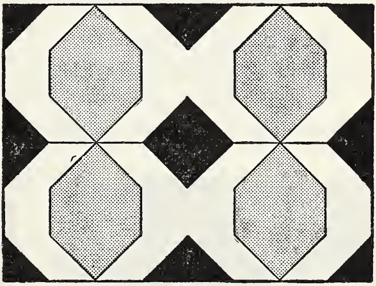




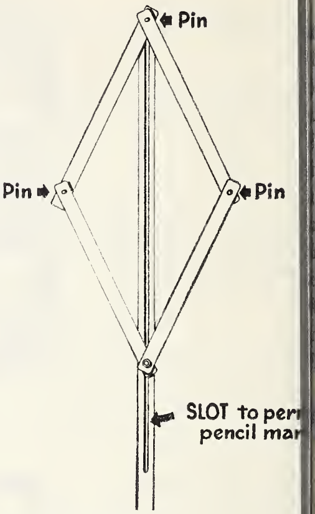


method of drawing lesign is indicated in igure *ABCD*.





A general utility homemade instrument. No doubt you are niliar with some of those ingenious gadgets sold to house-ves—the kind that can do about ten different operations. ell, here is a simple piece of apparatus that will make your trument-box green with envy. It is made of four strips of celluloid, plexiglass or cardboard, each about 7½ inches long, and one strip about 15 inches long. The four short strips are fastened together by pins as indicated in the sketch. One of the pins moves in a slot which is just wide enough to permit the making of a pencil mark. This piece of apparatus, besides familiarizing you with some properties of the rhombus, enables you

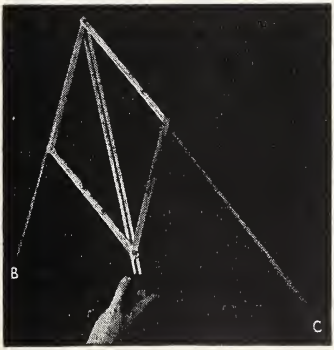


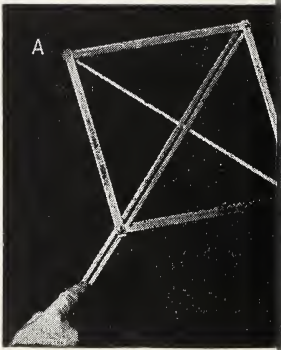
1. to bisect a straight line at right angles;

2. to bisect an angle;

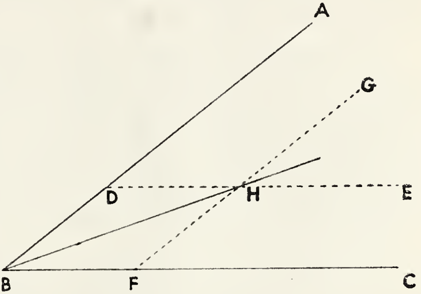
3. to erect a perpendicular to a straight line at a point on the line;

4. to draw a perpendicular to a straight line from a point on the line. Make one of these instruments and give it a t





How to bisect a given angle using only a ruler with parallel es. Let ABC be angle which is to isected. Lay the r carefully along and draw the DE. Then lay long AB and ob- FG. Join B to he point of inter- ion of DE and Measure the les DBH and H.



Try to explain why BH is the bisector. What kind of figure HFB?

EXERCISES

(Any instruments may be used)

1. A group of treasure hunters worked from the following ctions:

"N 22½° E from oaken stump due N of Skull Rock.
E from stump.
10 feet from E side of N 22½° E line; 8 feet from N side of E line; dig."

Assume that the treasure seekers have arrived at Skull Rock have found the oak stump. By means of a scale drawing, w how they would find where to dig.

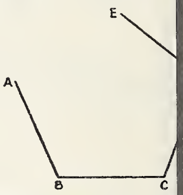
2. Construct a rectangle of length 3 inches and width 2½ es. Divide it into two equal rectangles (a) by a line parallel he longer sides, (b) by a line parallel to the shorter sides.

3. The sum of the sides of a rhombus is 8 inches and the le between two adjacent sides is 60°. Construct the rhombus.

4. Two sides of a parallelogram are 2 inches and $1\frac{1}{2}$ inch length. The angle between these sides is 45°. Construct parallelogram. Measure the remaining three angles.

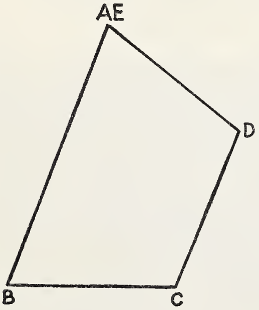
5. Construct (*a*) a square, (*b*) a rhombus, (*c*) a general angle, (*d*) a general parallelogram. From your own meas ments and from those of the other members of the class o probably true answers to the following questions for each o four kinds of figure: (i) Are the diagonals equal? (ii) Do diagonals bisect each other? (iii) Are they perpendicular to other? (iv) Do they bisect the angles through which they p

A few new terms. The figure *ABCDE* is composed number of straight lines which are not in the same straight line. *ABCDE* is called a ***broken line.***



In this figure, if *A* and *E* were to coincide, the figure would be ***closed.*** A closed broken line is called a ***polygon,*** which means literally 'many angles' (Gk. *polus*, many; *gonia*, angle).

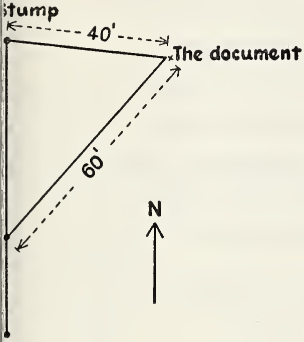
A polygon of four sides is called a ***quadrilateral,*** (*quattuor*, four; *latus*, side). N four kinds of quadrilateral to w you have had the pleasure of b introduced. Must a quadrilatera a plane figure?



* * *

Three-sided polygons are ca triangles. Mathematicians have amazing amount of information ab them, but you won't be expected acquire it all immediately.

The spy and the triangle. A spy who had buried an imp tant document feared that he would be unable to remember ct spot. Accordingly, he measured the distance from the tre of a tree stump to the place where the paper was hidden. This distance was 40 feet. Then he located the N-S line through the stump and drove in a peg on the line. Next he measured a distance of 50 feet from the stump along the N-S line and inserted a second peg. From this peg to the document was a distance of 60 feet.



He withdrew the pegs, returned to his hotel, and made a scale-drawing from his measurements.

Representing 20 feet by 1 inch, make a plan such as the drew.

In this exercise, you were given the lengths of the three sides he triangle and were required to construct the triangle.

EXERCISES

1. Try to draw a triangle ABC from each set of measure- nts:

(a) $AB = 2\frac{1}{2}$ in.; $BC = 3\frac{1}{2}$ in.; $CA = 2$ in.
(b) $AB = 3$ " ; $BC = 4$ " ; $CA = 1$ ".
(c) $AB = 3$ " ; $BC = 1\frac{1}{2}$ " ; $CA = 1$ ".
(d) $AB = 3\frac{1}{2}$ " ; $BC = 2$ " ; $CA = 1$ ".

2. After some additional attempts to draw triangles with es of varying lengths, decide what word should be written in blank space to give a true statement.

The sum of any two sides of a triangle must be ——— n the third side.

3. Construct a triangle ABC from the following meas
ments:

$$AB = 2\tfrac{1}{2} \text{ in.}; \ BC = 3 \text{ in.}; \ CA = 2\tfrac{1}{2} \text{ in.}$$

Measure the angles B and C.
What is peculiar about this triangle?

A triangle which has at least two of its sides equal is calle ***isosceles*** *triangle*, (Gk. *isos*, equal; *skelos*, leg).

4. Construct a triangle from the following data:

$$AB = AC = 2 \text{ in.}; \ BC = 1\tfrac{1}{2} \text{ in.}$$

5. Construct a triangle ABC from the following meas
ments:

$$AB = 3 \text{ in.}; \ BC = 3 \text{ in.}; \ CA = 3 \text{ in.}$$

Measure the angles A, B and C.
What is peculiar about this triangle?

A triangle which has its three sides equal is said to be ***eq*** ***lateral.*** (Lat. *aequus*, equal; *latus*, side). Is an equilat triangle isosceles?

In his *New Ballad of Sir Patrick Spens*[1], Sir Arthur Quil Couch described the construction of an equilateral triangle a line AB.

> " And he has set his compass foot
> Until the centre A,
> From A to B he's stretched it oot—
> 'Ye Scottish carles, give way!'
>
> " Syne he has moved his compass foot
> Until the centre B,
> From B to A he's stretched it oot,
> And drawn it viz-a-vee.
>
> " The ane circle was BCD,
> And ACE the tither.
> 'I rede ye well,' Sir Patrick said,
> 'They interseck ilk ither.'

---

[1] By permission of the Oxford University Press, London, England.

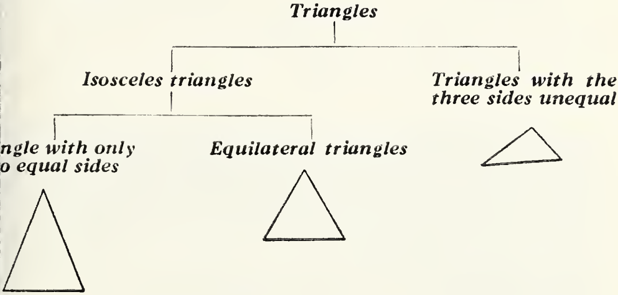
" 'See here, and where they interseck—
To wit with yon point C—
Ye'll just obsairve that I conneck
The twa points A and B.

" 'And there ye have a little triangle
As bonny as e'er was seen;
The whilk is not isosceles,
Nor yet is it scalene.[2]' "

You have 'obsairved' that, in the poem, an equilateral tri- e is not regarded as isosceles. The equilateral triangle is considered to be a special type of isosceles triangle.

. Construct an equilateral triangle such that the sum of its s is 7½ inches.

classification of triangles. The following is one classifica- of triangles:

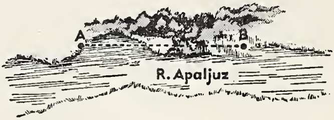

Triangles
Isosceles triangles
Triangles with the three sides unequal
ngle with only o equal sides
Equilateral triangles

problem in measurement. An engineer required to know distances between two points A and B on a bank of the ljuz River. Although he was on the bank containing A and a curve, a swamp and a clump of trees made it inconvenient ieasure AB directly. The engineer measured two lines and ngle. Then he made a scale drawing and from it found the iired distance.

---

[2] The name *scalene* is sometimes applied to triangles with three unequal

Try to discover how he obtained the distance AB. After have tried your best and have (or have not) found a solu turn to page 409.



* * *

Now you have learned how to construct a triangle when know the lengths of two sides and the angle included (or tained) between them.

EXERCISES

1. In a triangle DEF, name (*a*) the angle contained betw ED and EF, (*b*) the angle included between DE and DF, (*c*) angle included between DF and EF.

2. In a triangle PQR, name the sides which include (*a*) (*b*) $\angle Q$; (*c*) $\angle R$.

3. Construct a triangle XYZ from the data:

$$\angle Y = 38°; \quad XY = 3 \text{ in.}; \; YZ = 3 \text{ in.}$$

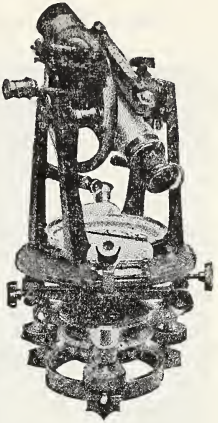
Measure the angles X and Z.

4. Construct a triangle RST from the data:

$$TR = 2\tfrac{1}{2} \text{ in.}; \quad TS = 2\tfrac{1}{2} \text{ in.}; \quad \angle T = 90°.$$

Bisect the angle T. If the bisector meets RS at the p P, measure TP, RP and SP.

How to measure angles out of doors. When measuring an out of doors, the engineer uses an instrument called the th dolite. You may find it of interest to construct an instrum which can be used for obtaining rough measurements of an in your school grounds or in the neighbourhood of your sch

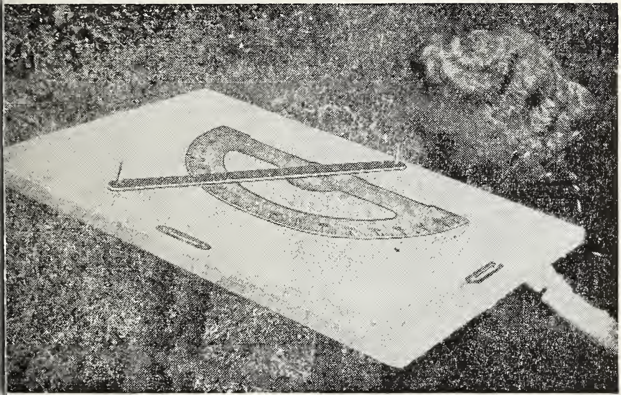


Courtesy of Instruments Ltd.

A theodolite such as is used by engineers.

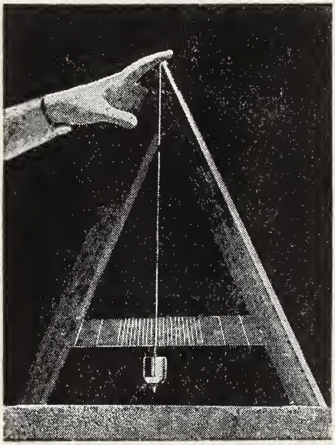
Take a piece of very stiff cardboard describe on it a circle of radius about ches. Draw a diameter of the circle. ut three-quarters of an inch from the neter, draw a line parallel to the diter. Cut out the figure. Graduate semicircle as if it were a protractor. a narrow strip of cardboard and ugh its axis of symmetry fasten it to centre of the semicircle. It should possible to rotate the strip easily. g the axis place a foresight and hindt. Phonograph needles embedded in oden base may be used. These pieces ood should be glued to the strip of board. There should be no difference e heights of the needle points.

)bviously, it is desirable to place the e-measurer on a level surface or



Pupil using a home-made angle-measure.

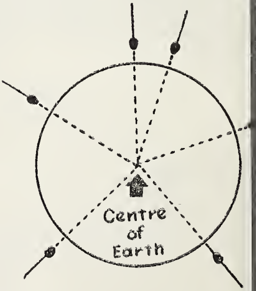
plane-table when making measurements. A drawing board



ported on four or, prefer three legs is used. Is it sible for one edge of the to be horizontal while an is oblique? How would use a spirit-level to e that the table is in a zontal plane?

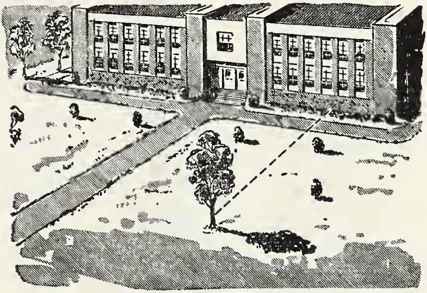
You may wish to cons your own levelling instru In the following paragra a description of the p level which was used in times and which is e made.

The plumb level. framework may be cons ted of wood or metal. It is essential that the two long p be of equal length. When the plumb-line passes along middle of the cross-piece, the line on which the feet rest is horizontal. In this position the instrument exhibits symmetry.



What is its axis? A plumb-line, if produced, would pass through the centre of the earth. However, the earth's radius is so great, that at points fairly close together on the earth's surface, freely suspended plumb-lines are regarded as parallel. A line perpendicular to a freely-suspended plumb-line is horizontal

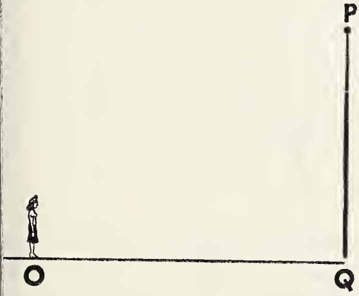
Using your angle-measurer. Here is a little problem on
ch to sharpen your wits. Let us suppose that you are re-
red to find how
it is from a fairly
ant tree or shrub
the front of the
ol. You must
go more than 20
from the build-
Compare your
hod with that on
e 410.



* * *

Now you should be able to construct a triangle when you are
n one side and the angles adjacent to that side.

How to find the height of a flagstaff if you are a poor climber.
r problem is to find the number of feet in *PQ* without leaving the ground.



From your previous experience in measuring distances, you might reasonably suspect that the trick consists in drawing a triangle to scale after obtaining certain measurements.

What measurement is already made?
t is the size of the angle between the flagstaff and the
l ground? What measurement of distance can you make
conveniently?

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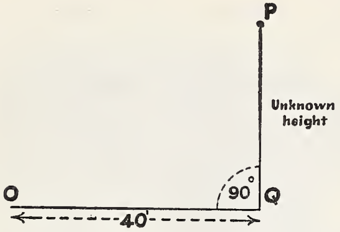
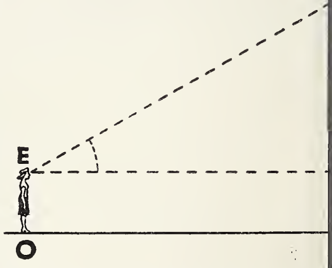
OF ALBERTA



Let us make a r
sketch showing the dist
from O to Q (40 feet) an
angle between the flag
and the ground. What
ditional piece of informa
is needed in order to ob
the height of the flagsta

* * *

However, you have probably noticed that there is a sl
precaution to be observed. The angle measured was *not*
angle included between OP and OQ, but the angle between
and EF, where E is the eye of the observer, and EF is a h
zontal line. To what line is EF equal? Instead of dra
the triangle POQ to scale, what triangle do you actually draw? What length must be added to PF to obtain the height of the flagstaff?



It should be noted that, in drawing the triangle PEF, you had the following data:—one side (EF) and the two angles adjacent to EF (PEF and PFE).

In the following illustration is shown a simple instrum
which may be used to measure angles in the vertical pl
Construct a similar piece of apparatus and use it in helping
to find the height of a flagstaff, church steeple, or high build

ng an angle measurer to find
the height of a flagstaff.

EXERCISES

1. In a triangle PQR, name the angles adjacent to (*a*) PQ, QR, (*c*) RP.

2. A man wished to find the width of a river which it was ossible for him to cross. With the aid of a figure, explain he could find the width.

3. Construct a triangle ABC from the measurements:

$$AB = 3\tfrac{1}{2}\text{ in.};\ \angle A = 47°;\ \angle B = 43°.$$

Measure $\angle C$.

4. Construct a triangle ABC from the measurements:

$$\angle B = 70°;\ \angle C = 70°;\ BC = 3\text{ in.}$$

Measure (*a*) $\angle A$, (*b*) AB, (*c*) AC.

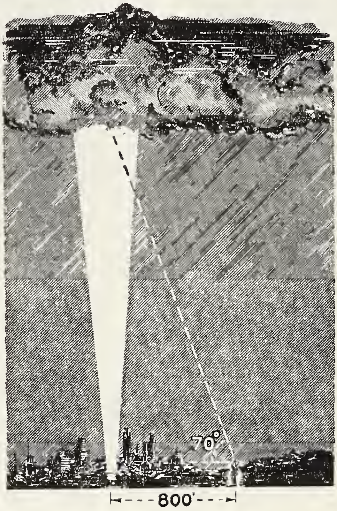
5. Construct a triangle ABC from the measurements:

$$\angle C = 60°;\ CA = 2\tfrac{1}{2}\text{ in.};\ \angle A = 60°.$$

Measure (*a*) $\angle B$, (*b*) AB, (*c*) BC.

6. The illustration shows a method of obtaining the he
of the base of a cloud. A beam of light is projected vertic
upwards. The bright patch on the cloud is viewed at a kn
distance from the source of light, and the angle between the
of sight and the ground is measured.

Find the height of the cloud base from the informa
given in the illustration.



EXERCISES IN SPATIAL IMAGINATION

Imagination is useful in geometry as it is in everyday life. T
exercises may be helpful to some of you in developing the abilit
visualize points, lines, surfaces and solids. They are to be so
mentally without the aid of models or diagrams.

1. Imagine that you are making capital letters with matc
To make the word 'eat' in capital letters would require

ches. How many are required for each of the following ds?

(*a*) tan	(*d*) animal	(*g*) little
(*b*) axe	(*e*) maintain	(*h*) kettle
(*c*) zeal	(*f*) twenty	(*i*) nineteen

2. Call to mind a mental picture of a box with rectangular s. Count the number of edges.

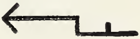
3. Which of the following letters would appear unchanged en in a mirror? O P Z Y H R

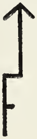
4. A triangle was drawn on each of the sides of a triangle. v many lines were there in the resulting figure?

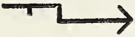
5. At approximately what time between 8 and 9 o'clock are hands of a clock opposite each other?

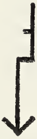
6. What is the least angle through which the letter F must urned to give its mirror image?

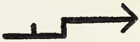
7. If this drawing were rotated 270° in a kwise sense, which of the following draw- would it appear like?











EXERCISES ON CHAPTER I

A

1. It is 12.4 miles from Stratford to Mitchell. On a m the distance between these places is 3.1 inches. What is scale of the map?

2. Using compasses and straight edges, construct angle (*a*) $112\frac{1}{2}°$, (*b*) 315°.

3. (*a*) What is meant by symmetry? Give four exam from nature.

(*b*) Is a square with one diagonal drawn a symmetr figure? If so, what are the axes of symmetry?

(*c*) ABC is a triangle with three unequal sides. bisector of the angle A meets BC at D. Is the figure symr rical? If so, what is the axis of symmetry?

(*d*) DEF is a triangle in which $DE = DF$. The bise of the angle D meets EF at G. Is the figure symmetrical? so, what is the axis of symmetry?

4. Construct a circle and in it draw two diameters AB CD at right angles to each other. (Use straight edge and c passes.) Join AC, CB, BD and DA. Give a description of figure $ACBD$.

5. A street is 50 feet wide between the curbs. A cl wishing to post a letter, crossed the street at an angle of to the street's direction. In doing so, she walked directl the letter box instead of crossing the street at the intersec where she was standing, turning through a right angle walking to the box. How many feet did the child save taking this dangerous short cut?

6. Construct a triangle ABC from the following data:

$$AB = 4 \text{ in.} \quad BC = 5 \text{ in.} \quad CA = 3 \text{ in.}$$

Bisect the angle BAC. If the bisector meets BC a measure AD, BD and DC.

7. Draw a triangle MNP in which $NM = 3$ in., $NP = 2\frac{1}{2}$
$\angle N = 90°$. What is the distance from N to MP?

Using your straight edge and compasses, draw MX par-
l to NP. Using a set square, draw PY parallel to NM.
Q be the point of intersection of MX and PY. Measure
), PQ and the $\angle MQP$. What kind of figure is $MNPQ$?

is instrument, called an astrolabe, used by Champlain for measur- the angle between the horizontal the line from one of the heavenly es to his eye. From this angle btained his latitude. The astro- was lost in 1613 and was found Cobden, Ont. in 1867.

8. Construct a triangle XYZ in which $YZ = 3$ in., $\angle Y =$
and $\angle Z = 54°$. Bisect the angles Y and Z. Let the bi-
ors meet at O.

(*a*) Measure the angles YXZ and YOZ.

(*b*) From O draw a perpendicular to YZ meeting YZ in
With centre O and radius OW, describe a circle.

B

9. An engineer marked on the ground a line AB 30 feet long
ning due south. From B, the south end of this line, he laid
n BC, a line 40 feet long running due east. At C, he marked
, a line 30 feet long running north. Then he laid down a line
n D to A.

(*a*) Make a plan in which 1 inch represents 10 feet.

(*b*) What is the direction of DA?

(*c*) What is the length of DA on your plan?

(*d*) What is the length of DA on the ground?

(*e*) How far is it on the ground from C to A?

(*f*) What is the bearing of C from A?

(*g*) What kind of figure is $ABCD$?

10. (*a*) Wherein does a square resemble a general rhombu

(*b*) What are the differences between a square an general rhombus?

(*c*) What are the resemblances between a general recta and a general parallelogram?

(*d*) What differences exist between general rectangles general parallelograms?

(*e*) In what respects do a square and a general recta resemble each other?

(*f*) In what respects do a square and a general recta differ?

(*g*) What are the resemblances between a general rhom and a general parallelogram?

(*h*) What are the differences between a general rhom and a general parallelogram?

(*i*) What features are possessed in common by the squ rectangle, rhombus and parallelogram?

11. The length of a building is 125 feet. On a plan of building, the length measures $12\frac{1}{2}$ inches. If the actual widt 60 feet, what is the width of the plan?

12. From the centre O of a circle draw three radii OA, and OC such that $\angle AOB = \angle BOC = 120°$. Join AB, BC CA. What kind of triangle is ABC?

13. How many axes of symmetry has (*a*) an isosceles triangle, an equilateral triangle, (*c*) a general rectangle, (*d*) a square, a general rhombus, (*f*) a general parallelogram, (*g*) a circle? each case, draw a lettered figure and state what the axes are.

14. Using compasses and straight edge, construct an angle (*a*) $202\frac{1}{2}$°, (*b*) $292\frac{1}{2}$°.

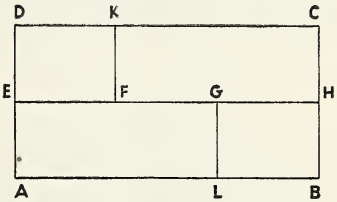
C

15. The bearings of two lighthouses from a ship are found be N 15°E and N 68°W. The bearing of one lighthouse from other is S 65°W, and their distance apart is 12.5 miles. By ans of a scale drawing find how far the ship is from each thouse.

16. From the centre O of a circle draw three radii, OA, OB OC such that $\angle AOB = \angle BOC = 60°$. Produce AO, BO CO to meet the circle again at D, E and F respectively. Join , BC, DC, DE, EF and FA. Give a description of the figure $CDEF$.

17. (*a*) It has been noted that an axis of symmetry is a line ng which a figure may be said to balance. Is a line of bal- e necessarily an axis of symmetry? With the aid of ex- ples, give an explanation.

(*b*) $ABCD$ is a rectangle which $AE = ED = BH$, $EF = FG = GH$. FK perpendicular to DC and is perpendicular to AB. Is the figure symmetri- If so, what is the axis ymmetry?

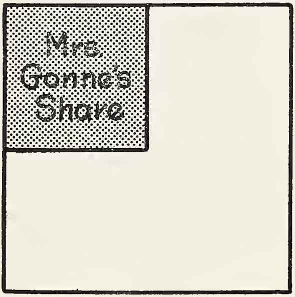


18. Using compasses and straight edge construct an angle of $11\frac{1}{4}$°, (*b*) 405°.

19. For each of the following, give the *greatest* number points of intersection: (*a*) two straight lines, (*b*) three strai lines, (*c*) four straight lines, (*d*) five straight lines.

RECREATIONS

1. The late Mr. Gonne left his square country estate to widow and four daughters. Mrs. Gonne was to receive quarter. The remainder wa be divided equally among girls. It was stipulated in will that the four parts left the girls should have the sa shape. The family lawyer sol the problem in five minu Can you do as well as he?



2. A man planted ten strai rows of trees, each row conta ing three trees. He managed do this using only nine trees. How did he do it?

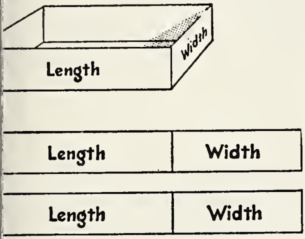
3. Construct nine squares with sides (measured in inch $\frac{1}{4}$, 1, $1\frac{3}{4}$, 2, $2\frac{1}{4}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $3\frac{3}{4}$, $4\frac{1}{2}$. Cut these squares out and arra them to form a rectangle. How long are its sides?

CHAPTER II

LENGTH, AREA AND VOLUME

Perimeter

How far is all the way around a parallelogram? Mr. Brown getting ready to paper a rectangular room. To find how much wallpaper border he should buy, he measured the lengths of the four walls and added his results. Mrs. Brown, who had been watching her husband at work, was heartless enough to chuckle. When he enquired what the cause of the merriment was, Mrs. Brown removed the sides from a choc- e box and demonstrated that the sum of the length and th is half the total distance around the box. She could not ain from asking if it was really necessary to measure the th and width of the room twice.



The sum of the sides of a figure is called its ***perimeter*** (Gk. , around; *metron*, measure). If the length and width of a le top are 3 feet and 2 feet respectively, then the perimeter is

$$3 + 2 + 3 + 2 \text{ feet,}$$

as Mrs. Brown pointed out, 3 + 2 multiplied by 2.

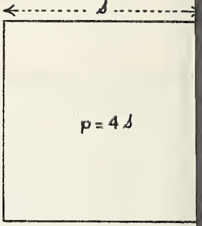
If we represent the number of units in the perimeter of a angle by p, the number in the length by l, and the number he width by w, then we can use the following 'shorthand' for stating what the perimeter is equal to:

$$p = l + w + l + w$$

$$\text{or,} \quad p = 2 \times l + 2 \times w$$

or, to avoid confusing the multiplication sign with the lette

$$\boxed{p = 2l + 2w.}$$



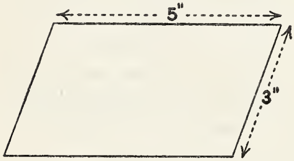
You remember, no doubt, that a square is a special kind of rectangle in which the length and width are the same. What is the perimeter of a square whose side is 2 feet?

In general, if there are s units of length in the side of a square, the perimeter is $4 \times s$ units. More briefly,

$$\boxed{p = 4s.}$$

* * *

What is the perimeter of the parallelogram in the illustratio



If the length of one side is units, the length of the other si n units, and the perimeter units, complete the followi 'shorthand' statement in yo work book:

$p =$

EXERCISES

1. The length of a rectangle is 4 feet and its width is 3 fe Find its perimeter.

2. The length of a rectangle is 2 feet and its width is 9 i ches. Find the perimeter.

3. A rectangular garden is m feet long and n feet wide. Ho many feet of wire netting would be required to enclose it?

4. What is the perimeter of a rhombus a side of which 4 inches in length?

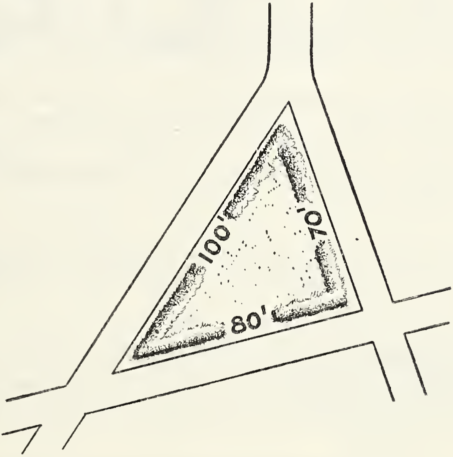
5. If the side of a rhombus is r feet in length, what is the imeter?

6. The side of a square is 14 inches in length. What is the gth of the perimeter in feet?

7. If the side of a square is k inches in length, what is the imeter in inches?

8. What is the length in inches of the side of a square the imeter of which is 2 feet 4 inches?

9. What is the difference between the perimeter of a rectangle iches long and 2 inches wide and that of a square with a side iches long?



The triangle. There is no mystery about finding the perimr of a triangle. If it were necessary to erect an iron railing und the triangle in the illustration, the total length would be nd by adding the lengths of the three sides, namely

$$100 + 80 + 70 \text{ feet.}$$

In general, if the number of units in the sides of a trian are a, b and c, then the number of these units in the perime is $a + b + c$. To express this in short form,

$$\boxed{p = a + b + c.}$$

If we are dealing with an equilateral triangle the length each of whose sides is a, then $p = a + a + a$ or $3a$.

If the triangle has only two equal sides, then

$$p = b + b + a \text{ or } 2b + a.$$

EXERCISES

1. The sides of a triangular garden are 80 feet, 40 feet a 60 feet in length. Find the perimeter in yards.

2. The sides of a triangular plot of ground are a yards, b yar and c yards in length. Give the perimeter in feet.

3. Find in yards the perimeter of an equilateral triangle side m miles.

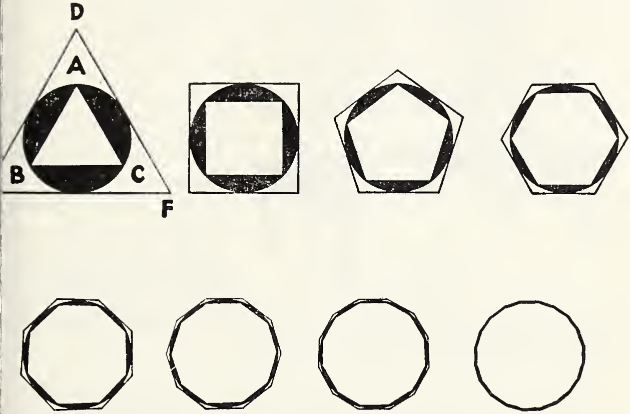
4. What is the difference, in feet, between the perimeter a square of side 4 feet and that of an equilateral triangle of si 4 feet?

The perimeter of a circle. The rubber ruler is not includ in sets of mathematical instruments, so you can readily imagi that finding the perimeter of a circle has caused a considera amount of trouble.

As you see from the following drawings, the perimeter of polygon ***inscribed*** in a circle is less than the perimeter (or, as i more frequently called, the ***circumference***) of the circle. Y observe also that as the number of sides of the inscribed polyg increases, the polygon's perimeter more and more nearly a proaches the circumference.

Similarly, as the number of sides of the circumscribed polygon reases, its perimeter approaches more and more closely to the umference.

It is said that in the Middle Ages a man calculated "with unspeakable labour" the perimeter of a regular circumscribed polygon of 1,073,741,824 sides. In the sixteenth century, a man named Van Ceulen beat this record by using polygons of 32,212,254,720 sides!

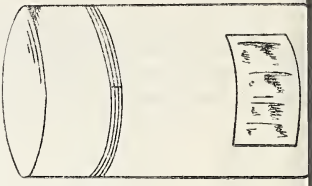


You may say, "Why all this fuss about the circumference of ircle? Why take for homework the unspeakable labour of nputing the perimeter of a polygon with a thousand million es? Give me a piece of fine white thread and a tin can and I l do the trick."

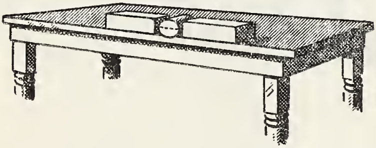
Good! And so you may.

EXERCISES

Closely wrap a fine white thread a number of times around ircular can or bottle. With a fine ink mark indicate the successive complete turns. (A strip of paper may be used and pricked with a pin.) Measure the distance between the ginning of the first and the end of the last turns and divide the number of turns. In this way you find the ***average*** for the length of the circumference. What is assumed, or taken for granted, when you regard the distance between the ink marks or pin pricks as the sum of a number of measures of the circumference?



Now find out how the circumference compares with the len of the diameter. Pl the cylinder between t bricks or blocks of wo Make sure that bricks are parallel. T measure the distance tween them three or f times and find the average. Pool your results with those other pupils and enter them in a table.



Circumference in inches	***Diameter in inches***	$\frac{\textit{Circumference}}{\textit{Diameter}}$

If a number of circles of different radii have been used, th should be some interesting results in the third column. member that it would be rash to jump at general conclusio after making measurements on four or five circles.

* * *

If the measurement has been careful, the numbers in the last umn should be close to 3.1.

The actual number is a very famous one. Archimedes, the greatest mathematician of the ancient world, using inscribed and circumscribed polygons, found that it lay between $3\frac{10}{71}$ and $3\frac{10}{70}$. The Hebrews used the value 3.

> "*And he made a molten sea, ten cubits from the one brim to the other: it was round all about, and his height was five cubits: and a line of thirty cubits did compass it round about.*" I Kings vii, 23.

Ludolf van Ceulen, to whom we referred previously, calculated the number to 35 places of decimals. Part of his answer was 3.14159265358979323846. When he died, the number was carved on his tombstone. In 1873, an Englishman named Shanks performed a Marathon calculation by carrying the number to 707 decimal places. It is estimated that, without the aid of modern computing devices, it would require ten years to obtain 1000 places.

For short, this number is called π, the first letter in the Greek d περιφερια (peripheria), meaning circumference. (Look up meaning of 'periphery' in your dictionary.) It has been ved that, no matter how many figures are worked out in the imal expression for π, there will always be further figures to ow and that these cannot be obtained without the great our of working them out. Even if we knew the first 1000 ures, that would give no clue concerning the 1001st figure.

The following approximations to the value of π are used, the roximation chosen being determined by the degree of accuracy ired in the answer: $3, 3.1, 3.14, 3\frac{1}{7}, 3.142, 3.1416, 3.14159, \frac{355}{113}$.

When the diameter and circumference of a circle are expressed he same units,

$$\frac{\text{Circumference}}{\text{Diameter}} = \pi.$$

s may be written,

$$\text{Circumference} = \pi \times \text{diameter},$$

n this form:

$$\text{Circumference} = \pi \times 2 \times \text{radius}.$$

If we denote the number of units in the circumference b
the number in the diameter by d, and the number in the rad
by r, we can use the following concise statements:

Circumference of circle

$$c = \pi d$$
$$c = 2\pi r$$

Thus, if the radius of a circle is $3\frac{1}{2}$ inches in length,
number of inches in the length of the circumference is ab
$2 \times \frac{22}{7} \times \frac{7}{2}$ or 22.

* * *

By practice indoors and outdoors, acquire a reasonable deg
of accuracy in the estimation of lengths.

12 inches = 1 foot
3 feet = 1 yard
$5\frac{1}{2}$ yards = 1 rod
[40 rods = 1 furlong]
[8 furlongs = 1 mile]
5280 feet = 1760 yards = 320 rods = 1 mile
[3 miles = 1 league]

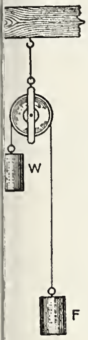
The length of a tennis court is 78 feet; the width is 36 fe
The length of a rugby field is 110 yards.
The side of a baseball diamond is 90 feet.

EXERCISES

1. Find the circumference of a teapot stand the radius
which is $2\frac{5}{8}$ inches. ($\pi = 3\frac{1}{7}$)

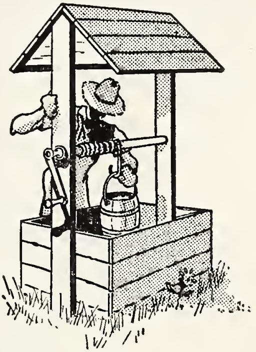
2. Calculate the length of a semicircular arch over a do
way 4 feet 1 inch in width. ($\pi = 3\frac{1}{7}$)

3. What should be the width of a strip of sheet iron from
ich a stove pipe six inches in diameter is to be made? Allow
f an inch for overlapping. ($\pi = 3.1$)

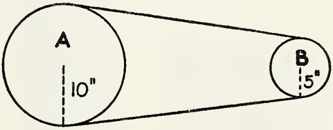


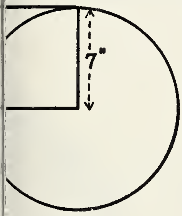
4. If the diameter of the pulley is 1″, how far will *W* descend when the pulley makes two rotations in the counter-clockwise sense? ($\pi = 3.1$)

5. How many feet will the bucket descend in 10 turns if the radius of the axle is $3\frac{1}{2}$ inches? ($\pi = 3\frac{1}{7}$)



6. The illustration shows
) wheels *A* and *B* con-
ted by a belt. When the
eels are in motion, how
s the rate of rotation of *A*
ipare with that of *B*? Explain.



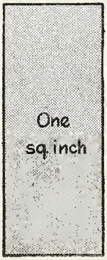


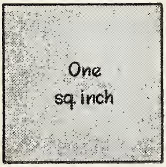
7. By how many inches is the circumference of the circle greater than the perimeter of the square? ($\pi = 3\frac{1}{7}$)

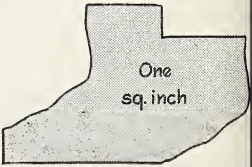
Area

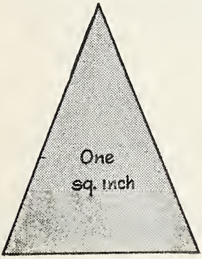
By the ***area of a polygon*** we mean the amount of surf bounded by the figure. Before we can deal with amount must have a unit. There are many units of area, each with special value. We express the area of Canada in square mil of a farm in acres, of a garden in square rods or square yar of a lily-pool in square feet, and of the face of a brick in squ inches.

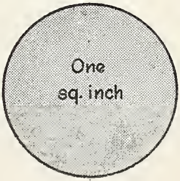
> *When we speak of 75 square rods, the* **unit** *employed is 1 square rod, the* **measure** *(or number of units) is 75, and the* **quantity** *is 75 square yards. Note that* **measure** *is a pure number.*

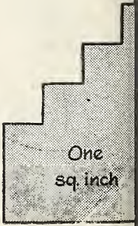












Do not make the mistake of some people who think that a iare mile or a square inch must be square in shape. The istration shows a square inch in a variety of shapes.

The area of a rectangle. Let us recall how to find the area a rectangle.

Draw rectangles with the following dimensions: (a) 5 inches, iches; (b) 4 inches, 2 inches; (c) 3 inches, 3 inches; (d) 2 inches, iches.

Divide each into squares the sides of which are one inch long. mplete the following table:

ectangle	*Number of inches in length*	*Number of inches in width*	*Number of inches in length × number of inches in width*	*Number of squares of area 1 square inch*
a				
b				
c				
d				

For each rectangle, compare the product in the fourth column h the number of squares of one inch side.

Clearly, the number of surface units in the area of a rectangle qual to the number of units of length multiplied by the num- of units of width.

Let A be the number of units of area, and l and w the num- s of units in the length and width respectively.

Area of rectangle

$$A = lw$$

In a square, the length and width contain the same num of units, say s. Hence we may write

$$A = ss.$$

Instead of writing ss to represent $s \times s$, we nearly alw write s with a little 2 above it thus, s^2, and call it "s square

The shorthand note for the area of a square is

$$A = s^2.$$

EXERCISES

1. (*a*) Find in square feet the area of a rectangular bor 30 feet long and 4 feet wide.

(*b*) What is the perimeter of the border?

2. The side of a square room is 15 feet in length. (*a*) W is the perimeter of the room? (*b*) What is the area of the flo

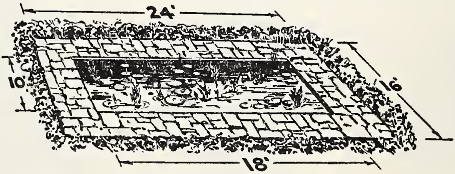
3. When buying a pane of glass, why do we give the len and width instead of simply stating the area?

4. Draw a diagram to show how many times the area square is greater than the area of a square on (*a*) half the si (*b*) one-third of the side, (*c*) one-fifth of the side.

5. (*a*) A rectangle is 6 inches long and 3 inches wide. W is its area?

(*b*) If the length is increased by 2 inches, what is the a of the new rectangle?

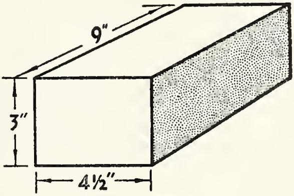
(*c*) Draw a diagram to illustrate (*a*) and (*b*).



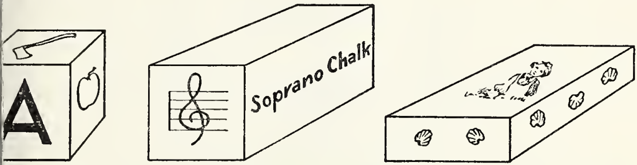
6. Find the area of the walk around the formal pool shown the illustration. Find this area in two different ways.

7. What is the total area of the faces of a brick 8 inches long, nches wide and 2 inches thick?

8. Find (*a*) the area of the shaded face of the brick shown, the total area of the six faces.



A brick is an example of a ***rec-ngular solid.*** Other examples a chalk box and a match box. rectangular sheet of paper is a ry thin rectangular solid. If the faces of a rectangular solid square, the solid is called a ***be.***

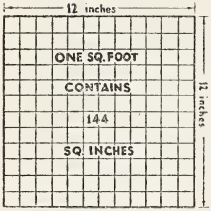


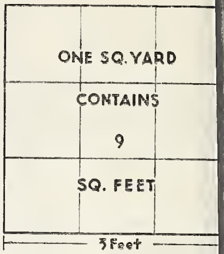
Some rectangular solids

9. The perimeter of one face of a cube is 20 inches. Find the al area of the faces.

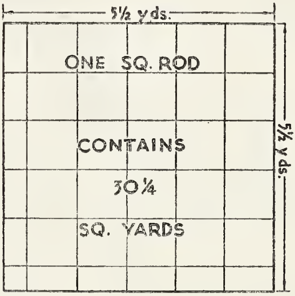
10. The length of the edge of a cube is 1 inch. What is the al area of the faces?

Some of the other units of area. A square of side 1 foot, that to say an area of 1 square foot, contains 144 square inches. we construct a square of side 1 yard, we have an area of 1 ıare yard. Obviously, 1 square yard is equivalent to 9 sq. feet. You will recall that there are $5\frac{1}{2}$ yards in 1 rod. An area of quare rod is obtained when a square of side 1 rod is construc-

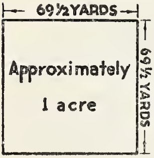

12 inches
ONE SQ. FOOT
CONTAINS
144
SQ. INCHES
12 inches

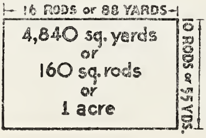

ONE SQ. YARD
CONTAINS
9
SQ. FEET
3 Feet

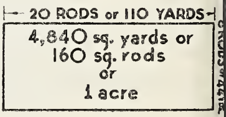
ted. Count the number of whole squares and the number fractional squares in the accompanying illustration and you w be convinced that there are $30\frac{1}{4}$ square yards in each square r


5½ yds.
ONE SQ. ROD
CONTAINS
30¼
SQ. YARDS
5½ yds.

A square of side $69\frac{1}{2}$ yards would contain $4830\frac{1}{4}$ square yard or almost 1 acre. (The acre is equal to 4840 square yards.) field 40 rods square contains 10 acres.


69½ YARDS
Approximately
1 acre
69½ YARDS

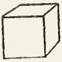

16 RODS or 88 YARDS
4,840 sq. yards
or
160 sq. rods
or
1 acre
10 RODS or 55 YDS.

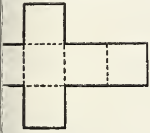

20 RODS or 110 YARDS
4,840 sq. yards or
160 sq. rods
or
1 acre

The estimation of areas. It is suggested that you lay out following areas in your school grounds: a square yard; a are rod; an acre. You should acquire by practice a reason-e degree of accuracy in the estimation of areas and should be iliar with the following table:—

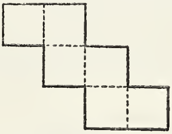
144 sq. inches = 1 sq. foot
9 sq. feet = 1 sq. yard
$30\frac{1}{4}$ sq. yards ≐ 1 sq. rod
[40 sq. rods = 1 rood]
[4 roods = 1 acre]
4840 sq. yds. = 160 sq. rods = 1 acre
640 acres = 1 sq. mile

* * *

If this paper cube  were cut along some of its edges l the paper laid flat on the table the result might be this

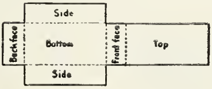


or this



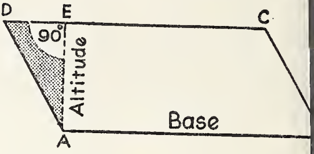
. The latter two wings are called ***nets*** of the solid.

A net of this rectangular solid is the following ure:



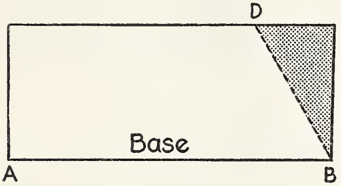
The area of any parallelogram. Construct a fairly large eral parallelogram $ABCD$ and erect the perpendicular AE. he length of AE is the ***altitude.***) Then cut off the tri-gle DAE.

Place the triangle *DAE* so that it takes the position sh in the second diagram. What kind of figure has been formed? How does the area of this parallelogram compare with that of the original parallelogram?


D
E
C
90°
Altitude
Base
A

* * *

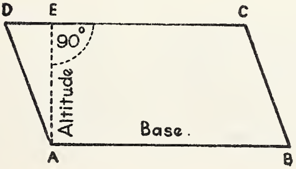
You have seen that the area of the parallelogram *ABC* equal to that of the rectangle which has the same base as parallelogram and which a width equal to the perp dicular height (or altitude) the parallelogram.

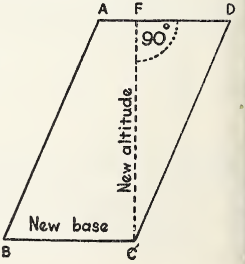


Let *A* be the number units of area. Let *b* and *h* the numbers of units length in the base and a tude. Then we may compress the procedure for finding t area into the following 'tabloid':

> ***Area of a parallelogram***
>
> $A = bh$

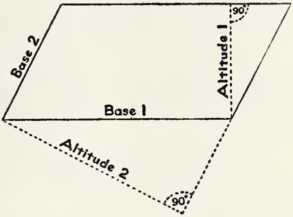
What base? What altitude? Construct a parallelogra *ABCD*. Then turn your book so that the base is *BC*.


D
E
C
90°
Altitude
Base.
A
B


A
F
D
90°
New altitude
New base
B
C

Draw the two perpendiculars AE and CF. (You may find ecessary to extend DA.)

Find the area of the parallelogram in two ways, using the mula $A = bh$. If your construction has been accurate, the numbers obtained should be nearly equal. (There is always ertain amount of unavoidable or in making measurements. e careful worker eliminates all idable errors.)



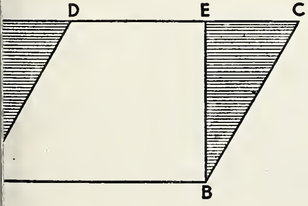
Clearly, in finding the area of arallelogram, it does not matter ich side you select as base, vided that you use the appro- ate altitude. You may use e 1 and Altitude 1, or Base 2 and Altitude 2.

WARNING!

When finding the area of any parallelogram, be careful to use the same unit in measuring the lengths of the two required lines.

EXERCISES

1. If the triangle FAD is a replica of the triangle EBC, what inference may be made concerning the parallelogram $ABCD$ and the rectangle $ABEF$?



2. (*a*) Using straight edge and compasses, construct a parallelogram $ABCD$ in which $DA = 2$ in., $AB = 3$

in., $\angle DAB = 135^\circ$. Find its area (*a*) using AB as base, using DA as base.

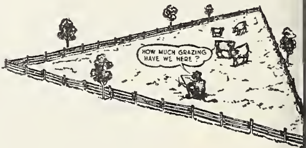
3. (*a*) Using straight edge and compasses, construct a pa lelogram $ABCD$ in which $DA = 2\frac{1}{2}$ in., $AB = 2\frac{1}{2}$ in., a $\angle A = 45^\circ$.

(*b*) Construct a rectangle equal to the parallelogr $ABCD$ and find its area.

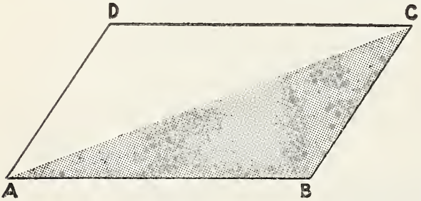
4. In many parts of Canada, the roads divide the land i squares of side $1\frac{1}{4}$ miles. How many 100-acre farms could laid out in one of these squares?

How to find the area of a triangle. Make a general paral ogram $ABCD$.

In future when we refer to a parallelogram without specifying what kind it is, take for granted that a *general* parallelogram is meant. When we say, "Draw a rectangle," draw the general type of rectangle, not the special one called a square.

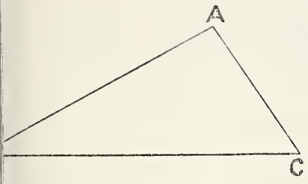

HOW MUCH GRAZING HAVE WE HERE ?

Draw the diagonal AC. Cut out the parallelogram. Then out the triangle CA Will the triang CAB fit exactly the triangle ACD? How many time the area of a pa lelogram greater th the area of each angle formed by drawing a diagonal?



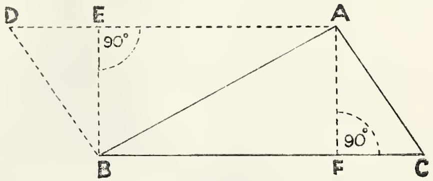
You have learned how to find the area of the general pa lelogram. Hence, to find the area of a triangle, it would se necessary to complete the parallelogram (of which you m imagine the triangle to be one half), find its area, and div the answer by 2. Let us do that.

ABC is a triangle the area of which we require. Through *B* draw a parallel to *CA*. Through *A* draw a parallel to *BC*. The area of the resulting parallelogram *BCAD* is obtained from the length of the base *BC* (which you will remember is also the



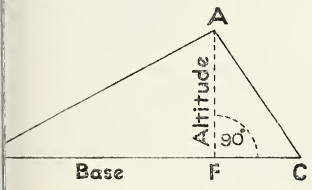
e of triangle *ABC*) and the length of the perpendicular *BE*.

But, surely it is unnecessary to take the trouble of construc-



g the parallelo- m *BCAD* when we d only its altitude . The line *AF*, ich is the altitude he triangle, is the e length as *BE*, altitude of the parallelogram.

Thus we arrive at a rule. To find the number of units of area in a triangle, multiply the number of units of length in its base by the number of units of length in its altitude and divide by 2. Let *A* be the number of units of area, *b* the number of units of length in



base, and *h* the number of units of length in the altitude.

Area of a triangle

$$A = \tfrac{1}{2} bh$$

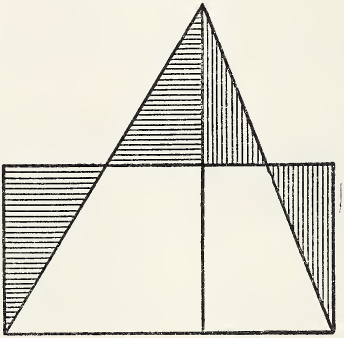
mple. If the base and altitude of a triangle are 3 and 4 hes in length respectively, find the area.

We have $b = 3$ and $h = 4$. It is required to find A.

$$\begin{aligned} A &= \tfrac{1}{2}\,bh \\ &= \tfrac{1}{2} \times 3 \times 4 \\ &= 6. \end{aligned}$$

Hence, the area is 6 square inches.

EXERCISES



1. Compare the area of a triangle with that of a rectan on the same base and with half the altitude.

2. Draw a triangle ABC in which $AB = 2\frac{1}{2}$ in., $BC = 3\frac{1}{2}$ i and $CA = 2\frac{3}{4}$ in. From A, draw AD perpendicular to BC a meeting BC at D. From B, draw BE perpendicular to CA a meeting CA at E. From C, draw CF perpendicular to AB a meeting AB at F.

Copy the following table in your work book and compl the entries:

de	*Length of side in inches*	*Perpendicular*	*Length of perpendicular in inches*	*Number of inches in side × number of inches in perpendicular*	*½ × no. of inches in side × no. of inches in perpendicular*
C	$3\frac{1}{2}$	*AD*		$3\frac{1}{2}\times\ldots\ldots$	$\frac{1}{2}\times 3\frac{1}{2}\times\ldots$ =
A					
B					

Give three pairs of measurements any pair of which will ble you to calculate the area of a triangle.

3. Draw a triangle ABC in which $BC = 3\frac{1}{2}$ inches, $CA = 3$ ies, and $\angle C = 90°$.

Using BC as base, find the area of the $\triangle ABC$.

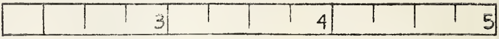
(Frequently, the symbol $\triangle$ is used instead of the word 'trile'. You say that you are more interested in the 'shorthand' the word 'parallelogram'? In future, you may write the reviation $||^{gm}$.)

4. Construct a $\triangle ABC$ in which $AB = 3$ inches, $BC = 3\frac{1}{2}$ ies, and $CA = 4$ inches. Measure BC and the altitude. culate the area.

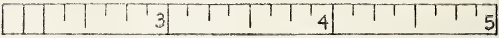
How good are your measurements? Let us suppose that need the area of the top of a table in your classroom. You e measured the length and width of the rectangle and have id them to be 43 inches and 26 inches, respectively, to the rest inch.

It may be that the length of the desk is nearer to $43\frac{1}{4}$ inches the width nearer to $25\frac{3}{5}$ inches. Since you were asked for e dimensions to the nearest inch, you did not include the tions. With great care you might have made the measurements to a sixteenth of an inch. Would you then have t exactly correct? Do you think that you could be sure of lengths to the tenth of an inch?

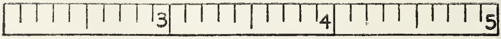
A



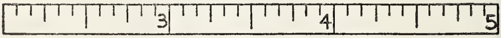
B



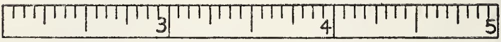
C



D

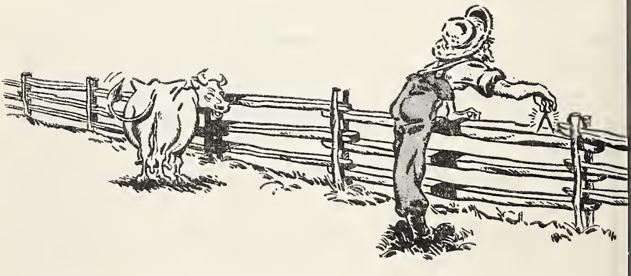


E



It is not difficult to see that any measurement we mak subject to error. Errors may arise from (*a*) inaccuracy in measuring instrument, or (*b*) lack of care or inadequate trai of the person making the measurement.

When measuring, it is possible to aim at too high a degre



uracy. Who wants the length of a tablecloth to the thous-lth part of an inch? What would be gained by measuring to nearest inch the fence around a 15-acre pasture?

To measure with a degree of accuracy greater than the asion demands is a sheer waste of time. On the other hand, **accuracy less than the occasion demands is highly unsatisfactory.** A machinist who makes a metal part a little smaller than the size required wastes both time and material.

ccuracy less than the occasion demands."

Now come back to our table. When we report that the length is 43 inches, we are merely asserting that its measure is nearer to 43 than to 42 or 44. The actual measure might be 43.4 or 42.7. Indeed, you see that it might be any number between 42.5 and 43.5. Similarly, to report the width as 26 inches is to assert that its measure is between 25.5 and 26.5.

EXERCISES

1. If a native told you that "From here to Jones' Corners is odd miles," between what limits would you conclude that the tance lies?

2. If the weight of a bag of potatoes is given as 85 lb., how urate should this quantity be?

3. A piece of copper wire was weighed very carefully and the ght recorded as 18.6 oz. Between what limits would you clude that the weight lay?

4. The following are recorded measures of lengths. each, state the values between which the actual length l (*a*) 37 inches, (*b*) 5.2 feet, (*c*) 8.5 inches, (*d*) 18.54 yards, 13.7 miles, (*f*) 6.0 inches.

To return to our table, we shall say that the numbers 43 26 have two ***significant digits*** or that they are ***correct*** ***two figures.*** It is helpful to think of the first figure as exa correct and of the second figure as correct within half a u Thus, the measure of the length is some number between 42.5 43.5 and that of the width some number between 25.5 and 2 It follows then that the measure of the area of the rectangle i number between

$$42.5 \times 25.5 \qquad \text{and} \qquad 43.5 \times 26.5,$$

that is to say, between

$$1083.75 \qquad \text{and} \qquad 1152.75$$

Clearly, all we can say with assurance is that the area of rectangle is approximately 1100 square inches.

It may be helpful to look at this matter from another po of view. Taking the measures 43 and 26 and using the form $A = lw$, we have

$$A = 43 \times 26 = 1118.$$

With the fact in mind that the final digits 3 and 6 hav possible error up to $\frac{1}{2}$, let us look at the multiplication. I

$$\begin{array}{r} 4\underline{3} \\ 2\underline{6} \\ \hline 2\underline{58} \\ 8\underline{6} \\ \hline 11\underline{18} \end{array}$$

almost certain that the digits underli are in error. Hence, the last two di in the answer are worthless. This c clusion is in agreement with the previ statement, namely that, with the len and width measured only to the near inch, 1100 approximately is as close as can get to the measure of the area.

Rounding off numbers. Another way of stating our c clusion is that only the first two digits in the product are sig

nt and we report the answer as 1100, ***to two significant***
its. The two zeros are not significant; they might be re-
ced by dashes, in which case 11- - would stand for a number
the neighbourhood of 1100. In discarding the last two
thless digits in 1118, we say that we ***round off*** the number
wo digits. We may apply this notion to other numbers. In
nding off 2.54, 36.2 and 0.583 to two digits, we should get 2.5,
and 0.58; it is merely a matter of dropping the last digit.
w take 3.47, 52.6 and 0.389. These numbers, rounded off to
figures, would be 3.5, 53 and 0.39. How shall we round off
5 to two figures? There seems to be as much reason for 7.2
for 7.3. In order that we shall all be consistent, let us agree
7.3 and state a rule.

Rule for rounding off a number

When the first digit discarded is less than 5, make no change in the digits which are retained. When the first digit discarded is 5 or more, then increase the last digit retained by 1.

If we round off the number 6.965 to three digits, we get 6.97
d if we round it off to two digits we have 7.0. Shall we drop
s last 0 as we would if the number 7.0 were exact? You see
t the 0 here is a significant digit and should be retained. The
proximate number 7.0 means a number between 6.95 and 7.05,
ereas the approximate number 7 means a number between
and 7.5.

EXERCISES

1. If you are told that the distance to Ottawa is roughly
ee hundred and eighty miles, to how many significant figures
you being given the distance?

2. The sun is ninety-three million miles from the earth.
ite the number and name the significant and the non-signif-
nt digits.

3. Round off to 3 digits: 7.34, 5.837, 6.7248, 4.999, 52. 326.5.

4. Round off 3.14159, 2.71828, 57.3684, 777.777, 59.0 and 8.99999 to (*a*) 5 digits, (*b*) 4 digits, (*c*) 3 digits, and (digits.

How good are your products? When we took a rectan the measures of whose sides were known to two figures, we fo that we could depend on the area only to two figures. Bef jumping to conclusions, we had better try some other examp Observe these two products which result from the multi

$$\begin{array}{r}
5.46\\
62.\overline{\underline{4}}\\
\hline
2\underline{184}\\
109\underline{2}\\
327\underline{6}\\
\hline
340.\underline{704}
\end{array}
\qquad\qquad\qquad
\begin{array}{r}
5.82\\
7.\overline{\underline{4}}\\
\hline
2\underline{328}\\
407\underline{4}\\
\hline
43.\underline{068}
\end{array}$$

cation of approximate numbers. The figures underlined almost certainly in error so that the first product should rounded off to 341 and the second to 43. These examples s gest to us the following rule.

Rule for the number of digits in the product of approximate numbers

After multiplying two or more numbers, round off the product, keeping only as many digits as there are in the number with the fewest significant digits.

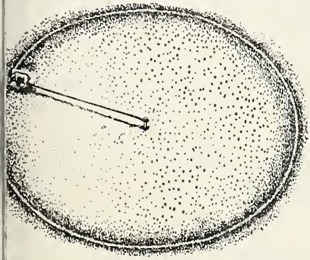
EXERCISES

The following numbers are approximate. Find the products icated and round them off to the necessary number of digits.

1. 65 × 47 2. 45.8 × 56.3 3. 74.2 × 6.4
4. 50.6 × 3.48 5. 6.24 × 5.30 6. 6200 × 450 (each
h 2 significant digits.)

The area of a circle. You will recall that a circle is a curved e lying on a plane and such that every point on it is a fixed tance from a point called the centre. The ***area of a circle*** nderstood to be the amount of surface bounded by the line. much by way of introduction.

A certain goat, whose real name we shall conceal under the udonym Butta, was tethered to a stake in a field. She looked mournfully at the stake, rope and pasture, wondering how long it would take her to eat her grass ration.



After deciding to mark out her boundary, she walked to the limit permitted and, daintily goatish, grazed a very narrow strip all the way around. Butta then made a shrewd estimate of the length her rope and became absorbed in the problem of calculating e number of square feet of grass allotted to her.

This problem of finding the area of a circle if you know the gth of the radius is an ancient one. Its whiskers are longer n Butta's.

To find the area of a polygon is an easy matter. However ny sides the polygon may have, one can always construct a uare equal to it in area. So the age-old problem reduced elf to this—how to make a square of the same area as a given cle, using only straight edge and compasses.

For thirty-five centuries curious people have sharpened their
on this problem—one of the Three Famous Problems of Antiq
Archimedes weighed circular objects and compared their weights
the weights of squares of the same material and thickness whose s
were equal to the radii of the circles. That threw some light on
matter but did not solve the problem. It was Archimedes who fo
that the area of a circle is equal to the area of a triangle with an alti
equal to the radius and a base equal to the circumference. That se
promising until you remember that before you can find the circumfer
you must find the elusive π.

For centuries the circle squarers laboured—in relays. They
inscribed and circumscribed polygons of many sides. (See page
But, although they were able to calculate the area of a circle with g
accuracy, they fell short of solving the problem. In a textbook publis
in 1616, we read, "The preciſe ſquaring of a circle was never yet fo
out."

In 1775, the French Academy passed a resolution that it w
examine no more claims of circle squarers.

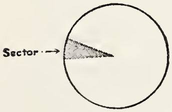
In 1882, it was finally proved that the problem could not be sol
with only straight edge and compasses. However, that did not pre
amateur mathematicians from struggling on with an obstinacy equa
only by that of the seekers of perpetual motion. One man offered a
ward of a thousand dollars to anyone who could discover a flaw in
solution. The flaw was discovered and the court compelled the ma
pay the money.

As H. Schubert[1] has said, "The race of circle-squarers, unmindf
the verdict of mathematicians, the most infallible of arbiters, will n
die out as long as ignorance and the thirst for glory remain united."

* * *

Let us consider the following method of obtaining roug
the relationship between the area of a circle and its radius.

Draw a circle of radius, say 2 inches. Divide it into a nu
ber of equal sectors—sixteen is a convincing number. Cut



the circle. Then almost cut out the sect
leave them joined close to the circumferen
Divide the sectors into two sets, each c
taining eight. Then fit one set into the ot
as indicated in the illustration. You
observe that the resulting figure is v

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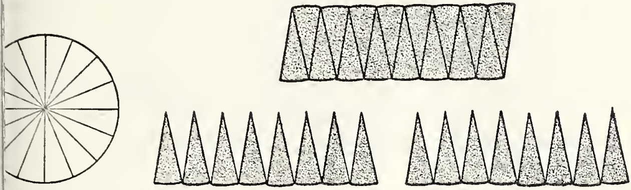
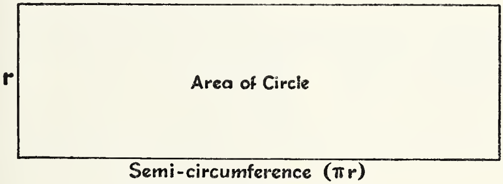
[1] *Mathematical Essays* (Open Court Publishing Co., Chicago).

ghly a rectangle. If you could have divided the circle into)00 sectors, the arcs in the 'rectangle' would be less pro- nced and the width would be approximately equal to the ius. If you could spare the time, you would arrive at a still er approximation by using 100,000,000 sectors!



The width of the 'rectangle' would differ very little from the ius and its length would be close to that of the semi-circum- nce.



The number of units in the area of the rectangle, that is to say, he area of the circle is $\pi r \times r$ or πr^2. That is,

$$\boxed{A = \pi r^2.}$$

A Word About Accuracy of Measurements

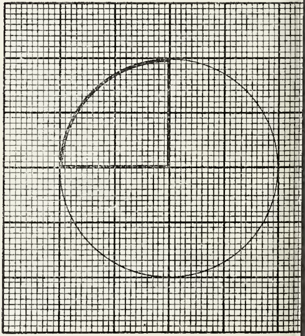
In the exercises which follow, you will often be given data obtained from measurements. When you are told that the side of a square is 6 inches or the radius of a circle is 7 feet, you can assume that reasonable care has been taken in the measuring. ***This means that such measurements justify 2-figure accuracy.***

Hence a measurement reported as 6 in. can be taken as 6.0 in. which involves a possible error of 0.05 in. Likewise a radius of 7 feet can be interpreted as 7.0 ft. Where measurements justify greater accuracy, it will always be indicated in the data.

EXERCISES

1. (*a*) Butta's rope was 20 feet long. Over how ma square feet of grass could she roam? ($\pi = 3.1$)

(*b*) If her rope were doubled in length, how would the new area compare with the old?

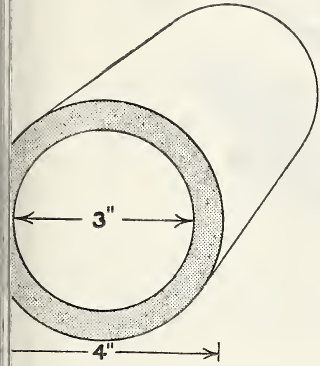


Compare the area of the quadran with the area of a circle whos radius is half that of the give circle.

2. The diameter of a cylindrical can is $2\frac{1}{2}$ inches. Calculate to the nearest ten square inches the number of square inches of metal required for the tops and bottoms of 100 cans. Disregard any creases in the metal. Disregard also the wastage involved in punching circular discs from a sheet of metal.

($\pi = 3\frac{1}{7}$)

. Thirty-five centuries ago, Ahmes, an Egyptian, gave the owing rule for finding the area of a circle: Diminish the neter by one-ninth of itself. Then find the area of the are with that length as side.

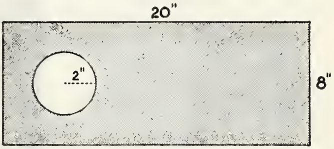


Find to the nearest hundred square inches the area of a circle of diameter 5 feet 3 inches (*a*) by Ahmes' method, (*b*) by using πr^2 where $\pi = 3\frac{1}{7}$.

By how many square inches is the one answer greater than the other?

4. The illustration shows a drainage tile. Calculate the area of the shaded portion. ($\pi = 3\frac{1}{7}$)

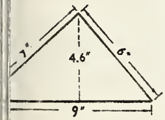
5. Find the area of the ded piece of metal work. = 3.1)

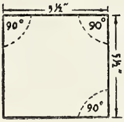


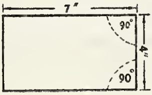
6. (*a*) For each of the owing figures find (i) perimeter, (ii) the area. ($\pi = 3\frac{1}{7}$)

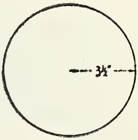
(*b*) Of these four figures, which has the greatest area?

(*c*) Would you be justified in concluding that of all figures qual perimeter the circle has the maximum area?

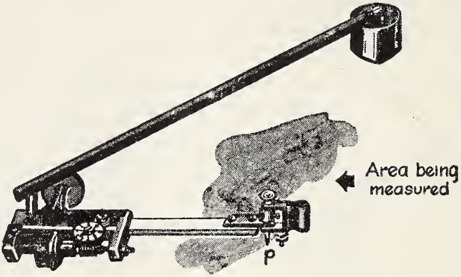








The planimeter. The planimeter is an ingenious instrument for me ing areas.



The tracer point P is moved around the boundary of the surface t measured. No matter how irregular the boundary may be, the area ca calculated readily from readings on the instrument.

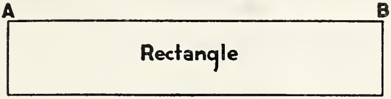
The planimeter is used mainly in map work. For the sake of those are interested in scientific agriculture, an unusual use of the instrument may be mentioned.

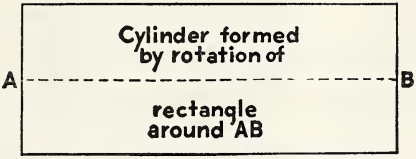
In trying to discover if the percentage of lean meat in a hog carcass were inherited, research workers at the Central Experimental Farm, Ottawa, made tracings of the lean and fat areas in certain cuts of pork. The areas of lean and fat in the cross-sections were obtained from the tracings by means of the planimeter.

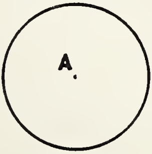
The area of a cylinder's faces. In the following list, th is *one* article that does not 'belong'. Which is it?

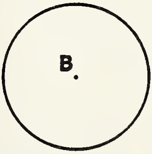
Unsharpened round pencil; salmon can; cigarette; tennis b straight piece of wire: a straight drainage tile; an unused piece chalk.

Excellent! You recognized the six ***cylinders,*** and threw out tennis ball!









If the rectangle could rotate around the side AB, the solid duced by this rotation would be a cylinder which is shown he second of our four drawings. (Use a rectangular piece of dboard and some wire, and observe the effect of the rotation.) e two plane faces of the cylinder have also been drawn.

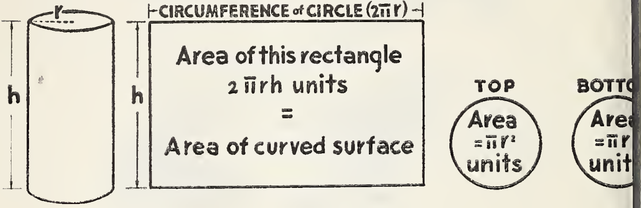
To find the area of the plane faces is an easy matter for you v. However, the problem of obtaining the area of the curved face may give you something to think about. Don't depend the textbook; show your initiative and resourcefulness.

* * *

If you are completely baffled, obtain a can and wrap a piece paper around it. Now make another attempt to obtain the a.

* * *

It is a simple affair to cut a piece of paper of such dimensi that it can just cover the curved surface of the cylinder. Op the paper and lay it on a flat surface. Describe its shape.


r
h
CIRCUMFERENCE of CIRCLE (2πr)
h
Area of this rectangle
2πrh units
=
Area of curved surface
TOP
Area
=πr²
units
BOTT
Are
=πr
unit

Clearly, the length of this rectangular piece of paper is t circumference of the circle ($2\pi r$ units) and the width is the hei of the cylinder (h units).

Hence, the number of units of area in the curved surface $2\pi r \times h$ or $2\pi rh$.

The number of units of area (A) in the whole surface of t cylinder is $2\pi rh + \pi r^2 + \pi r^2$, or $2\pi rh + 2\pi r^2$.

Total surface of cylinder

$$A = 2\pi rh + 2\pi r^2$$

Example. Find the whole surface area of a cylinder wh height and radius are 2 inches and $3\frac{1}{2}$ inches respectively.

We have $h = 2$, $r = 3\frac{1}{2}$. A is required.

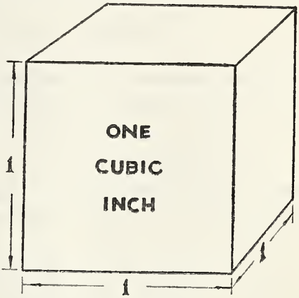
$$\begin{aligned} A &= 2\pi rh + 2\pi r^2 \\ &= 2 \times \tfrac{22}{7} \times \tfrac{7}{2} \times 2 + 2 \times \tfrac{22}{7} \times \tfrac{7}{2} \times \tfrac{7}{2} \\ &= 44 + 77 = 121. \end{aligned}$$

Hence, the area is 121 square inches, or 120 square inches two significant figures.

We measure volume. You have learned something ab the measurement and calculation of lengths and areas. Ho ever, as you are fully aware, there are many commodit ich are not bought and sold by the inch, foot or yard or by square inch, square rod or acre. What you want is bulk or we say ***volume.***

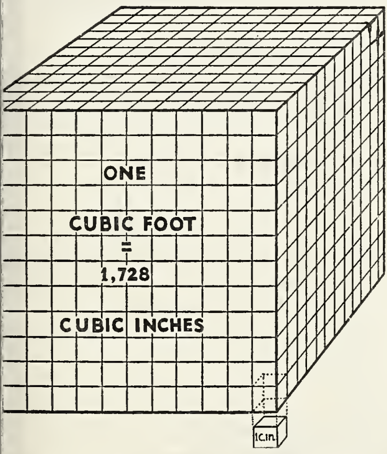
With most of the units of volume you are already quite iiliar. You will recall that a ical box 1 inch long, 1 inch le and 1 inch deep is the unit volume we call a ***cubic inch.*** course, a cubic inch need not cubical in shape.

If the box had originally been of milk and if all the milk were lled on the floor, its bulk, ig- ing the small amount which iered to the sides, would still 1 cubic inch.



A cubical box, 1 foot long, 1 foot wide and 1 foot deep, would tain a volume of one ***cubic foot.*** If you packed this box with 1-inch cubes, you would have a total of 12 layers with 144 cubes in each layer. Hence the total number of cubic inches in a cubic foot is 144 × 12 or 1,728, which seems astonishingly large.

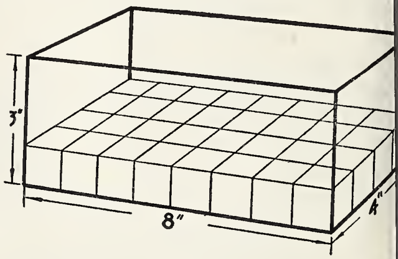
How many cubic feet are there in a ***cubic yard*** of sand? (Sometimes contractors and builders speak of "a yard of sand". You will un- stand that they mean a *cubic* yard.)



Think of the volume of air in an imaginary cube with a b
1 mile by 1 mile and a height of 1 mile. This volume is eq
to 147,197,952,000 cubic feet! With such immensity in mi
we may let our imagination roam with the R.C.A.F. pilot w

> “. . . . . *trod*
> *The high untrespassed sanctity of space.*”[1]

The volume of a rectangular solid. We have sketche
box with cellophane sides, 8 inches by 3 inches by 4 inch
Let us find its volume.



If we packed one-inch cubes into this box, each layer would contain 4 × 8 cubes. There would be 3 layers. Hence, the total number of cubes would be 3 × 4 × 8. Thus, the volume of the box is 3 × 4 × 8 cubic inches.

In general, if the length of the solid contains l units, the wi
w units, and the height h units, the bulk is $l \times w \times h$ units
volume.

Volume of rectangular solid

$$V = lwh$$

Example. Find the volume of a rectangular solid the leng
width and height of which are respectively 6, 4 and 3 inches.

We have $l = 6$, $w = 4$, $h = 3$. V is required.

$$\begin{aligned} V &= lwh \\ &= 6 \times 4 \times 3 \\ &= 72. \end{aligned}$$

Hence, the volume is 72 cubic inches.

---

[1] *High Flight* by Pilot Officer John Gillespie Magee.

EXERCISES

(Assume that measurements have 2-figure accuracy.)

1. A brick 9 inches by $4\frac{1}{2}$ inches by 3 inches was dropped o a pail of water. How many cubic inches of water were placed?

2. The edge of a cube is 3 inches in length. Find (*a*) the al area; (*b*) the volume.

3. The edge of a cube is 4 inches in length.

(*a*) Calculate the total area of the cube's faces.

(*b*) If the length of the edge of the cube were doubled at would be the total area? How many times is the new area ater than the old?

(*c*) Calculate the volume of the original cube.

(*d*) If the edge were doubled, what would be the volumc? w many times is the new volume greater than the old?

4. (*a*) A rectangular solid is 3 inches long, 4 inches wide and ches deep. What is its volume?

(*b*) If each edge were increased by one inch, what would new volume be?

5. (*a*) A rectangular water cistern is 5 feet long, 5 feet wide 4 feet deep. How many cubic feet of water can it contain?

(*b*) What weight of water can it contain? (A cubic foot vater weighs $62\frac{1}{2}$ pounds.)

(*c*) How many gallons can it contain? (1 cubic foot = $6\frac{1}{4}$ ons.)

6. Mr. Lowe has a bin 4 feet by 5 feet by 6 feet. How many hels of grain can be stored in it? (1 cubic foot = 0.8 bushel.)

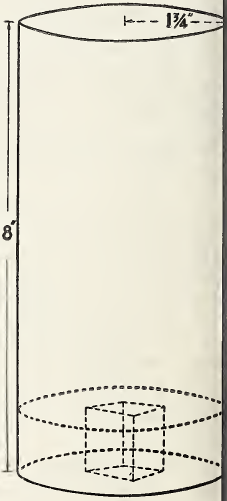
7. (*a*) The external dimensions of a lidless rectangular box as follows:—length, 1 ft. $2\frac{1}{2}$ in.; width, $8\frac{1}{2}$ in.; depth, $5\frac{1}{4}$ in. he wood of which the box is made is a quarter of an inch k, calculate the volume of the interior of the box.

(*b*) How many cubic inches of wood are in the sides a bottom?

8. A cellar 40 feet long, 28 feet wide and 9 feet deep is quired. If the top of the cellar is to be at the ground level, h many cubic yards of earth must be removed?

9. Mr. Smith bought a house and offered to purchase the c remaining in the rectangular bin. He levelled the coal in bin and made some measurements. The length of the bin 10 feet and the width, 4 feet. The depth of the coal was inches. What should he pay for the coal at $16 per ton? (C ton of coal occupies approximately 35 cubic feet.)

The volume of a cylinder. Let us find the volume of the cylinder shown in the accompanying illustration.



The water in the cylinder rests on a circular base and has a depth of one inch. Consider 1 square inch of this base. The volume of the cube of water standing on this square inch is clearly 1 cubic inch. Hence, for each square inch of base there is 1 cubic inch of water. Let us find the area of the base.

The radius of the base is 1$\frac{3}{4}$ inch or $\frac{7}{4}$ inch. Therefore, the base contains $\pi \times \frac{7}{4} \times \frac{7}{4}$ square inches, or $\frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}$ square inches.

Therefore, the one-inch layer of water has a volume of $\frac{22}{7} \times \frac{7}{4} \times \frac{7}{4}$ cubic inches.

Since the height of the cylinder is 8 inches, the number of one-inch layers that the cylinder c hold is 8.

Therefore, the volume of the cylinder is $8 \times \frac{22}{7} \times \frac{7}{4}$ cubic inches, i.e., 77 cubic inches.

You will have noticed that this process consists of (*a*) finding number of units of area in the base, (*b*) multiplying this mber by the number of units of length in the height, and expressing the product in units of volume. Let V be the mber of units of volume, h the number of units of length in height, and r the number of units of length in the radius.

Volume of cylinder

$$V = \pi r^2 h$$

It is hardly necessary to mention that a mixture of units st be avoided. One should express all units of length in hes, or all in feet, or all in yards; then the volume would be ubic inches, or in cubic feet, or in cubic yards respectively.

ample. Find the volume of a cylinder whose height and ius are 16 inches and $8\frac{3}{4}$ inches respectively.

We have $r = 8\frac{3}{4}$ and $h = 16$. It is required to find V.

$$\begin{aligned} V &= \pi r^2 h \\ &= \tfrac{22}{7} \times \tfrac{35}{4} \times \tfrac{35}{4} \times 16 = 3850. \end{aligned}$$

Hence, the volume is 3850 cubic inches, or 3900 cubic inches wo significant figures.

* * *

A 'shorthand' statement such as $A = \pi r^2$ or $V = \pi r^2 h$ is called ***rmula.***

EXERCISES

these exercises take $3\frac{1}{7}$ as the value of π. Assume that measurements justify 2-figure accuracy.)

1. How many cubic inches of water could be contained in a ndrical pail with the following dimensions: diameter, 7 in., ght, 15 in.?

2. (*a*) A closed cylindrical metal tank is 4 feet in length $1\frac{3}{4}$ feet in radius. How many gallons of gasolene can it h (1 c. ft. = $6\frac{1}{4}$ gallons.)

(*b*) If each square foot of the sheet metal used in mak the tank weighs 5 pounds, what is the weight of the empty ta

3. A cylindrical hot water tank has a radius of 8 inches a height of 5 ft. 10 in. How many gallons of water doe contain? (277 c. inches = 1 gal.)

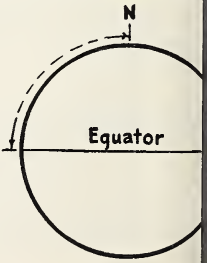
4. (*a*) Find to the nearest thousand cubic inches the volu of a mile of copper wire the radius of which is one-eighth of inch.

(*b*) Calculate to the nearest hundred pounds the wei of copper. (1 c. inch of copper weighs 0.3 lb.)

5. Water is flowing through a pipe of diameter 2 in. at rate of 14 feet per second. How long will it take to fill a recta ular cistern 11 feet long, 7 feet wide and 5 feet deep?

The Metric System of Units

Units of length. Up to the present, we have used the fami units of our everyday life. You should, however, have so knowledge of other units of length, area and volume employed in scientific work and in many non-English-speaking countries. We refer to the ***metric*** units. The metre is very nearly one ten-millionth of the distance from the equator to the north pole of the earth measured along a meridian. Roughly speaking, the metre is $39\frac{1}{3}$ inches in length.



It is advisable to be familiar with three other metric units length: the millimetre, the centimetre and the kilometre. e relationships are given in the following table:

10 millimetres = 1 centimetre
100 centimetres = 1 metre
1000 metres = 1 kilometre

Comparison of English and metric units of length

1 inch = 2.54 cm.
1 cm. = .394 in.
1 yd. = .914 m.
1 m. = 1.09 yd.
= 39.4 in.
1 mile = 1.61 km.
1 km. = .621 mile
8 km. = 5 miles

The relationships given are all approximate.

EXERCISES

1. (*a*) Draw lines of the following lengths: (i) 8 cm.; (ii) cm.; (iii) 66 mm.; (iv) 42 mm.

(*b*) For each of the lengths, give the maximum error ch could creep into the work of a careful pupil using a reasony accurate ruler.

2. Express to the nearest millimetre the number of millires in an inch.

3. Express to the nearest centimetre the number of centires in one foot.

4. Express (*a*) 220 yards in metres, (*b*) 100 metres in ya

5. A boy was riding a bicycle at the rate of 10 miles per h What was his speed in kilometres per hour?

6. An aircraft is flying with a speed of 360 kilometres hour. Express this speed in English units.

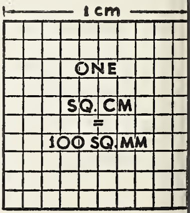
7. What is the height in metric units of a girl 5 feet 1 i tall? Express your answer in centimetres to the nearest ce metre.

8. A rectangular flower bed is 9.8 metres long and 1.7 me wide. What is its perimeter (*a*) in metres, (*b*) in feet, to nearest foot?

9. (*a*) Draw a triangle in which $AB = 6.8$ cm., $BC =$ cm., and $CA = 4.9$ cm.

(*b*) Using straight edge and compasses, draw the per dicular from A to BC, meeting BC in E. Measure AE.

10. The distance between two towns on a map is 11.2 If the map is drawn on a scale of 1 inch to 2 miles, how far a are the two towns? Give your answer to the nearest half m



Units of area. It is unnecessary for your present purposes to be familiar with more than two or three metric units of area.

A square with a side 1 centimetre[2] long has an area of one ***square centimetre*** or 100 ***square millimetres.***

If the side of a square is 1 metre in length, the enclosed area is a ***square metre*** or 100 × 100 or 10,000 square centimetres.

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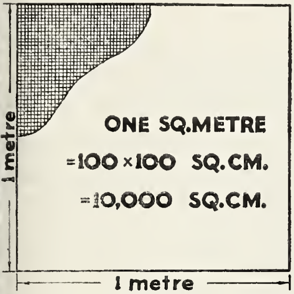
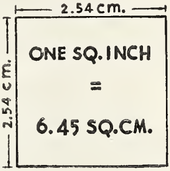
[2] The figures on pp. 102-104 are not drawn to scale.

In the metric system, large land areas are measured in *uare kilometres.*


ONE SQ.METRE
=100 × 100 SQ.CM.
=10,000 SQ.CM.
1 metre
1 metre


2.54 cm.
2.54 cm.
ONE SQ. INCH
=
6.45 SQ.CM.

Comparison of English and metric units of area

1 sq. in. = 6.45 sq. cm.
1 sq. cm. = 0.155 sq. in.
1 sq. ft. = 929 sq. cm.
1 sq. yd. = 0.836 sq. m.
1 sq. m. = 10.8 sq. ft.
= 1.20 sq. yd.
1 sq. mi. = 2.59 sq. km.
1 sq. km. = 247 acres
= 0.386 sq. mi.

The relationships given are all approximate.

EXERCISES

1. The length of a rectangle is 8.7 cm. and its width is 4.3 cm. culate its area to the nearest square centimetre.

2. The altitude of a triangle is 6.8 cm. and its base is 4.6 Find its area (*a*) to the nearest square centimetre, (*b*) to nearest tenth of one square inch.

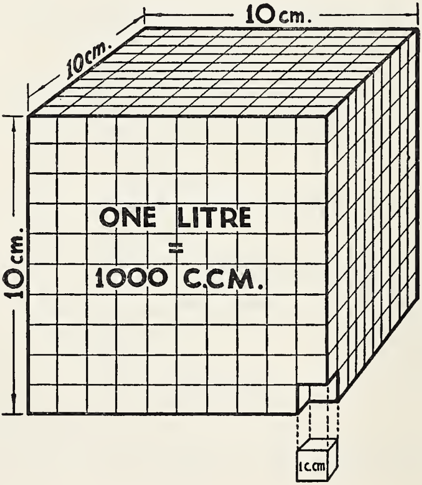
3. A pupil measured the side of a square and obtained cm. as the length, whereas the correct length was 3.5 cm. was required to find the area to the nearest square centime

(*a*) What was the error in the area obtained by the pu

(*b*) If the square represented an area on a map drawn a scale of one centimetre to a kilometre, by how many squ kilometres did the obtained area exceed the true area?

(*c*) By how many acres did the obtained area exceed area? Give the answer to the nearest ten acres.

Units of volume. If a cubical box is 1 centimetre lon centimetre wide, and 1 centimetre deep, its volume is a cu centimetre.



A cubical box of edge 10 centimetres has a volume of × 10 × 10 or 1,000 cubic centimetres. This is a well-known t of volume called the ***litre.*** It is roughly equivalent to $1\frac{3}{4}$ ts.

Comparison of English and metric units of volume

1 c. in. = 16.4 c.cm.
1 c.cm. = 0.006 c. in.
1 c. ft. = 28.3 litres
1 litre = 61.0 c. in.
= 0.035 c. ft.

These relationships are only approximate.

EXERCISES

1. Find the volume of a rectangular box the dimensions of ch are (*a*) 8 in., 5 in., 4 in., (*b*) 8 cm., 5 cm., 4 cm.

When using formulas, we deal, not with quantities, but ith measures or pure numbers. The calculations in (*a*) *nd* (*b*) *above were the same because the measures involved ere identical. The raw material of the formula consists f measures; the finished article is also a measure. When e finished article is obtained, the unit is assigned according the system used. For both* (*a*) *and* (*b*) *the measure of the lume is 160. However, the quantities are respectively 60 cubic inches and 160 cubic centimetres.*

2. Find, to the nearest 10 litres, the volume of a rectangular rn 4 feet long, 4 feet wide and 5 feet deep. (Assume that have 3-figure accuracy.)

3. (*a*) Calculate to the nearest hundred cubic centime the volume of a cylinder with the following dimensions: heigh 30.3 cm.; diameter = 21.0 cm. ($\pi = 3\frac{1}{7}$)

(*b*) Find the volume to the nearest half litre.

4. A pupil measured the edge of a cube and obtained result 5.0 cm. instead of the correct length 4.9 cm.

(*a*) What was his error?

(*b*) He calculated the area of a face to the nearest squ centimeter. By how much was he in error?

(*c*) Then he calculated the volume of the cube to nearest cubic centimetre. What was the size of his error?

EXERCISES IN SPATIAL IMAGINATION

(These exercises should be performed mentally.)

1. How many triangles are formed when the diagonals rectangle are drawn?

2. If a triangle be inscribed in a circle, how many cl figures are there, each having a curved side?

3. How many rectangles of different area could you f from 10 matches? All the matches are to be used in e rectangle.

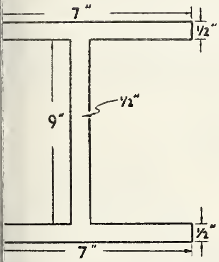
4. A box contains two boxes, each of which contains boxes, each of which contains two boxes. What is the t number of boxes?

5. In one of the Sherlock Holmes stories, the famous de tive examined the tracks made by the tires of a bicycle. T he declared that he knew in which direction the cyclist going. He based this inference on the fact that the track m by the back tire is more distinct than that made by the front because the back wheel carries the greater part of the rid weight.

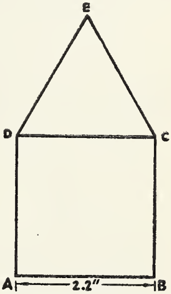
Had Holmes justification for his inference? Is it possible to
from the tracks of a bicycle the direction in which it is going?
sume that the treads of the two tires are identical.)

EXERCISES ON CHAPTER II

A

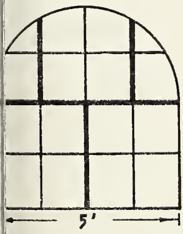


1. The illustration shows the cross-section of an I-beam. Calculate the cross-sectional area. (Assume exact data.)



2. (*a*) *ABCD* is a square and *EDC* an equilateral triangle. Calculate the
meter of the polygon *EDABC*.

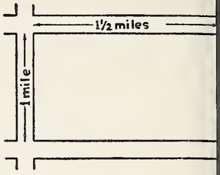
(*b*) Using the compasses and straight
only, construct the figure *EDABC*
calculate its area to the nearest tenth
square inch.



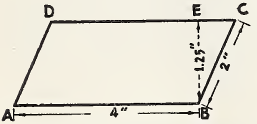
3. Find (*a*) the perimeter to the nearest inch, (*b*) the area to the nearest tenth of a square foot of the window shown in the illustration. The upper portion is semicircular. (Assume 3-figure accuracy. Take $\pi = 3.14$.)

4. Construct a rhombus of side 4
with one diagonal 7 cm. in length. Find its area. Con-
ct a rectangle equal in area to the rhombus.

5. How many quarter-sections of land are enclosed in the rectangle between the roads? By means of a sketch show the most convenient arrangement of the farms. (Each farm contains a quarter-section or 160 acres.)



6. Construct a parallelogram $ABCD$ in which $AB = 4$ $BC = 2$ in., and the altitude B 1.25 in. (You may use a set squa



(*a*) Calculate the perimeter the parallelogram.

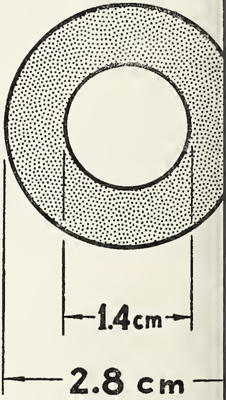
(*b*) Find the perimeter of $\triangle C$

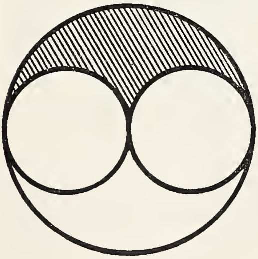
(*c*) Find the area of $\triangle EBC$.

(*d*) Calculate the area of the parallelogram $ABCD$.

7. (*a*) Calculate to the nearest square centimetre the area of a face of the washer. ($\pi = 3.14$)

(*b*) If the washer were divided into two portions by cutting along a diameter, what would be the perimeter to the nearest tenth of a centimetre of one of the halves?





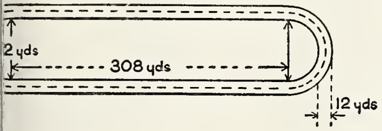
✓**8.** Each of the small circles ha radius half that of the large on

(*a*) In your opinion, what fr tion of the large circle is the sha part? Check the accuracy of y estimate by giving values to lengths of the radii.

(*b*) Is the whole figure symmetrical? If so, indicate the (or axes) of symmetry.

If the figure were not shaded, give the axis (or axes) of ımetry.

9. The ends of the racing track are semicircular.



(*a*) If a runner kept to the middle of the track, how far would he go in making a complete circuit? ($\pi = 3\frac{1}{7}$)

(*b*) Calculate to the nearest hundred square yards the area he cinder track.

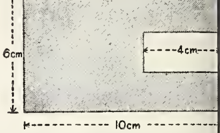
10. A mailing tube has a length of 24 inches and an external us of 3.0 inches. Calculate, correct to two figures, the num- of square inches in the external surface.

11. (*a*) When a pupil measured the edge of a square, the imum error to which the measurement was subject was half illimetre. If the length was found to be 7.6 centimetres, t was (i) the least length that the pupil might have obtained, the greatest length he might have obtained?

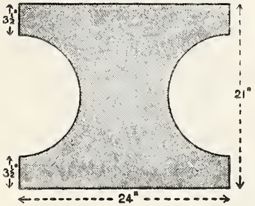
(*b*) The pupil calculated to the nearest square inch the of a face of the square. Calculate to the nearest square inch the least possible value of the area, (ii) the greatest possible e of the area.

12. A cylindrical brass rod is 80 centimetres long and has a neter of 1.4 centimetres. Find, correct to two figures, (*a*) its l surface area; (*b*) its volume. ($\pi = 3.1$)

13. Calculate the area of the figure shown in the acc panying illustration.



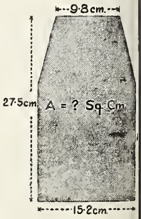
14. A circular pool has a diameter of 14 feet. At \$2.00 per square yard, what would be the cost of making a concrete walk $2\frac{1}{2}$ feet wide around it? ($\pi = 3\frac{1}{7}$)



15. (*a*) Calculate to the nea ten inches the perimeter of the fig shown in the illustration. curved portions are semicircu ($\pi = 3\frac{1}{7}$)

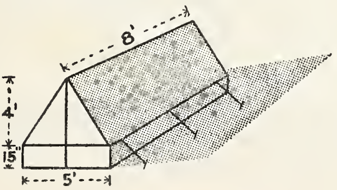
(*b*) Find its area in square ches to the nearest ten square inc

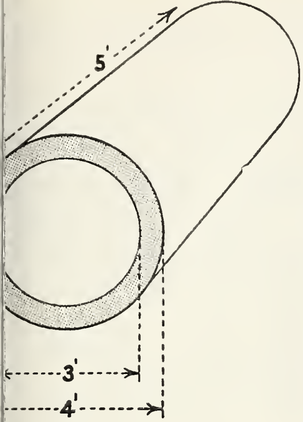
16. A man enclosed a square plot of ground with a fence feet long. Do you think that it would have been possible him to have used the same length of fence to enclose a recta ular plot of greater area?



17. Calculate the area (A) of the figure shown in the illustration. (Assume 3-figure accuracy.)

✓ **18.** Find the number of square feet of canvas required for the construction of the tent. Disregard the amount needed for seams and reinforcements. (We have not given you all the dimensions you need, but we believe t you have enough ingenu to solve the problem.)

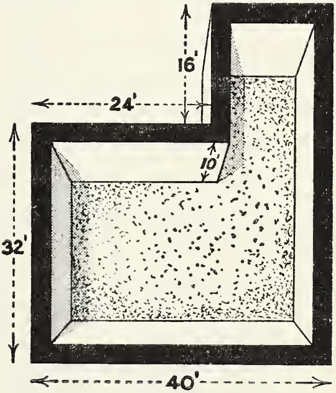




19. Calculate to the nearest cubic foot the volume of the material in the tile. ($\pi = 3\frac{1}{7}$)

20. (*a*) The top of the concrete foundation is level with the surface of the ground. How many cubic yards of earth were removed in making this excavation? Express your answer to the nearest ten cubic yards.

(*b*) If the walls are 12 inches thick, how many cubic feet of material were required to construct them?



21. A belt passes around
ulley 7 inches in dia-
er. If the pulley makes
revolutions per second,
t is the speed of the
(*a*) in feet per second,
n miles per hour?

($\pi = 3\frac{1}{7}$)

C

22. Copy the following tables in your workbook and fil the blanks.

(*a*)

Triangle	*Length of base in cm.*	*Length of altitude in cm.*	*Area in sq. cm.*	*Area of triangle compare with that △ABC*
ABC........	48	12	288	Same
DEF........	24	24	288	....
GHK........	12	48	....	....
LMN........	6	96	....	....
PQR........	3	192	....	....

(*b*)

Triangle	*Length of base in cm.*	*Length of altitude in cm.*	*Area in sq. cm.*	*Area of triangle compare with that △ABC*
ABC........	x	y		
DEF........	$\frac{x}{2}$	$2y$		

(*c*) What conclusion can be drawn from these tables

23. Construct a square. Make a rectangle equal to i area, but with one side double the side of the square.

24. (*a*) Compare the circumferences of two circles the leng of whose radii in centimetres are (i) $2x$ and x, (ii) $3x$ an (iii) $4x$ and x.

(*b*) Compare the areas of two circles the lengths of wh radii in centimetres are (i) $2x$ and x, (ii) $3x$ and x, (iii) $4x$ an

✓ **25.** (*a*) The length of a rectangle is $4k$ cm. and its wi $2k$ cm. Find (i) its perimeter, (ii) its area.

(*b*) Find the area of a square with the same perimeter hat of the rectangle.

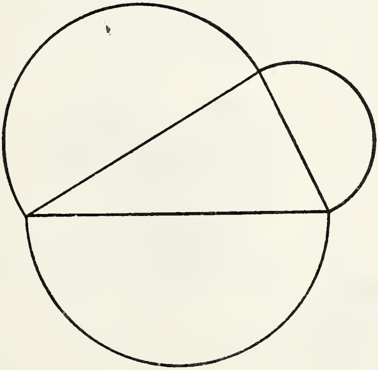
26. The length and width of a rectangle are each increased 10 per cent. What is the percentage increase in the area? diagram may be helpful.)

27. A cylindrical container 5 feet 10 inches high and 4 feet liameter was full of water. The water was emptied into a angular tank 11 feet long and 3 feet 4 inches wide. What the depth of water in the tank? ($\pi = 3\frac{1}{7}$)

28. Show that the total area of a cylinder the height and us of which are h and r units respectively is the same as that rectangle the length and width of which are $2\pi r$ and $r + h$ s respectively.

29. The circumference of an elephant's fore-leg is 60 inches. at is the diameter to the nearest inch? ($\pi = 3.1$)

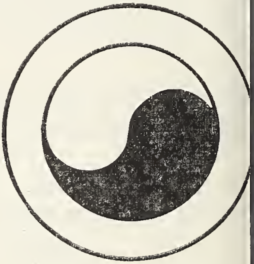
30. Show that the sum of the perimeters of the three semiles is $\frac{\pi}{2}$ times the perimeter of the triangle.

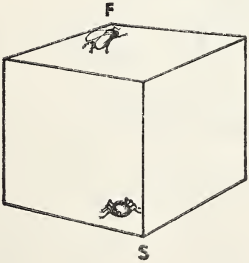


Recreations

1. Imagine that there are very small intelligent being living on the surface of a spherical ball. They know th value of π to a high degree of accuracy. Like many cleve creatures they are consumed with intellectual curiosit One question that keeps them awake of nights is whethe the surface on which they live is flat or round. Show tha by measuring the radius and circumference of a relativel large circle on the surface of their sphere, they could solv their problem.

2. The sketch shows the Great Monad, a Chinese symbol and one of the most ancient in the world. The upper boundary of the blackened portion consists of two semicircles, each of radius equal to half the radius of the inner circle. Prove that the shaded and unshaded portions of the inner circle are equal in area.





3. A spider of much intelligence is at one vertex of cubical box and has arrived a the correct solution to the problem of what route she shoul take to reach the fly F mos quickly. What route did sh decide to take?

CHAPTER III

THE BEGINNING OF ALGEBRA

> "*In mathematics he was greater*
> *Than Tycho Brahe or Erra Pater;*
> *For he by geometric scale,*
> *Could take the size of pots of ale;*
>
> * * *
>
> *And wisely tell what hour o' the day,*
> *The clock does strike, by algebra.*"
>
> Samuel Butler.

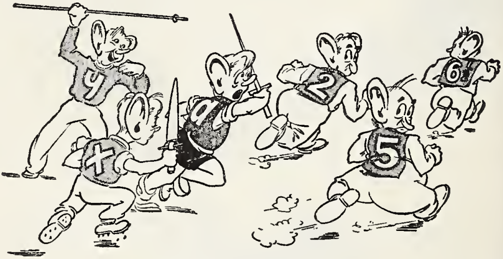
Let x be the Number

ou have already made some shorthand statements or ulas and used them in problems. No doubt you have rved that a formula is a sort of key which opens the way he solution of a number problem. A single formula, more-

over, may be the key to several doors; the solutions of a nu of problems may be revealed by a skilful use of the key.

> ***The power of the formula arises from our use of letters to stand for numbers.***

A great teacher, Felix Klein, has said that "real mathem begins with operations with letters." These letter-num enable us to work with number relations without thinking a ***particular*** numbers. For example, the formula $A = \pi r^2$ giv the relation between the radius and the area of ***any*** circle. W we want the area of a circular disc or cover, we simply subst a *particular* number for the *general* number r. Now that have begun to use letters to stand for numbers, you can tell friends that you have crossed the border line from arithmet algebra.



Letter-numbers take the field.

Let us consider some of the advantages of the metho algebra. Take, for instance, the problem of finding the ar a rectangle. You meet it everywhere—in dealing with pan glass, rugs, linoleum, tile flooring, beaver board, lumber, roo and a host of other things. In Chapter II you experime with a number of special rectangles, finally concluding that

can be found from the length and the width. Your results
ıt be summed up like this:

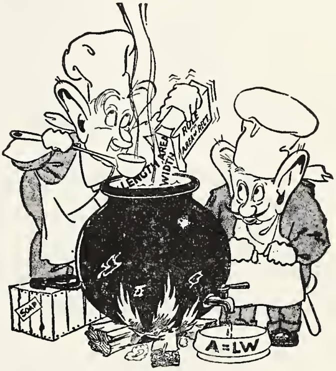
Rule of Arithmetic

The number of units in the area of a rectangle equals the number of units in the length times the number of units in the width.

is a useful rule; but how much shorter it is if we write the
using the number symbols of algebra!

lgebraic Formula

$$A = lw$$



ou can think of the
ula as a translation of
tatement from English
algebra. This gives
n very concise form,
elation connecting the area, length, and width of ***any*** rec-
le.

ever forget that the letters A, l, and w stand for **numbers.**

is not the area but the ***number*** of units of area. Similarly,
he number of units of length, and *w* the number of units of
h. This use of numbers to give the measures of quantities
e of the great services of the number system to mankind.

"The Shape of Things to Come"

Formulas are much more than shorthand statements of ru
A single formula may be the key to several problems.
example, you have learned that the volume of a tin can is gi
by $V = \pi r^2h$. Well, this formula enables us to deal with th
problems about tin cans:

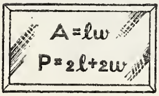
(1) If r and h are known, we can find V.
(2) If V and r are known, we can find h.
(3) If V and h are known, we can find r.

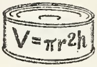
Example. If the volume of a can of paint is 1540 cu. in. and
radius of its top is 7 in., find its height.

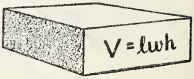
Since $V = \pi r^2h$,
we have $1540 = \frac{22}{7} \times 7 \times 7 \times h$.
Therefore $1540 = 154h$,
and $h = 10$.
Hence, the height is 10 inches.

We Give Meanings to Some Words

When letters are used to represent numbers, they are so
times called ***literal numbers*** (Lat. *litera*, letter) to disting
them from ordinary numbers like 4. You have already met s
literal numbers in your formulas for perimeters, areas, and volu







The numbers lw, $2l$, $2w$, lwh, πr^2h are also called *terms*
term may be

(1) an ordinary number like 7,
or (2) a literal number like x,
or (3) the product or quotient of ordinary or literal numbers like $7x$, or $2xy/a$.

Here are some terms: 6, $\frac{3}{4}$, 5×2, $3a$, pqr, πr^2.

)f course, that example was easy. You may suspect that e complicated formulas handle harder problems just as well, , if you do, your suspicion is correct. When you learn to use e number symbols of algebra with ease, you will be able to e many interesting problems.

To this end, let us examine some of the processes of algebra. will meet some new words which should be added to your ıbulary. Moreover, you must always think about the *ınings* of the symbols. Recall, for example, that

bh means b times h,
r^2 means $r \times r$,
$2w$ means 2 times w, etc.

Similarly, $r \times r \times r = r^3$,
$m \times m \times m \times m = m^4$,
and $a \times b \times c = abc$.

e you know what the words and symbols mean, then *mastery e various skills comes with attentive practice.*

Target for Now

A new mathematical tool—the letter-number—is now in your nds. Before you can achieve worth-while results with it, you ıst learn how to use it. This is the task that now confronts u—to gain facility in adding, subtracting, multiplying, and viding these new numbers.

You may find some drudgery in the pages that follow. here is no royal road. What real achievement does not involve me hard work? The exercises are chosen to help you master rtain processes so that you can gain skill in a very useful bject.

Be patient. Do your best. The more you achieve, the more joyment you will get from your study, and the more interesting ideas that will be yours. "Civilization advances by extending number of important operations which we can perform witht thinking about them."[1]

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[1] \. N. Whitehead: *Introduction to Mathematics.*

EXERCISES

1. Why is $2l + 2w$ not a term?

2. What do the letters used in algebra stand for?

3. How many terms are there in the expression $a + b$

4. State the number of terms in each of the followin pressions: (*a*) $6 + 4a$, (*b*) $x^3 + y^3 + z^3 - k^3$, (*c*) $15m^2n^2r^3s^3$, (*d*) $10 - 7x$.

5. Make up a list of twenty different terms. (Use both li and ordinary numbers.)

Another name for a term is a ***monomial.*** (Monomial m *one number.*) The sum or difference of two terms is call ***binomial.*** (Binomial means *two numbers.*) Here are s binomials: $3x + 7$, $2 + 5a$, $4 - x^2$, $2l + 2w$, $abc - xyz$.

You have now seen various letter-and-number combinati each of which boils down to a number when numbers are stituted for the letters. A lazy man's way of referring to s combinations is to call them ***expressions.***

EXERCISES

1. In the following list, state which expressions are m mials, which are binomials, and which are neither.

(*a*) $\frac{3}{4}m$, (*b*) $x^2 + y^2$, (*c*) $3p^2q - 4q^2p$, (*d*) $m + n$
(*e*) $23ab/c$, (*f*) $3 - k + t$, (*g*) $u + v + w + z$, (*h*) $def - rs$

2. Make up a list of twenty different binomials, using b literal and ordinary numbers.

In a product like $4xy$, the numbers 4, x, and y are ca ***factors.*** We say that 4 is a ***numerical factor,*** and x and y ***literal factors.*** In the same way, the factors of πr^2 are π, r, an The product $6x$ has two factors, 6 and x. We often call on these factors the ***coefficient*** of the other one. Thus, 6 is coefficient of x; x is also the coefficient of 6. In $3b$, what is fficient of b? What is the coefficient of y in xy? What is the fficient of x in xy? What is the coefficient of m in m?

If, in a product, you select one or more factors, you may ll the product of the remaining factors the **coefficient** *of the e or more selected.*

It is usual in a product like $4x$ to think of the ordinary number the coefficient of the literal number, that is, to think of 4 as coefficient of x. In a given problem, it is usually clear which t of a term is to be regarded as the coefficient.

amples. In $7ab^2$, the coefficient of b^2 is $7a$, the coefficient of s $7b^2$, and the coefficient of ab^2 is 7. Similarly, in $9xyz$, the fficient of z is $9xy$, the coefficient of yz is $9x$, and that of xyz . The ordinary number (for example, 7 in the first case and the second) is sometimes referred to as ***the coefficient of*** ***e term.***

EXERCISES FOR ORAL ANSWERS

1. Name the coefficient of a in each of the following terms:

$3a$, $8ab$, $\frac{3}{4}ax$, $abcd$, $11axy$, $4ac^3$, ma, $25x^2ab$, $8a/b$,
$0.5ar$, $6a/2$, a.

2. Give an example of a monomial involving the letter x. me the coefficient of x in this monomial.

3. In the formula πr^2h, name the coefficient of r^2 and that h.

4. How many terms has the expression $3x^2 - 5xy + y^2$? What he coefficient of x^2 in the first term? Of x in the second term? xy in the second term? Of y^2 in the third term?

If two terms have the same literal factor or factors, we call them ***e terms.*** The terms $3b$, $7b$, and b are like terms; so also are y, $8x^2y$, and $12x^2y$. The terms $4a$ and $3b$ are *not* like since the ral factors are different.

EXERCISES

1. Write 5 terms like $4m$.

2. Write 3 terms like x^2y and 4 terms like $0.3abc$.

3. Select those of the following terms which are like:
$3xy$, $5x$, $7y$, xy, $6x^2y$, $11xy$, $4xy^2$, $\frac{3}{4}xy$, $0.2xy$, $8yx$, xyz.

4. Write 10 monomials involving the letter k. For ea monomial select the coefficient of k.

5. Write an expression with 2 terms, and one with 3 ter

6. State the number of like terms in each of the followi expressions:

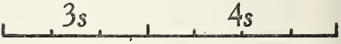
(*a*) $7a - 3b + 2a$, (*b*) $1.8xy + 2.3yx - 1.3xyz$,
(*c*) $a^2 - 6a^2b - 5a^2b^2$.

7. The terms $2x$ and x^2 have a race when x goes from 0 to They both start from scratch. Which takes the lead at fir For what value of x does the other overtake it? Which wi the race?

How to Add Like Terms

We noted on page 64 that the perimeter of an equilate triangle is $3s$. In other words,

$$s + s + s = 3s.$$



It follows that

$$\begin{aligned} &3s \quad + \quad 4s \\ =\ &s + s + s + s + s + s + s \\ =\ &7s. \end{aligned}$$

ilarly,

$$\begin{aligned} 3x^2 + 4x^2 &= 7x^2, \\ 3lw + 4lw &= 7lw, \\ 5x + 8x &= 13x, \\ 2a + 4a + 9a &= 15a, \text{ and so on.} \end{aligned}$$

Notice that in each addition, the sum has the same literal t as each of the added terms; that is, the sum of like terms term like those added. Moreover, the coefficient of the sum he sum of the coefficients of the like terms.

> ***The sum of like terms is another like term whose coefficient is found by adding those of the given terms.***

en groups of like terms occur in an expression, we "melt" all hem into a single term. This is sometimes called ***collecting e terms.***

mple. Add: ab, $5ab$, $7ab$.

The sum is a like term so its literal part is ab.

The coefficient of the sum is $1 + 5 + 7 = 13$.

Therefore,

$$ab + 5ab + 7ab = 13ab.$$

$$\begin{array}{r} ab \\ 5ab \\ \underline{7ab} \\ 13ab \end{array}$$

work can be arranged horizontally, as above, or vertically t the right of the page.

EXERCISES

d the following sets of like terms.

1. $3x$, $5x$, $12x$.

2. ab, $4ab$, $20ab$.

3. $9x^2y$, $7x^2y$, $14x^2y$.

4. k^3, $6k^3$, $3k^3$, $5k^3$.

5. $20pqr$, $18pqr$.

6. $9.5z^3$, $2.5z^3$.

7. $\frac{1}{4}s$, $\frac{3}{4}s$.

8. $0.3mn$, $1.2mn$, $2.7mn$.

9. $4y^2$, $3.2y^2$, $1.8y^2$.

10. $\frac{1}{2}r$, $\frac{3}{4}r$.

11. $\frac{9}{4}u^2v^2$, $\frac{2}{3}u^2v^2$. **12.** $6a$, $3.5a$, $\frac{1}{4}a$.

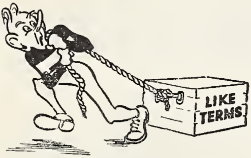
13. The sides of an equilateral triangle are each $2s$. Find perimeter.

14. What is the perimeter of a tablecloth whose length is inches and whose width is $3x$ inches?

15. The sides of a triangle are $4a$ in., $7a$ in., and $10a$ i respectively. Find its perimeter.

16. How many rods of fence would be needed to go aroun square field whose sides are $40.5m$ rods long?

17. Find the sum of the areas of three circular trays w radii r, $2r$, and $3r$ inches, respectively. Compare your ansv with the area of a single tray with radius $6r$ inches.



How to Subtract Like Terms

When we write $x - y$, we mean that the number y is to subtracted from the number x; also, $5m - 3m$ means t difference between $5m$ and $3m$. Can you suggest what t difference is?

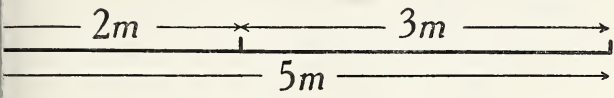
* * *

We can get some help by referring the problem to arithmet Every problem in subtraction can be solved by changing it in a problem in addition. Thus, $5 - 3 = 2$ since $3 + 2 = 5$; other words, the difference between 5 and 3 is the number whi must be added to 3 to yield 5. Did you ever notice how m clerks in stores use this idea to make change? They *a* enough change to the value of the purchase to equal the val of the money given to them. In the same way, we have

$$3m + 2m = 5m,$$

that $5m - 3m = 2m.$

$$\begin{array}{r} 5m \\ \underline{3m} \\ 2m \end{array}$$



Recall that the usual way of checking the accuracy of subction is to add the remainder to the number above it in the tical arrangement; if the work is correct, the sum equals the number.

You can readily see that

$$11xy - 7xy = 4xy,$$

l $$9x^2 - 8x^2 = x^2, \quad \text{etc.}$$

e coefficient of the remainder is found by ***subtracting the efficients*** of the like terms. Make up some more examples l satisfy yourself about this rule:

> ***The difference of like terms is another like term whose coefficient is found by subtracting those of the given terms.***

ample. Subtract $4ab^2$ from $17ab^2$.

The common literal factor is ab^2, and the difference of the coefficients is $17 - 4 = 13$. The result may be written

$$17ab^2 - 4ab^2 = 13ab^2,$$

or in a column as shown at the right.

$$\begin{array}{r} 17ab^2 \\ \underline{4ab^2} \\ 13ab^2 \end{array}$$

EXERCISES

Perform the subtractions indicated in the following. Do as ny as you can orally.

1. $10x - 6x.$

2. $24y^2 - 17y^2.$

3. $7ab - 6ab.$

4. $8m^2n - 5m^2n.$

5. $13klm - 9klm$. **6.** $45a^3 - 32a^3$.

7. $8.5t - 4.4t$. **8.** $0.46xy - 0.38xy$.

9. $\frac{9}{4}p^2q^2 - \frac{5}{4}p^2q^2$. **10.** $\frac{1}{3}s - \frac{1}{4}s$.

Simplify the following expressions by collecting like ter

11. $8k + 12k - 7k + 4k - k$.

12. $a^2 + 4a^2 + 8a^2 - 13a^2$.

13. $10xyz - 4xyz + 3xyz - xyz$.

14. $12.3a + 4.5a - 8.9a$.

15. $\frac{3}{4}mn - 0.5mn + \frac{3}{8}mn$.

16. Is $8 \times 5 + 4 \times 5$ equal to 12×5? Give your ans without multiplying out the terms.

17. Give oral answers for the following differences:

(i) $98 \times 5 - 96 \times 5$, (ii) $1278 \times 9 - 1277 \times 9$.

18. The perimeter of a triangle is $14s$ in. and two of its si are $3s$ in. and $5s$ in., respectively. Find the third side.

19. The length of a rectangle is $7.5x$ feet and its perimete $26x$ feet. Find its width.

Substitution

It's about time we reviewed an important process wh we shall use a great deal. You have been reminded alre that ***the letters of algebra represent numbers.*** If, i term like $5xy^2$, we know the particular numbers that x an stand for, we can find the numerical value of $5xy^2$. This placing of letters by their numerical values is called ***subs***

tion. You have done some substituting in using the formulas Chapter II.

Substitution is one method of checking the accuracy of your rk. If you add $8a$ and $5a$ to get $13a$, your answer holds no tter what number a stands for. Let us give a some value, 2; the addition becomes

$$8a + 5a = 8 \times 2 + 5 \times 2 = 16 + 10 = 26,$$

d $13a = 13 \times 2 = 26$, which provides a check.

> *When substituting, be careful to keep in mind the* ***meanings*** *of the symbols.*

ample 1. If $a = 2$ and $b = 5$,

then $3ab = 3 \times 2 \times 5 = 30,$

and $2b^2 = 2 \times 5 \times 5 = 50,$

and $a^2 + b^2 = 2 \times 2 + 5 \times 5 = 29.$

ample 2. Add $6m^2n$ and $9m^2n$, and check the work when $m = 2$ and $n = 3$. The vertical arrangement of the work is convenient.

$$\begin{array}{rcrcl} & & \text{Check: } m = 2, n = 3 & & \\ \hline 6m^2n & & 6 \times 2 \times 2 \times 3 & = & 72 \\ 9m^2n & & 9 \times 2 \times 2 \times 3 & = & 108 \\ \hline 15m^2n & & 15 \times 2 \times 2 \times 3 & = & 180 \end{array}$$

EXERCISES

If $x = 1$, $y = 2$, $z = 3$, find the values of the following exssions:

1. $x^2 + y^2$.
2. $z^2 - y^2$.
3. $xy + yz - zx$.
4. $3x + 2y$.
5. $5z - 7$.
6. $4y - 3x$.
7. $x^3 + 5x$.
8. $y^3 + xyz$.
9. $3xy^2 - 2x^2y$.
10. $\frac{1}{2}y^2 + \frac{1}{3}z^2$.
11. $0.4x + 0.6$.
12. $0.3xyz + 0.5y^2$.
13. $12y^2z - 10x^2y$.
14. $4x^3 + 27$.
15. $\frac{1}{4}y^2 + \frac{1}{9}z^2$.

16. $3.5x + 1.4y + 0.3$. **17.** $2x^2y^2 + 3y^2z^2 + 4z^2x^2$.

18. $x^3 + y^3 + z^3 - 3xyz$.

19. If $a = 2.5$, $b = 1.4$, and $c = 3.2$, find the value $2a + 3b - c$.

20. If $m = 3/4$ and $n = 1/8$, find the value of $2m^2 + 7n$.

21. Add $8p^2q$ and $1.5p^2q$. Check when $p = 1$, $q = 2$.

22. Subtract $12x^3$ from $15x^3$. Check when $x = 2$.

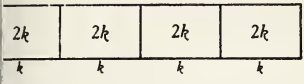
23. The area of a sphere is given by $4\pi r^2$. Find the area a rubber ball with radius 3.5 in. (Take $\pi = 22/7$.)

24. The volume of a cylinder is given by $\pi r^2 h$. If a cyli drical tomato can has $r = 5$ and $h = 4$, find its volume. (Ta $\pi = 3.1$.)

25. Here is one way of getting your number. Consider yo surname as a formula in which the first consonant stands for the second consonant stands for 3, and the third for 5, the four for 7, and so on. The first vowel stands for 2, the second for the third for 6, and so on. Also, any letter occurring for t second time has twice its original value, any letter occurring f the third time has three times its original value, and so o Find your number.

About the Product of Two Monomials

Our rule for finding the area of a rectangle gives us inform tion about the product of two monomials. Thus the produ of 2 and $4k$ must equal the area of a rectangle 2 units wide a $4k$ units long. The figure gives us the answer.

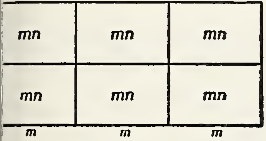


$$2 \times 4k = 8k$$

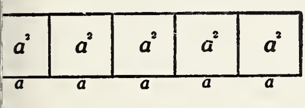
Draw a figure to illustrate the product of b and $4k$.

* * *

Now look at the other figures and you will see that each figure gests the answer to the monomial product beside it.



$$3m \times 2n = 6mn$$



$$a \times 5a = 5a^2$$

Each of these results can be checked by substitution. For mple, if $m = 1$ and $n = 2$, then

$$3m \times 2n = 3 \times 1 \times 2 \times 2 = 12,$$

l $6mn = 6 \times 1 \times 2 = 12$, which checks our answer.

Now try to find for yourself the product of $7x$ and $4y$; of $3a^2$ l $10b^2$; of $2z$ and $9z^2$. State a rule for multiplying two monols.

* * *

The product of two monomials equals the product of their numerical factors times the product of their literal factors.

Example 1. Find the product of $2r$ and $5rs$.

Since the product of the numerical factors is 10, and that of the literal factors is

$$r \times r \times s = r^2s,$$

we have

$$2r \times 5rs = 10r^2s.$$

$$\begin{array}{r} 2r \\ 5rs \\ \hline 10r^2s \end{array}$$

Check: $r = 1$, $s = 2$.

$$\begin{array}{lcl} 2r & = & 2 \times 1 = 2 \\ 5rs & = & 5 \times 1 \times 2 = 10 \\ \hline 10r^2s & = & 10 \times 1 \times 1 \times 2 = 2 \end{array}$$

The product can be arranged in vertical form and checked substitution as shown.

Example 2. Multiply $4a^2b$ by $5ab^2$, and check the work for $a =$ $b = 2$.

The product of the numerical factors is 20, a that of the literal factors is

$$a^2bab^2 = aaabbb = a^3b^3.$$

Hence the answer is $20a^3b^3$. The work can arranged thus:

Check: $a = 1$, $b = 2$.

$$\begin{array}{rcl} 4a^2b & \quad & 4 \times 1 \times 1 \times 2 = 8 \\ 5ab^2 & & 5 \times 1 \times 2 \times 2 = 20 \\ \hline 20a^3b^3 & & 20 \times 1 \times 1 \times 1 \times 2 \times 2 \times 2 = 1 \end{array}$$

EXERCISES

Find the product of the following monomials. Do as ma as you can orally.

1. $4, 3x$.
2. $2m, 4n$.
3. $8, 3y$.
4. $3r, r$.
5. $2z^2, 2z$.
6. $5x, 5x$.
7. $0.5, 6a$.
8. $13xy, 3x^2y^2$.
9. $8ab, 2cd$.
10. $4, 3.14r$.
11. $0.5m, 0.5m^2$.
12. $7k, 9$.
13. pk, kr.
14. $\frac{1}{2}s, \frac{3}{4}st$.
15. $2.4ab, 0.3a$.
16. $\frac{1}{3}hk, \frac{1}{5}mn$.
17. $5, 7, 2a$.
18. $2, 3x, y$.
19. $3, 4x, 2y$.
20. $2a, 3ab, 4bc$.
21. $x^2, 2.3xy, 2y^2$.

22. Find the area of a pane of glass whose length is $5pr$ inches width $2r$ inches. Illustrate your answer with a diagram like se on page 129.

23. Find the area of a table-top whose dimensions, in feet, $2x^2$ and $3xy$. Check your work for $x = 2$, $y = 1$.

How many square inches are contained in sheets of paper h the following dimensions, in inches:

24. 8 by $3x$? **25.** $2k^2m$ by $5m$? **26.** $0.3a$ by $0.8ab$?

27. Find the product of $4ab$, $3.5ac$, bc. Check for $a = 1$, 2, $c = 3$.

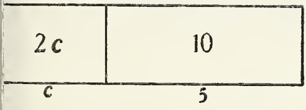
28. Find the volume of a rectangular cardboard carton which x in. long, $\frac{1}{2}y$ in. wide, and $4x$ in. high.

he Product of a Monomial and a Binomial

Mary has a rectangular flower-bed which is c feet long and et wide. What is the area of the flower-bed? If Mary adds additional 5 feet to the length, what is the new length? at area is added? What is the new area of the flower-bed?

* * *

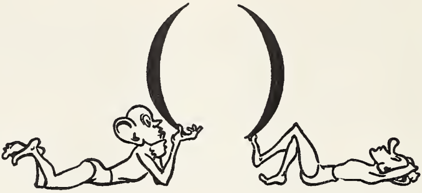
The figure helps us to answer these questions. The original a was $2c$ sq. ft. The new length is $c + 5$ feet and the area led is 10 sq. ft. Since the width is 2 feet and the new length 5 feet, the number of square feet in the new area is $c + 5$ ltiplied by 2.



$$\begin{array}{r} c + 5 \\ 2 \\ \hline 2c + 10 \end{array}$$

If you were to try to indicate this product by $2 \times c + 5$, it ght be taken that you wished to multiply only c by 2 and add

5 to the result. To avoid such confusion, we use ***parenthes*** They look like this:



Thus we write the foregoing product as $2(c + 5)$. This ca attention to the fact that $c + 5$ is to be regarded as ***one*** numb symbol.

Suppose now that, instead of adding 5 feet to the length of t flower-bed, Mary had added b feet to the original length of c fe What would the area have become?

* * *

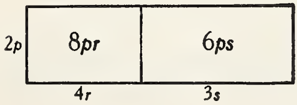
Make diagrams to show that

$$2(c + b) = 2c + 2b,$$

and that

$$k(c + b) = kc + kb.$$

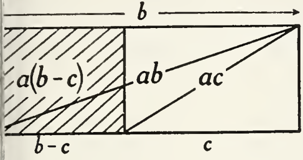
These examples suggest that we can get the correct result multiplying each number within the parentheses by the numb outside and adding the results. It is easy to see that hard examples yield to the same method. For example, the produ $2p(4r + 3s)$ is the area of a rectangle $2p$ units wide and $4r +$ units long. The figure shows that this area equals $8pr + 6p$



Thus we have

$$2p(4r + 3s) = 8pr + 6ps.$$

When the sign within the binomial is negative, the operation rried out in a similar way. The faithful rectangle again gives



us our cue. The product $a(b - c)$ is the area of a rectangle a units wide and $b - c$ units long. This area (shaded) is the difference between the area ab of the large rectangle and the area of the smaller rectangle. Hence,

$$a(b - c) = ab - ac.$$

Try to state a rule for multiplying a binomial by a monomial.

* * *

mple 1. The product	Check: $x = 1, y = 2$.
$7x(2x + 5y)$	$7(2 + 10) = 84$
$= 14x^2 + 35xy$	$14 + 70 = 84$

Make a diagram to illustrate this product.

mple 2. The product	Check: $k=3, m=2, n=1$.
$4m(3k - 5n)$	$8(9-5) = 32$
$= 12mk - 20mn$	$12\times2\times3-20\times2\times1=72-40=32$

To multiply a binomial by a monomial, multiply each term of the binomial by the monomial and add or subtract the products as indicated.

EXERCISES

Find the following products, and make a rectangle to illus- e each:

. $r(s + t)$. **2.** $2m(3m + n)$. **3.** $x(y - z)$.

Find the following products, and check your answers $a = 1, b = 2$.

4. $5a(4a + 2b)$.
5. $3b(2b + 5)$.
6. $9(a + 3b)$.
7. $7a(a^2 + b^2)$.
8. $a(b + a)$.
9. $0.4a(2a - 0.5$

Find the following products.

10. $8(10 + 5)$.
11. $7(9 - 3)$.
12. $4(r - 2s)$.
13. $6(2m + 3n)$.
14. $6a(b - c)$.
15. $x^2(2x - y$
16. $a(x + y + z)$.
17. $1.5k(2m + n)$.
18. $20a^2(s + t$
19. $\frac{1}{4}x^2(\frac{1}{3}a - 2b)$.
20. $3b(2.4m^2 - 0.8m)$.
21. $\frac{1}{2}h(a + b)$.
22. $2a(x + 2y - z)$.
23. $7m(2m + n + 4)$.
24. $2\pi(R^2 - r^2$

25. Find the area of a rug whose dimensions, in feet, are and $5p + 2r$.

26. Find the area of a triangular piece of tin with base inches and altitude $(3x + 5)$ inches.

27. A group of m men on a construction job worked for t month of August except for 4 Sundays and r rainy days. H many days' work did the group do?

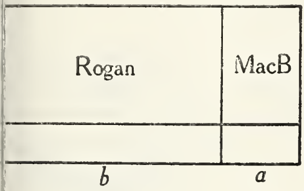
28. Find the total weight, in ounces, of $2n$ bricks each which weighs p pounds and q ounces.

29. If a guinea is worth x dollars and y cents, what is t value, in cents, of g guineas?

The Product of Two Binomials

On page 132, you learned that a binomial may be consider as one number. We use parentheses to indicate this fa Suppose now that you wish to multiply the number $a + b$ b number $c + d$. How would you indicate this in order to id confusion? * * * Yes, it would be neces- y to write it thus: $(a + b)$ $(c + d)$. What objection might raised to writing the product thus: $a + b \times c + d$?

It is to be hoped that by now you suspect a way of finding product of $a + b$ and $c + d$. See if you can figure it out yourself. * * * If you are having difficulty, the following story about Mr. Rogan and Mr. MacBain may help.



Rogan and MacBain bought adjacent lots of land. (See figure.) Both lots were d feet deep with an extra c feet of depth at the front to be set aside for a dway. Rogan's frontage was b feet and MacBain's, a feet. w much of Rogan's lot was used for roadway? How much MacBain's? What was the total depth of the lots? What the total frontage? What was the area of each lot, dis- rding roadway? Now write down the total area of both and satisfy yourself that

$$(a + b)(c + d) = ac + ad + bc + bd.$$

Similar examples lead to the following rule:

To multiply two binomials, multiply the terms of one by the terms of the other, in order, and add the results.

mple: The product

$$(2x + y)\ (x + 3y)$$
$$= 2x^2 + 6xy + xy + 3y^2$$
$$= 2x^2 + 7xy + 3y^2.$$

Check: $x = 1$, $y = 2$.

$$(2 + 2)\ (1 + 6) = 4 \times 7 = 28$$
$$2 \times 1^2 + 7 \times 1 \times 2 + 3 \times 2^2$$
$$= 2 + 14 + 12 = 28$$

Instead of arranging the work horizontally as above, you
set it up vertically, thus:

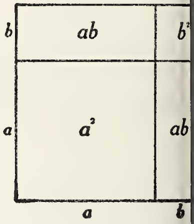
$$\begin{array}{r} x + 3y \\ \underline{2x + y } \\ 2x^2 + 6xy \\ \underline{ xy + 3y^2} \\ 2x^2 + 7xy + 3y^2 \end{array}$$

In this arrangement, you first multi the terms of $x + 3y$ by $2x$, then m tiply them by y, being careful to pl like terms in the same column. adding the columns, you collect like terms.

* * *

A very useful product is that of $a + b$ and $a + b$, or, as it is usually written, $(a + b)^2$. We have

$$\begin{aligned} (a + b)^2 &= (a + b)\,(a + b) \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2. \text{ (See figure.)} \end{aligned}$$



This is worth remembering:

$$\boxed{(a + b)^2 = a^2 + 2ab + b^2.}$$

EXERCISES

Find the products of the following binomials:

1. $(8+7)(5+4)$.
2. $(t+1)(t+1)$.
3. $(x+4)(x+7)$
4. $(x+y)(m+n)$.
5. $(a+2)(a+8)$.
6. $(k+m)(k+m$
7. $(2r+3)(3r+2)$.
8. $(3x+1)(3x+1)$.
9. $(2a+b)(a+3$
10. $(m+2n)(m+2n)$.
11. $(3a+7)^2$.
12. $(2x+5)(x+3$
13. $(x+2y)(3x+2y)$.
14. $(c+2d)(c+3d)$.
15. $(b^2+1)(b^2+2$
16. $(s+5)^2$.
17. $(3a+2b)(2a+5b)$.
18. $(4x+3y)(x+$
19. $(2k+m)^2$.
20. $(x+3y)^2$.
21. $(3a+2b)^2$.
22. Check exercises 9 and 17 when $a = 1$, $b = 2$.

Check exercises 8 and 18 when $x = 2$, $y = 1$.

Draw rectangles to illustrate the products in exercises 6 and 14.

State in words a rule for writing the product $(a + b)(a + b)$ or $(a + b)^2$.

Multiply 74 by 52 by writing the product as $(70+4)(50+2)$, and then multiplying the two binomials.

Changing the Form of an Expression

A good way to gain ease in dealing with literal numbers is to nsform some expressions. You can do this by

(1) multiplying out products (if they appear), and then

(2) collecting like terms.

netimes these operations give you a simpler expression than one you started with. The operations may then be said to *plify the expression.* However, they do not always produce plification. As you learn more about algebra, you will find er methods of simplifying.

For the present, we shall make use of the two processes men- ned above in changing the form of our expressions.

*lways think of the **meanings** of the symbols and operations.*

call that x^2 means x times x, and ab means a times b, etc. ink also about the nature of the operation and the method of cedure. For example, consider the following:

Expression	*Operation*
(*a*) $7x^2 + 2x^2$	Add like terms.
(*b*) $8ab - 3ab$	Subtract like terms.
(*c*) $2x(3x + 7y)$	Multiply the binomial by the monomial.
(*d*) $(5m + 3n)(2m + n)$	Multiply two binomials.

An expression like $2 + 4 \times 5 + 6$ needs some comment. A we to multiply and then add, or add first and then multipl According to our definition, 4×5 is a term. This term cann be added to the other terms until it is multiplied out. For th reason, we shall agree that in situations of this kind

multiplication precedes addition and subtraction.

Study the operations indicated in the next two expressions.

Expression	*Operation*
(*e*) $4a \times 7b + 3 \times 2ab$	Multiply the monomials and add the products.
(*f*) $2r(5p + 2q) - 3r(p + q)$	Multiply the binomials by the monomials and subtract the products.

Now note carefully the solutions of the following two example the arrangement of the work on the page is important.

Example 1. The expression	Check: $m = 1,\ n = 2.$
$3m \times n + 4 \times 8mn - 20mn$	$3 \times 2 + 4 \times 8 \times 1 \times 2 - 20 \times 1 \times$ $= 6 + 64 - 40 = 30$
$= \quad 3mn + 32mn \quad - 20mn$	
$= \quad 35mn - 20mn \quad = 15mn$	$15 \times 1 \times 2 = 30$

Example 2. Simplify $2r(5p + 2q) - 3r(p + q)$.

The work can be arranged thus:

$$\begin{array}{lcl} 2r(5p + 2q) & = & 10pr + 4qr \\ \underline{3r(p + q)} & = & \underline{3pr + 3qr} \\ \text{Subtract:} & & 7pr + qr \end{array}$$

EXERCISE FOR ORAL ANSWERS

State the meaning of each of the following symbols:

ab, (b) m^2, (c) $2m$, (d) xyz,
x^3, (f) $3x$, (g) a^3b^3, (h) $(a + b)$.

State the operation indicated in each of the following expres-
ns:

$3(a + b)$, (b) $4x^2 + 3x^2 - x^2$, (c) $2x(a + 4b)$,
$(a + 3b)(a + b)$, (e) $4xy \times 7x$, (f) $15xy + 3xy$,
$3a \times 2b$, (h) $3a + 2b$, (i) $(3a)(2b)$.

EXERCISES

Simplify the following expressions by multiplying out pro-
ts (where necessary) and collecting like terms.

$5 \times 4a + 2 \times 3a$. **2.** $7ab + 3ab$.
$14x^2 + 8x^2 - 12x^2$. **4.** $4(2m + n) + 5m$.
$5x \times 3y - 2x \times 4y$. **6.** $17xyz - 4x \times 4yz$.
$x(y + z) \times 4xy$. **8.** $2xy^2 + 7y^2x$.
$4ab \times 6bc - 7a \times 3b^2c$. **10.** $8xyz - 3yzx + zxy$.
$2m(3m + 7n) - 6m^2$. **12.** $8(4x^2y) - 4x^2(7y)$.
$(5m + 3n)(2m + n)$. **14.** $(7r + s)(2r + 3s)$.
$b^2 + b(b + c)$. **16.** $7(2a) + 5(4a)$.
$r(5p + 3q) + 2r(p + q)$. **18.** $2x(x + 3y) - x(x + y)$.
$5m(3a + 2b) - 3m(a + 3b)$. **20.** $(a + b)^2 - 2ab$.
$(x + 2y)^2 + x(x + y)$. **22.** $(m + n)^2 - m^2$.

Check Ex. 5 when $x = 1$, $y = 2$.

Check Ex. 19 when $m = 1$, $a = 0$, $b = 2$. Then check the same exercise for $m = 0$, $a = 1$, $b = 2$. Do you think that 0 is a good number to use in checking?

One blackboard slate measures $7x$ in. by $4y$ in., and another measures $2x$ in. by y in. Find the sum and the difference of their areas.

26. The base of a triangle is $3a$ in. and its altitude is $4b$ in. second triangle has base $5b$ in. and altitude $3a$ in. Find sum of their areas.

Division by a Monomia

From arithmetic, we know that the problem of dividi 6 by 2 is that of finding the number which, when multiplied 2, will give 6. Thus, we see that

$$6/2 = 3, \qquad \text{because } 2 \times 3 = 6,$$

and
$$24/6 = 4, \qquad \text{because } 4 \times 6 = 24.$$

In the same way, we have

$$\frac{6a}{2} = 3a, \qquad \text{because } 2 \times 3a = 6a,$$

and
$$\frac{ab}{a} = b, \qquad \text{because } a \times b = ab,$$

and
$$\frac{12mn}{6m} = 2n, \qquad \text{because } 6m \times 2n = 12mn.$$

Since $a^2 = a \times a$, it follows that $a^2/a = a$; also, $a^3/a =$ Now study the following additional examples:

$$(1)\ \frac{14abc}{7ab} = 2c; \qquad (2)\ \frac{8xyz}{x} = 8yz; \qquad (3)\ \frac{32r^2s}{8rs} = 4r.$$

Can you find a method of arriving at these quotients? * *
The examples suggest that we should

divide factors of the denominator into factors of the numerator, where possible.

course, numerical factors divide into numerical factors, and ral factors into literal; then the product of the quotients the numerator gives the result. For instance, we have

$$\frac{\overset{m \cdot 1}{5\cancel{m^2}\cancel{n}}}{\cancel{m}\,\cancel{n}} = 5 \times m \times 1 = 5m,$$

ce the factors m and n of the denominator divide, respecely, into m^2 and n in the numerator.

Another way of obtaining the foregoing results is to make use a basic principle for dealing with fractions. Here it is.

The terms of a fraction may be divided by the same number without changing the value of the fraction.

Note. The numerator and denominator of a fraction are led its ***terms***. This is a different use of the word *terms* from t given on page 118 in connection with the terms of an pression.

If we make use of this basic principle, we can work the last mple by dividing the terms of the fraction by m and n, thus:

$$\frac{\overset{m \cdot 1}{5\cancel{m^2}\cancel{n}}}{\underset{1 \cdot 1}{\cancel{m}\,\cancel{n}}} = \frac{5 \times m \times 1}{1 \times 1} = 5m.$$

her method gives

$2rs/2s = r$; $9a^2/3a = 3a$; $4x^2y/2xy = 2x$; and so on.

You can apply the same reasoning to the division of $(2x + 4y)$ 2.

have $\dfrac{2x+4y}{2} = x + 2y$, since $2(x + 2y) = 2x + 4y$.

ain, $\dfrac{ab - ac}{a} = b - c$, since $a(b - c) = ab - ac$.

> ***To divide a sum or difference by a monomial, divide the terms separately and add or subtract the quotients as indicated.***

Examples. (1) $\dfrac{2ab + 8ac}{2a} = b + 4c$

(2) $\dfrac{3x^2 - 12xy}{3x} = x - 4y$

EXERCISES

(Do the first nine orally.)

Divide:

1. $6x$ by 3.
2. $10ab$ by 2.
3. $3xy$ by x.
4. $5m$ by m.
5. 8×7 by 7.
6. n^2 by n.
7. $2x^2y$ by $2x$.
8. b^3 by b.
9. r^3 by r^2.

Find the following quotients.

10. $6mn \div mn$.
11. $\pi r^2 \div r$.
12. $25a^2b^2 \div 5ab$.
13. $24r^3s^2 \div 6r^2s^2$.
14. $0.4s^2/4s$.
15. $3t^2/0.3t$.
16. $4abcd/2ad$.
17. $pq \div p$.
18. $8x^3 \div 8x$.
19. $\frac{2}{3}mn \div 2m$.
20. $7a/7a$.
21. $2.4z/3$.
22. $(bm + cm)/m$.
23. $xyz \div x$.
24. $(Km - Kn)/$
25. $(6m + 4n)/2$.
26. $(5x + 10y)/5$.
27. $(5x^2 - 9x)/x$.

28. The area of a sheet of paper is $21x^2y$ sq. ft. and its lengt is $7xy$ ft. Find its width.

29. The opening for a barn door has an area of $15m^3 - 10m$ sq. ft. and its width is $5m^2$ feet. Find its length.

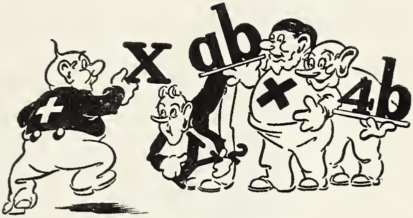
30. A company pays a total of $14x^2 - 7x$ dollars in wages ever week. If there are $7x$ men on the payroll, find the averag weekly wage per man.

31. How many times is the area of a square of side s include in that of a rectangle $3s$ by $4s$?

32. The length of a bookshelf is $3a^2 - 12ab$ inches. How man books each $3a$ inches thick can be placed on it?

Things to Keep in Mind

You have met a number of different kinds of problems in chapter. The exercises which you have done were selected s to give you practice in one new skill at a time. However, real test of mastery is provided by mixed sets of exercises ch involve all the ideas and skills of the chapter. Such a set now be provided for you. Before beginning you will be l advised to review the ideas and methods so as to be sure t you have the necessary tools at your disposal. Recall:



Combined operations

(1) the use of letters to stand for numbers;
(2) the meanings of symbols like ab and x^2;
(3) the definitions of ***literal numbers, monomial, binomial, coefficient, term, like terms, terms of a fraction***;
(4) the rules for adding and subtracting like terms;
(5) the method of substituting;
(6) the rules for multiplying monomials and binomials;
(7) the methods of changing and simplifying expressions;
(8) the order of operations;
(9) the rules for dividing by a monomial.

You should find the Exercises marked *A* quite easy. Some hose in the *B* part will offer more resistance, and the *C*'s will a real test of your progress in algebra.

EXERCISES ON CHAPTER III

A

1. What is the coefficient of x in each of the following term

$$3x,\ 4ax,\ \tfrac{3}{4}c^2x,\ yx,\ xs,\ 0.6a^2bx,\ x?$$

2. What is a *term*? Give the number of terms in each the following expressions:

$$(a)\ 3x^2yz,\ (b)\ 3x^2 + yz,\ (c)\ 3 - x^2 + yz,\ (d)\ 3x^2/yz.$$

Add:

3. $\begin{array}{l} 11rs \\ 0.3rs \\ \hline \end{array}$

4. $\begin{array}{l} 6x \\ 3x \\ \hline \end{array}$

5. $\begin{array}{l} 0.06t^3 \\ 0.21t^3 \\ \hline \end{array}$

6. $\begin{array}{r} 2a^2b \\ 7a^2b \\ a^2b \\ \hline \end{array}$

7. $\begin{array}{l} \frac{1}{4}a \\ \frac{1}{2}a \\ \hline \end{array}$

8. $\begin{array}{l} 0.4km \\ 1.5km \\ 2.6km \\ \hline \end{array}$

Subtract:

9. $\begin{array}{r} 7x^2 \\ 5x^2 \\ \hline \end{array}$

10. $\begin{array}{r} 8pq \\ 7pq \\ \hline \end{array}$

11. $\begin{array}{l} 7r \\ 7r \\ \hline \end{array}$

12. $\begin{array}{r} 8lw \\ 0.5lw \\ \hline \end{array}$

13. $\begin{array}{r} \frac{3}{4}x^2y \\ 0.25x^2y \\ \hline \end{array}$

14. $\begin{array}{r} 18a + 7 \\ 14a + 2 \\ \hline \end{array}$

Multiply:

15. $\begin{array}{r} 6k \\ 8 \\ \hline \end{array}$

16. $\begin{array}{r} 4ab \\ 2a \\ \hline \end{array}$

17. $\begin{array}{l} xy^2 \\ x^2y \\ \hline \end{array}$

18. $\begin{array}{l} 0.3m \\ 0.5m^2 \\ \hline \end{array}$

19. $\begin{array}{l} \frac{3}{4}x \\ \frac{3}{4}x \\ \hline \end{array}$

20. $\begin{array}{r} 2r + 5s \\ 3 \\ \hline \end{array}$

21. $\begin{array}{l} a + 7 \\ a + 2 \\ \hline \end{array}$

22. $\begin{array}{l} x + y \\ x + y \\ \hline \end{array}$

23. $\begin{array}{r} c + 3d \\ 2c + d \\ \hline \end{array}$

form the divisions:

4. $3x^2/3x$. **25.** $1.5abc/0.3bc$. **26.** $12mn/6n$.

7. $(4m^2+4n^2)/4$. **28.** $(2x^2-4xy)\div 2x$. **29.** $(12r^2+4r)\div 4r$.

$n = 1$, $n = 2$, find the value of each of the following:

0. $6m^2 - n^2$. **31.** $m^3 + n^3 - 3mn$.

B

2. If $a = 2$, $b = 3$, $c = 4$, find the value of

$$a^2b^2 + b^2c^2 + c^2a^2 - 10abc.$$

3. Subtract:

$$\begin{array}{r} 3x^2 + 8xy + 5y^2 \\ 2x^2 + 3xy + y^2 \\ \hline \end{array}$$

Verify your answer by addition.

implify the following:

4. $8xy + 14xy - 20xy + 5xy - 7xy$.

5. $2r(r^2 + 4s) - 2r^3$.

6. $8x^2y^2 + (3x^2)(5y^2) - 10x^2y^2$.

7. $(7a + 4b)(4a + 7b) - 65ab$.

8. $(a + b)^2 + (b + c)^2 - 2ab - 2bc$.

9. $5m + 6m^2n/2mn - 8m$.

0. $(5s + r)(3s + 2r) + 6r^2$.

1. $8a(2a + b) - 4(4a^2 + ab)$.

2. $(7x^2y + 3xy^2 - xy) \div xy$.

3. Multiply $(4x + 3y)$ by $(x + y)$. Check your work when

$$x = 1, y = 2.$$

4. Find the area of a rectangular cement path whose dimenns, in yards, are $2x$ by $3a + 2b$. Illustrate your answer with iagram like that on page 132.

5. Divide $8m^2n^2 - 2mn$ by $2mn$. Check your work when

$$m = 1, n = 3.$$

46. The area of a garage door is given by $3x^3 - 6x$ sq. ft. a its length is $3x$ ft. Find its width. Check your work for $x =$

C

47. The edge of a cubical cardboard carton is $3s^2$ inches. Fi (*a*) the area of its surface and (*b*) its volume.

48. The area of the outer surface of a tin can is given $2\pi r(h+r)$, where h is the height and r, the radius of its en Find the outer surface of a can whose height is 8 in. and radi 2 in. (Take $\pi = 3.14$.)

49. Find the value of r^3-2r^2+5r+4 when (*a*) $r=1$, (*b*) $r=$

50. Find the value of x^3-3x^2+3x-1 when (*a*) $x=1$, (*b*) $x=$

51. Multiply $(a + b + c)(a + b + c)$. State a rule for findi the product $(a + b + c)^2$.

52. Draw a diagram to illustrate the product $(5x+4y)(3x+5$ Use your diagram to find the product of the two binomials.

53. Simplify: $(a + b)^2 + 2a(4a + 5b) - (3a + b)(3a + b)$

54. If $m = 1$, $n = 2$, $r = 3$, find the value of

$$0.3m^3 + 1.2n^3 + 0.1r^3 - 1.5mnr.$$

55. Simplify:

$$\frac{12x^2 + 6xy + 24x}{6x} - \frac{10xy + 5y^2 + 20y}{5y}.$$

56. The area of a triangular opening is $6s^3 - 3s^2 + 9s$ sq. and its altitude is $3s$ ft. Find its base.

RECREATIONS

If the letters of the word *pepper* were part of an algebr formula, we would abbreviate it to $\mathrm{p^3e^2r}$. How would y abbreviate *minimum*? Now guess what words the followi abbreviations came from:

(*a*) $\mathrm{n^2i^3s^2pg}$; (*b*) $\mathrm{mi^4s^4p^2}$; (*c*) $\mathrm{m^3a^2l}$; (*d*) $\mathrm{r^2e^3m^2b}$.

Try to decode the following message:

$\mathrm{m^2a^2t^2heics}$ $\mathrm{as^4it}$ $\mathrm{s^2c^2ie^2n}$ $\mathrm{pr^2oges^2}$.

CHAPTER IV

THE EQUATION AND ITS USES

I'm very well acquainted too with matters mathematical,
I understand equations, both the simple and quadratical."

W. S. Gilbert

What is an Equation?

Mrs. Birdwhistle was very enthusiastic about the results of Music Festival sponsored by the Ladies' Aid. "We gave nty-five dollars, or one-third of the proceeds, to the Red ss," she said. Junior B. was not impressed. "I thought you ld make more than seventy-five dollars," he replied.

Before answering his mother, Junior had rapidly performed operation the origin of which goes back to the remote ages. solved an equation! Yes, Junior made use of the fact that number equalled another and drew a conclusion therefrom. thought as follows:

one-third of the number of dollars raised is twenty-five,
so the total number of dollars raised is seventy-five.

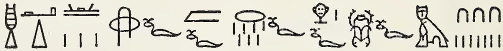
course, Junior may have known a little algebra and worked problem thus:

$$\tfrac{1}{3}\, d = 25, \qquad \text{therefore} \quad d = 75.$$

People have been interested in such problems for thousands years. A famous Egyptian papyrus, written by a scribe, nes, about 1800 B.C., gave directions for getting information n equations. For example, Ahmes showed a method of solv- this problem:

A number plus two-thirds of itself plus one-half of itself plus one-seventh of itself equals 37. Find the number.

This is a little more difficult than the problem Junior B. wor out, but you will soon be able to solve it with ease. Ahmes fo it difficult enough and no wonder! Here's how he wrote equation:



Your experience in mathematics has already involved s use of equations. In arithmetic, for example, statements s as $9 + 5 = 14$ are common. These statements of equality examples of one type of equation. Another type of equatio the formula; $A = lw$, $p = a + b + c$, $A = \pi r^2$, and $V = s^3$ equations. Note that here *the equation is used to state a rela among numbers.*

Formulas may give rise to equations of a different sort f those above. Suppose that the perimeter of a triangle is 24 and that two of its sides are 8 in. and 11 in., respectively. formula for the perimeter is

$$p = a + b + c.$$

By substituting, we get

$$24 = 8 + 11 + c$$

or

$$24 = 19 + c.$$

This last equation involves an unknown number c, and equality holds only when $c = 5$. Such an equation, which is t for only a certain value of the unknown number, is calle ***conditional equation.*** Thus

$$x - 6 = 4$$

is a conditional equation since it is true only when $x = 10$.

Each of the foregoing equations is ***a statement of equality of two numbers.*** In algebra, the word ***equati*** used by itself, usually means a ***conditional equation.*** An portant problem in algebra is that of finding a value of the known number which makes the equation hold true.

To ***solve*** *an equation means to find the value of the unknown number which makes the equation true.*

alue of the unknown which makes the equation true is called ***oot*** of the equation. You can easily see that the equation

$$x + 5 = 30$$

25 for its root.

Conditional equations, or more simply *equations*, make up important part of algebra. You will soon find out that they very useful for attacking problems. For this reason, we must rn how to solve them.

EXERCISE FOR ORAL ANSWERS

ve each of the following equations.

. $n + 7 = 10$. **2.** $n - 2 = 8$. **3.** $x + 4 = 12$.
. $a - 10 = 18$. **5.** $m + 0.5 = 2$. **6.** $k - \frac{3}{4} = 5$.
. $x + 4 = 4$. **8.** $x + 90 = 100$. **9.** $x - 1.5 = 1.5$.
. $12 + m = 13$. **11.** $x - 7 = 2$. **12.** $s + \frac{1}{4} = \frac{1}{2}$.

Before reading the next section, try to explain how you got utions for these equations.

* * *

How to Find Unknown Numbers

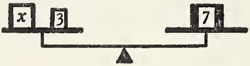
Perhaps you solved Ex. 1 by asking yourself: "What number ded to 7 will give 10?" Also Ex. 2 asks: "What number leaves fter 2 is subtracted from it?" Such questions enable you to upon the unknown numbers quite readily. It was lucky for u that all the equations were simple ones. You would likely ve some difficulty in trying to apply such a method to an uation like

$$3x - 5 = 2x + 6.5.$$

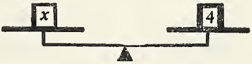
In order to find a way of solving this equation and others l it, we must seek out some basic principles which are useful the purpose. One way of getting at these principles is to thi of an equation as

two numbers in balance.

Imagine that the numbers on each side of the equation repres magnitudes of weights on a scale. Then the weights represent by the left side must balance those represented by the right si Thus the equation $x + 3 = 7$ can be pictured as in the figure.



The object is to get x alone on the left side and nothing b known numbers on the right, and still maintain the balance. W can do this by removing 3 units from each side of the scale. Th gives us the equation $x = 4$ which tells us the value of the u known number. (See figure.)

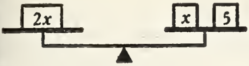


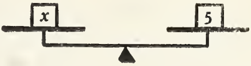
You can remove equal weights from each side of a scale wit out disturbing the balance. This operation gives us a clue to important principle for solving equations:

> *The same number may be* ***subtracted*** *from each side of an equation without disturbing the equality.*

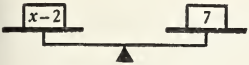
The next figure illustrates the method of solving $2x = x +$ What number is subtracted from each side in this example?

* * *

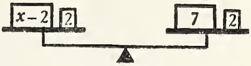




w study the equations represented below.



$$\boxed{x - 2 = 7.}$$



$$\boxed{\begin{aligned} & x - 2 + 2 &= 7 + 2 \\ \text{or}\quad & x + 2 - 2 &= 9 \\ \text{or}\quad & x &= 9. \end{aligned}}$$

ıat has been done to the balanced scales? Is the balance ·cted? What happens when equal weights are ***added*** to each e of the balance? Before reading further, draw from this fact onclusion about the handling of equations.

* * *

> *The same number may be* **added** *to each side of an equation without disturbing the equality.*

Remember that the number must be added to or subtracted m ***both*** sides of the equation. Here are some examples of ıations solved by the use of the two principles of this section.

ample 1. Solve $x + 8 = 17.$

$$\begin{aligned} \text{We have}\quad x + 8 - 8 &= 17 - 8. \quad \text{(Subtract 8)} \\ \text{Therefore}\quad x &= 9. \end{aligned}$$

If 9 is substituted for x in each side of the given equation, get:

$$\begin{aligned} \text{Left side} &= 9 + 8 = 17. \\ \text{Right side} &= 17. \end{aligned}$$

ıis verifies our solution.

Example 2. Solve $3a - 7 = 2a + 20.$

$$\begin{array}{llll} \text{We have} & 3a - 2a - 7 & = 2a - 2a + 20, & \text{(Subtract } 2a \\ \text{whence} & a - 7 & = 20. & \text{(Collect)} \\ \text{Also} & a + 7 - 7 & = 20 + 7. & \text{(Add 7)} \\ \text{Therefore} & a & = 27. & \end{array}$$

By substituting this value for a in the two sides of the giv equation, we get the following verification.

$$\begin{aligned} \text{Left side} &= 81 - 7 = 74. \\ \text{Right side} &= 54 + 20 = 74. \end{aligned}$$

EXERCISES

Solve the following equations for the unknown number. F the first six problems, indicate the principle used as in the exar ples just given. Verify all solutions by substitution.

1. $x+32=60.$ **2.** $a-11=8.$ **3.** $m+18=32.$

4. $x-3=11.$ **5.** $2x=x+6.$ **6.** $3c=2c+5.$

7. $k+0.5=4.5.$ **8.** $r-2=0.$ **9.** $15+y=23.$

10. $15+y=15.$ **11.** $x-3.5=2.4.$ **12.** $s+\frac{1}{3}=\frac{1}{2}$.

13. $10z=9z+3.$ 3 **14.** $13+4n=5n.$ 13 **15.** $3x-7=2x+2$

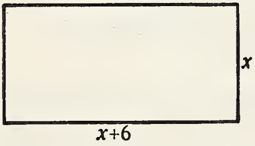
16. $5a+3=4a+12$ 9 **17.** $7z-1=6z+19.$ 20 **18.** $18=x-2.$ 20

19. $5x+7=4x+11.$ **20.** $2.5x-4=1.5x+1.$ **21.** $27+8x=9x-3$

4 5 [illegible]

Two More Principles

Consider a pane of glass $x + 6$ inches long and x inches wid Suppose that its perimeter is 60 inches. Then, we have



$$\begin{array}{lrl} & x+x+x+6+x+6 & = 60, \\ \text{or} & 4x+12 & = 60, \\ \text{or} & 4x & = 48. \\ & & \text{(Subtract 1} \end{array}$$

We can't find x in this equation by adding a number to, or sul tracting a number from, each side. However, it is easy to se t, if four times x is 48, then x must be 12. Hence the pane 2 inches wide and 18 inches long.

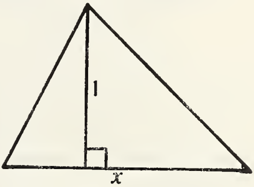
As another example, take a triangle with height 1 unit, base x ts, and area 20 units. Our formula for the area of a triangle is

$$A = \tfrac{1}{2}bh.$$

Substituting, we get

$$20 = \tfrac{1}{2}x \times 1,$$

or $20 = \tfrac{1}{2}x.$



re again the principles of the last section are not useful for ling x. It is easily seen, however, that if one-half of x is 20, n x must be 40, and the base of the triangle is 40 units.

EXERCISES FOR ORAL ANSWERS

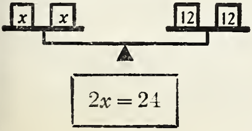
State the value of the unknown in each of the following ıations.

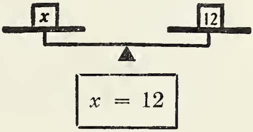
$3x = 6.$ **2.** $12a = 36.$ **3.** $\tfrac{1}{2}k = 10.$
$2x = 2.$ **5.** $7y = 28.$ **6.** $2x = 5.$
$\tfrac{1}{3}m = 4.$ 12 **8.** $5x = 20.$ **9.** $7s = 0.$ 6
$25a = 200.$ 4 **11.** $\tfrac{1}{5}n = 5.$ **12.** $9x = 81.$ 9

Before reading further, see if you can state two basic prinles that can be used to solve equations like these.

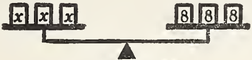
* * *

Let us once more think of the sides of an equation as repreting weights on the two pans of a balanced scale. Now it is dent that, if we take one-half of the weights from each pan, ance is preserved.

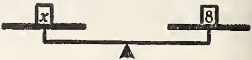




Similarly, if $\frac{1}{3}$ of the weights is left in each pan, the sc remains balanced.



$$\boxed{3x = 24}$$

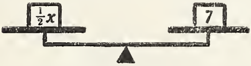


$$\boxed{x = 8}$$

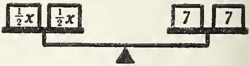
Further examples lead us to the conclusion that the balanc not disturbed when equal fractions of the weights are left on e side. This fact suggests the following principle for handli equations:

> *Each side of an equation may be* ***divided*** *by the same number without destroying the equality.*

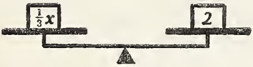
Can you now guess another principle? * * * Lo at the scales again.



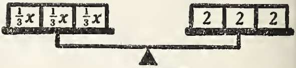
$$\boxed{\tfrac{1}{2}x = 7}$$



$$\boxed{x = 14}$$



$$\boxed{\tfrac{1}{3}x = 2}$$



$$\boxed{x = 6}$$

It is clear that the balance is not disturbed if the weights both sides are doubled, tripled, quadrupled, etc. Applying t fact to an equation, we are led to another useful principle.

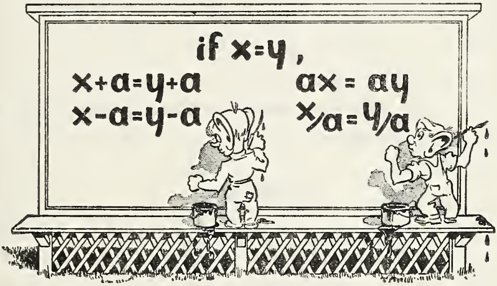
> *Each side of an equation may be* ***multiplied*** *by the same number without destroying the equality.*

You now have four basic principles for solving equations. sum up:

The equality of the two sides of an equation
is preserved if we

(1) ***add*** the same number to each side,
(2) ***subtract*** the same number from each side,
(3) ***multiply*** each side by the same number,
(4) ***divide*** each side by the same number.

haps you prefer to remember these principles by the method gested by the Pixie brothers (Paul and Percy) in this figure.



Study the following examples, noting the principle used at h step.

mple 1. Solve $5a - 4 = 7$.

We have $5a + 4 - 4 = 7 + 4$, (Add 4)
or $5a = 11$,
whence $a = 11/5$. (Divide by 5)
Substitute in the given equation and get:
Left side $= 5 \times \frac{11}{5} - 4 = 7$.
Right side $= 7$, which verifies the result.

Example 2. Solve $3x + 8 = 5x + 2.$

We have $3x + 8 - 2 = 5x + 2 - 2,$ (Subtract 2)
or $3x + 6 = 5x.$ (Collect)
Also $3x - 3x + 6 = 5x - 3x,$ (Subtract 3x
whence $6 = 2x$ (Collect)
and $3 = x.$ (Divide by 2

Verify the answer by substitution.

Example 3. Solve $\frac{1}{2}m + 9 = 2m - 6.$

We have $\frac{1}{2}m + 9 + 6 = 2m + 6 - 6,$ (Add 6)
or $\frac{1}{2}m + 15 = 2m.$ (Collect)
Hence $m + 30 = 4m,$ (Multiply by
and $30 = 3m$ (Subtract m)
and $10 = m.$ (Divide by 3)

Verify the solution by substituting 10 for m in the given equati

Observe that in the examples given above the four principl were used to ***get the unknown number by itself as o side of the equation.*** Keep this in mind as you try the f lowing exercises.

EXERCISES

Solve the following exercises, indicating the principle us at each step.

1. $2a = 6.$
2. $0.2a = 6.$
3. $\frac{1}{2}x = 5.$ 10
4. $7x = 5.$
5. $\frac{1}{3}n = 17.$ 51
6. $25x = 75.$
7. $2.5x = 75.$
8. $\frac{1}{5}r = 2.$ 10
9. $0.5m = 4.$
10. $0.1a = 1.$
11. $\frac{3}{4}x = 6.$ 8
12. $\frac{2}{3}a = 10.$ 15
13. $x + 5 = 11.$
14. $P - 3 = 8.$
15. $6 + a = 10.$
16. $x + 3.5 = 5.$
17. $k + \frac{7}{8} = 1.$
18. $b - 0.5 = 0.5.$
19. $8x - 16 = 0.$
20. $6n = 0.$
21. $1.5x = 15.$
22. $100s = 56.$
23. $1\frac{2}{3}x = 10.$ 6
24. $2x + 3x = 5.$
25. $12t - 7t = 3.$
26. $2x + 3 = 7.$
27. $2x - 3 = 7.$
28. $3n - 5 = 0.$
29. $\frac{1}{2}a + 1 = 5.$ 8
30. $\frac{1}{2}a - 1 = 5.$ 12
31. $2m = m + 9.$
32. $\frac{1}{2}c + 3 = c.$ 6
33. $\frac{1}{3}x + 5 = 10.$

$8x - 5 = 1.$ **35.** $4x+5=6x+3.$ **36.** $17a+18=20a.$

$3\frac{1}{7}r = 44.$ 14 **38.** $17a - 18 = 16.$ **39.** $7x - 2 = 5x+1.$

$9x = 6x + 2.$ **41.** $8n + 9 = 13.$ **42.** $4x-3=17+2x.$

$1.5x - 9 = 0.$ **44.** $16 + 45x = 39x + 310.$

$\frac{1}{2}a + 9 = 2a - 6.$ 10 **46.** $\frac{1}{3}x + 20 = x + 4.$ 2

$0.1m + 0.01 = 0.01m + 0.1.$ 14

$1.35y + 2.53y = 27.16.$ 2/

Translating English into Algebra

We have now had some practice in solving equations—that in finding the unknown number which makes the left side ial to the right side of the equation. This places in our hands ery useful tool for solving certain types of problems. For mple, if a problem requires the finding of an unknown nber, we may be able to get it from an equation as in the owing example.

ample. The Dramatics Club of Atomville High School pre- ted "Hilda's Uncle" to a packed auditorium. Every one of 400 seats was sold and the club received $162.50 from the e of tickets. Some days later an argument arose about the nber of adults who had seen the production. Charles, knowing t adults had paid 50 cents each for their tickets while children d only 25 cents for theirs, put an end to the discussion by ding the exact number of adults present. Here's how he did it.

Charles let the number of adults present be x. Then the number of children must have been $400-x$. Since each adult paid 50 cents, the number of cents received from all the adults was $50x$. Similarly, the children contributed $25(400-x)$ cents. This led to the equation:

$$50x + 25(400-x) = 16250.$$

en $$50x + 10000 - 25x = 16250,$$

$$25x + 10000 = 16250. \qquad \text{(Collect)}$$

us $$25x = 6250, \qquad \text{(Subtract 10,000)}$$

d $$x = 250. \qquad \text{(Divide by 25)}$$

> The show was attended by 250 adults and 150 childr
> You can easily verify Charles's answer by showing that t
> ticket sales, under these conditions, would amount
> \$162.50.

This solution hinged on Charles's being able to translate in algebra the English statement: *the number of cents received fr adults plus the number of cents received from children equals* 16,2 This ability to translate English into algebra is very necessa for skill in problem-solving. Some typical translations are list below; study them carefully. For the sake of convenience, shall confine ourselves to phrases and sentences which refer numbers. In each instance, we denote the unknown num by x.

English phrase or sentence	*Algebraic translati*
An unknown number increased by 5	$x + 5$
5 is added to an unknown number	$x + 5$
An unknown number decreased by 5	$x - 5$
An unknown number doubled	$2x$
Half of a certain number	$\frac{1}{2}x$
A certain number tripled and the result increased by 8	$3x + 8$
One quarter of an unknown number is taken and the result decreased by 2	$\frac{1}{4}x - 2$
Three times a certain number equals the number increased by 4	$3x = x + 4$
5 less than twice an unknown number equals 7 more than three times the number	$2x - 5 = 3x +$
A certain number is squared and the result multiplied by 3	$3x^2$
7 times the cube of an unknown number	$7x^3$

EXERCISES FOR ORAL ANSWERS

If x represents an unknown number, translate the following ases and statements into algebra. Which translations are ations and which are expressions?

1. 17 is added to an unknown number.
2. A certain number decreased by 12.
3. A certain number tripled.
4. An unknown number squared.
5. An unknown number is doubled and the result increased 2.
6. An unknown number is increased by 2 and the result is bled.
7. A certain number is squared and the result increased by 2.
8. A certain number quadrupled and the result decreased 7.
9. One-quarter of a certain number is increased by 9.

0. One-half of an unknown number is decreased by 3.

1. 14 is added to twice a certain number.

2. If 8 is added to a certain number, the result is 12.

13. If 20 is subtracted from a certain number, the result is 45.

4. If double an unknown number is increased by 12, the ult is 32.

5. A certain number is doubled when 8 is added.

6. A certain number is tripled when 14 is added.

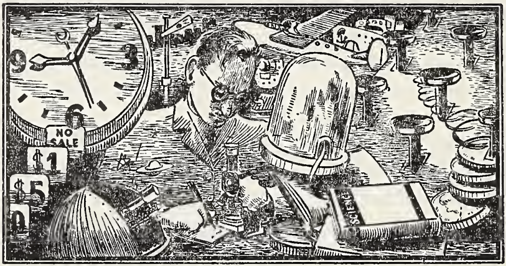
7. If the square of a certain number is increased by 6, the ult is 31.

8. Three times the cube of a certain number equals 24.

Using Equations to Solve Problems

In the last section, you saw how a problem was solved by the of an equation. This problem required the finding of a nber. You have doubtless realized by now that our basic nciples are very useful for finding the unknown numbers in ations. The thought may have occurred to you that this tter of finding unknown numbers is all right in its way, but it has little interest for hard-headed practical folk. If you ha wondered about this, just pause and think about the vast arra of real-life problems that have to do with *counting* and *measurin* Numbers are used to answer all questions of "How many?" an "How much?"

For example, we talk about 5,000 aircraft, 83 people, 40 miles per hour, $5,000,000, 1.3 grams, 2240 volts, 1.5 light-year 18,000 horsepower, and so on. The list is very long. Numbe make contact with life at many points,—in science, in busines in games,—wherever it is necessary to count or measure.



Number problems arise in a wide variety of situations.

The method of handling number problems involves the tran lation of English statements into algebraic equations. Rea again the problem of the last section. Then consider the fo lowing examples.

Example 1. If a certain number is doubled and then 3 is adde to the result, the sum is 33. Find the number.

Let n be the number. The expression $2n + 3$ gives th result of doubling the number and adding 3. Therefor we have

$$2n + 3 = 33,$$

or $$2n = 30. \qquad \text{(Subtract 3)}$$

Hence, $$n = 15. \qquad \text{(Divide by 2)}$$

The number 15 is seen to satisfy the conditions of the problem

WARNING. Always verify your answer by seeing that it satisfies the itions of the problem. ***Substitution in the equation does not verify correctness of your answer.*** Do you see why? * * * Here is the on.

Suppose that we had made a mistake in forming our equation and got $+ 3 = 33$ instead of $2n + 3 = 33$. Then our solution to the equation ld have been $n = 10$. This answer, if substituted in the equation makes left side equal the right side. Yet it does ***not*** satisfy the conditions of the lem. If we were sure that our equation was the correct one for the lem, then substitution in the equation would check our result. Since we not be sure, it is best to go back to the problem itself and see whether answer satisfies the conditions.

mple 2. The length of a rectangular lot is 60 feet more than width, and its perimeter is 300 feet. Find the width.

Let the number of feet in the width be x. Then the length is $x + 60$ feet. Since the length plus the width equals half the perimeter, we have

$$\begin{aligned} & x + x + 60 = 150, & \\ \text{or} \quad & 2x + 60 = 150, & \\ \text{whence} \quad & 2x = 90, & \text{(Why?)} \\ \text{and} \quad & x = 45. & \text{(Why?)} \end{aligned}$$

erefore the width is 45 feet. This makes the length 105 feet the perimeter 300 feet as required.

NOTE. In these examples, n and x stand for numbers. In the first nple, we were looking for a number; in the second, the solution to the lem was a *quantity*, namely 45 feet, and x stood for the *measure* of this ntity.

ample 3. What sum of money invested at 3% per annum yield interest amounting to \$60.90 in one year?

Let the number of dollars be x.

Then the interest is 3/100 of \$$x$ or \$$0.03x$. But we are told that the interest is \$60.90. Therefore, we have

$$.03x = 60.90,$$

$$\text{whence} \qquad x = \frac{60.90}{.03} = 2030. \quad \text{(Why?)}$$

nce the required sum is \$2030. Verify this answer.

Example 4. Elmer has \$6 and saves 30 cents a week. Wilb has \$20 and spends 20 cents a week. In how many weeks w the boys have equal sums of money?

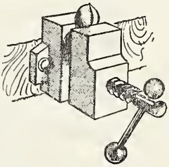
> Let the number of weeks be x. In order to arrive at equation, you might ask yourself the following questions:
>
> How many cents does Elmer save each week?
> How many cents will he have saved at the end of x week
> How many cents had he at the beginning?
> What will his total in cents be at the end of x weeks?
>
> The four answers in succession are 30, $30x$, 600, $600 + 3$ So much for Elmer. Now submit Wilbur to a simil investigation.
>
> How many cents does Wilbur spend each week?
> How many cents will he have spent at the end of x week
> How many cents had he at the beginning?
> How many cents has he left at the end of x weeks?
>
> The four answers in succession are 20, $20x$, 2000, $2000 - 20$ Since we are told that both boys have equal sums at the e of x weeks, we have

$$
\begin{aligned}
& & 600 + 30x &= 2000 - 20x, & \\
&\text{or} & 600 + 30x + 20x &= 2000 + 20x - 20x, & \text{(Why?)} \\
&\text{or} & 600 + 50x &= 2000. & \\
&\text{Therefore} & 50x &= 1400, & \text{(Why?)} \\
&\text{and} & x &= 1400/50 = 28. & \text{(Why?)}
\end{aligned}
$$

Thus in 28 weeks the boys will have equal sums of money. Veri this result by finding the amount of money which each boy h after 28 weeks.

A Challenge

Here follow three mathematical nuts for you to crack. The first has a soft shell and you should be able to handle it without much trouble. The second offers more resistance and is therefore more interesting. The third is a hard-shelled variety which cracks only under considerable pressure. See what you can do with them.



t No. 1. (A Peanut) When a certain number is tripled l then increased by 10, the result is 25. Find the number.

t No. 2. (An English Walnut) The perimeter of an isoses triangle is 28 inches. If the base is 12 inches less than the n of the two equal sides, find the length of one of the equal es.

t No. 3. (A Hickory Nut) An aircraft on patrol leaves base and flies due east at 225 m.p.h relative to the ground. e pilot knows that because of wind his returning speed will cut to 150 m.p.h. If he has enough gasoline for 10 hours, how g can he fly due east and have just enough gasoline to bring n back to his base?

A Suggested Plan of Attack

If you study the foregoing examples, you will see that each blem is solved by a series of somewhat similar steps. Perhaps you noted the need for some definite plan of attack when y tried to crack the three nuts. Did you arrive at a method taking the steps? They can usually be taken in the followi order:

(1) *Read the problem carefully to find out what is needed.* Som times a diagram is helpful in picturing the facts.

(2) *Let some letter, say x, stand for the unknown number, or the measure of the unknown quantity, as the case may be.*

(3) *Using* x, *translate the English statements into algebraic e pressions.*

(4) *Re-read the problem to find what expressions are equal.*

(5) *Form an equation from the equal algebraic expressions.*

(6) *Solve this equation for* x.

(7) *Test whether the answer satisfies the conditions of t problem.*

Each problem of the type you will meet in algebra this ye involves the finding of an unknown number from an equatio The suggested seven steps may be helpful in arriving at a sol tion. You will do well to think about the following commen on these steps.

In step (2), ***never forget that the letter stands for a number.*** F this reason, statements such as "Let the men be x," or "Let x the money," are not satisfactory. Can you improve them? T literal number often stands for the measure of a quantity; e. x years, x tons, $\$x$, x feet, etc. In such cases, *be sure that the prop unit is indicated.*

Steps (3) and (5) are the hardest for beginners. With pra tice, your skill in translation will improve. *Remember that ea problem gives rise to an equality*; you have to write the san number in two different ways. Thus, in Example 1, we obtain two different expressions for 33 and equated them; in Example we found two different expressions for the number of feet in ha the perimeter and equated them, and so on. Make sure that yo are using all the information given.

CAUTION. This last statement applies only to your work in elementary bra. In everyday life, you may not need all the information given you in nection with a problem.

Finally, note the method of verification. Read again the rning which accompanies Example 1. Be sure to substitute r answer in the problem rather than in the equation of p (5).

If you failed to crack one or more of the nuts, don't go back them yet. The following exercises will help you develop your cking powers gradually. The best possible preparation for se exercises is to recall your principles for solving equations. en study the worked examples on pages 160-162, bearing in d the seven suggested steps.

EXERCISES

A

1. What number is 10 greater than x?; 10 less than x?; x less n 10?; one-tenth of x?; 6 more than 3 times x?

2. In order to get x, what must you do to (*a*) $x + 5$? (*b*) $- 5$? (*c*) $5x$? (*d*) $\frac{1}{5}x$? (*e*) $2x + 8$? (*f*) $3x - 2$? (*g*) $\frac{1}{2}x + 7$? (*h*) x?

3. When a number is increased by 1.5, the result is 5.6. Find number, using an equation.

4. When a number is decreased by 0.4, the result is 4.8. Use equation to find the number.

5. If a number is doubled and the result increased by 4, the n is 49. Find the number.

6. If a number is tripled and the result decreased by 35, the nainder is 100. Find the number.

7. Jim's teacher told him that, if his marks in French were ubled and the product decreased by 12, the result would be). What was Jim's mark in French?

8. Fred's father is 3 times as old as Fred was two years ago. e sum of their ages today is 62. How old is Fred?

9. After the membership of a club increased by one-third, was found to have 128 members. How many members we there before the increase?

10. A chain 44 feet long is cut into two pieces so that t smaller piece is 3/8 of the larger. How long is the larger piec

11. A coat is sold at $32.50, which is 30% above the cost pri Find the cost price of the coat.

12. George has 6 times as much money as Charles and t gether they have $8.12. How much has Charles?

13. What sum of money will yield $26.25 in one year, at 3 simple interest?

B

14. Solve Ahmes' problem on page 147.

15. "What part of the day has disappeared, if the time left twice two-thirds of the time passed away."[1] (An old puzzle about the year 300 A.D.)

16. One angle of a triangle is twice the smallest angle and t third angle is 30° more than three times the smallest angle. Fi the angles.

17. One-half of a certain number is 12 more than one-thi of it. Find the number.

18. One side of a rectangle is 8 feet longer than the other a the perimeter is 36 feet. Find the dimensions.

19. A farmer has 300 rods of fence with which he wishes enclose a rectangular field. If one side of the field must be rods, find the other dimension of the field.

20. The perimeter of an isosceles triangle is 21 feet, and i base is 3 feet less than the length of an equal side. Find t length of one of the equal sides.

21. Crack Nut No. 2 on page 163.

22. Find two consecutive whole numbers whose sum is 73.

---

[1] Cajori: *A History of Elementary Mathematics.*

3. The larger of two numbers is 7 more than 5 times the ller, and their sum is 61. Find the numbers.

4. The length of a shelf is 6 times its width and the perimeter 4 inches. Find its dimensions.

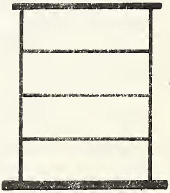
5. Two boys operate a hot-dog stand in the summer. Since n built the stand, it is agreed that he is to receive twice as ch of the profits as his partner, Bill. How should $412.50 be ided?

6. Mary wishes to save enough in the summer to enable her ave 75 cents a week for 40 weeks. Her father promises that will increase whatever she saves by one-third. How much d she save?

7. Harry has $3.90 now and saves $1.40 every week. In how ny weeks will he have saved enough to purchase a bicycle th $51.50?

8. A taxi-cab charges 15 cents for the first quarter-mile and nts for every quarter-mile thereafter. How far can you ride $1.40?

9. The inside height of a china cabinet 9 inches. Jim plans to put 3 shelves, h 1 inch thick, into the cabinet. If the lves are equally spaced, how far apart uld they be? (See figure.)



0. When George telephones Harriet long ance he pays 35 cents for the first 3 min-s and 10 cents for each additional minute. last call cost $1.55. How long did she talk?

1. Jack has $5.00 and saves 70 cents a week. Clarence has .00 and spends 80 cents a week. In how many weeks will y have the same amount?

C

2. A bag contains 27 coins all of which are either quarters dimes. If their total value is $5.55, how many coins of each d are there?

33. Henry is making a bookcase. The inside height is inches, and he wishes to add three 1-inch shelves so that spaces between the shelves get wider as you go from top bottom. If each space is 1 inch wider than the one above find the width of the bottom space.

34. Mr. Flatsole arrives at Square Corners only to find th his bus is not due for 2 hours. He walks along the road to m it. If Flatsole averages 3 miles per hour and the bus avera 33 miles per hour, how long will it be until he meets the bus

35. Find 3 consecutive numbers whose sum is 771.

36. A car leaves Roseburg at 9.00 a.m. and averages 32 m.p At 10.30 a.m., a second car leaves to overtake the first. If second car averages 48 m.p.h., at what time will it be abreas the first car? (Hint: The distance covered equals the aver speed times the time.)[1]

37. Crack Nut No. 3 on page 163.

38. The distance from Toronto to Montreal is about 334 mil At noon, a train sets out from each city bound for the other. the trains average 30 m.p.h. and 50 m.p.h., respectively, at wh time do they pass each other?

39. A grocer has tea at 90 cents per pound and tea at 75 ce per pound. He wishes to make up 50 pounds of a mixture to at 81 cents. How much of each kind of tea should he put in mixture?

40. How many minutes after 4 o'clock will the hands of clock be together?

41. Divide 72 into 3 parts so that the second part is 3 tin the first, and the third is 8 less than twice the second.

---

[1] This statement is an abbreviation for the correct statement: *The number of miles covered is the measure of the average speed times the nu ber of hours.*

RECREATIONS

. *An Old-timer about Ages.* Mary is 24 years old. Mary
ice as old as Anne was when Mary was as old as Anne is
. How old is Anne?

. A bottle and its cork cost $1.05. If the bottle costs $1
than the cork, how much does the bottle cost?

. *What Time is it?* Thirty minutes ago the number of
tes past noon was five times the number of minutes before
o'clock. What time is it?

. A gold miner arrives at a hotel where he engages room and
d for 60 days at $4 a day. His assets consist of three gold
ets worth $4 each, one worth $16, one worth $32, one worth
and one worth $128. The hotel keeper insists on daily pay-
ts and the miner in making change insists on receiving his
ge in nuggets. Show whether he can make his daily pay-
ts for 60 days.

. Show that by using weights of 1, 3, and 9 pounds in one
th scales of a balance, you can weigh any whole number of
ds up to 13. Find what four weights would suffice to weigh
whole number of pounds from 1 to 40.

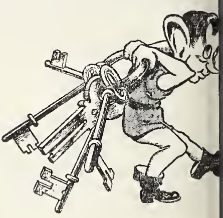
CHAPTER V

THE FORMULA AND ITS USES

> "*One cannot escape the feeling that these mathematical formulae . . . are wiser than we are, wiser even than their discoverers, that we get more out of them than was originally put into them.*"—Hertz.

What is a Formula?

In the **first** section of Chapter III, we used the exam $V = \pi r^2 h$ in order to find out something about formulas. compared them with keys which open doors to the solutions of many problems. Your growing acquaintance with formulas will justify this comparison.



Our use of formulas such as $A = lw$ and $V = \pi r^2 h$ has led us to two important conclusions:

(1) ***The formula is a symbolic statement of a ru*** To express the rule in algebraic symbols makes it c cise and clear. "The letter liberated algebra from slavery of the word."[1]

(2) ***The formula applies to any rectangle, or a cylinder, or any circle, etc.*** The literal numbers able us to talk about things in general rather than ab things in particular.

All formulas have these characteristics. For example,

$$A = \pi r^2$$

is a shorthand way of writing a rule which gives informati about ***any*** circle.

---

[1] Dantzig, T., *Number* (The Macmillan Company, New York, 1939).

There is another aspect of formulas. The formula $A = \pi r^2$ us that, for a given value of r, we can find an A to match. or example, $r = 14$, then $A = (22/7)(14^2) = 616$. Moreover, an't find A until we know r. Thus the formula reminds us *the area of a circle depends on its radius*, and that *the radius nds on the area.* So you may look upon this formula as the of the relation between the area of a circle and its radius. s is an important idea.

formula is a relation between the measures of two or more quantities.

The Use of Formulas

You are probably already convinced that formulas may be ul. To whom? Name some occupations in which formulas used. * * * We might mention carpentry, engi-ing of all kinds, architecture, machine work, contracting, so on.

> "*When it is considered how essential is their use in a vast range of trades and professions—from plumbing to dreadnought building—it is hardly extravagant to say that facility in the working, interpretation, and application of formulae is one of the most important objects at which early mathematical studies can aim.*"
>
> —Sir Percy Nunn

Many practical uses emphasize the convenience of the for-a in everyday life. Did you ever think that sometimes the ect use of a formula is a matter of life and death? In the struction of bridges, dams, and buildings accuracy in using ulas is vital. Here is an interesting example in which you asked to accept the truth of the formula on faith.

> Bill O'Brien, an aircraft pilot, is ordered to make a pa from his base out to sea along a certain course. There strong wind blowing. O'Brien knows that the wind will h him on the outward trip and hinder him on the return. fuel supply will keep the engines going for a known ti O'Brien is a bright lad who knows how to work out his sp over the water. (If you are an air-cadet, you will call t his *ground-speed*.) The important question to which he m calculate the answer is this: How long can he safely fly s wards before turning back?

A formula gives him the answer. Suppose that g is measure of the going speed, r the measure of the return sp and T the measure of the time for the whole trip. Then O'Br can find the time, t, for the outward trip from the formula

$$t = \frac{rT}{g + r}.$$

For example, if the speed going (g) is 200 m.p.h., the ret speed (r) 150 m.p.h., and the fuel supply good for 7 hours, th

$$t = \frac{150 \times 7}{200 + 150} = 3.$$

Under these conditions, O'Brien should fly seawards for no lon than 3 hours.

What happens if he guesses the time of turning and flies s wards for $3\frac{1}{4}$ hours? His outward mileage would be 200 × 1 or 650, and his inward mileage, until his fuel is exhausted, wo be 150 × 15/4 or 562.5. The error might mean disaster.

In the work that follows, most of the formulas used will old friends. You have already made or discovered them accepted them on faith. It is to be hoped that you are curio If so, you will likely prefer making your own formulas to hav them presented to you. The home-made formula is more sa fying and we feel more comfortable in using it. Let us exam some methods of constructing formulas.

Making Formulas

Before reading what follows, try to recall a method by which made formulas in Chapter II. What do formulas express? e some examples of formulas which you made. * * * n read the rest of this section.

We shall discuss three methods of constructing formulas.

(1) ***Translate a rule from English into algebraic symbols.*** This is a simple way. Its weakness is in assuming that you have a rule. Once a rule is known, we can easily represent the numbers involved by letters. Of course, you recall that this method stems from our first conclusion about formulas on page 170. We have used it freely in Chapter II.

mple. We are given the following rule: The number of are inches in the area of a circle equals the square of the nber of inches in the radius, times π. If we denote the number quare inches in the area by A, and the number of inches in radius by r, we can at once write

$$A = \pi r^2.$$

A Word about Units

If, in the foregoing example, the radius of the circle ha been r inches and the area A square feet, then we shoul have obtained the formula

$$144A = \pi r^2.$$

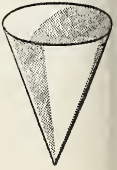
You see that the formula depends on the units used in th measurements. See page 105. Of course, it is usually convenient to measure areas in square inches when lengths ar measured in inches. Similarly, with lengths measured i centimetres, we usually associate areas in square centimetres and volumes in cubic centimetres.

The important fact is that every formula is used in association with a system of units. In most cases, common sense is a sufficient guide for determining the units *provided tha you are aware of the necessity of noting them.*

You will frequently hear such statements as the following: “area eq length times width”, or “the area is half the base times the height”. Th statements can be supported by a proper definition of the multiplicatio lengths. Indeed, in more advanced work, the definition of multiplicatio sometimes extended to other quantities as well as lengths. In this book, h ever, the operations of multiplication and division are confined to numb Hence, the statement “area equals length times width” should be underst as an abbreviation for “the number of units of area equals the numbe units of length times the number of units of width”.

EXERCISES

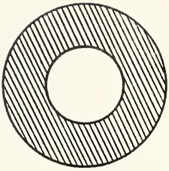
Make a formula for each of the following rules, giving t units associated with your formula.



1. The volume of a conical paper cup (see figure) is one-third of its vertical height times the area of its circular top.

2. The area of one side of a metal washer (see figure) is equal to the area enclosed by

uter circular boundary less the area en-
d by its inner circular boundary.



. The measure of the volume of a card-
d shoebox equals the product of the
sures of its length, width, and height.

. The area of the surface of a ball is 4π
s the square on its radius.

. If a stone is released from the edge of a precipice, the
nce, in feet, through which it falls is 16 times the square of
ime of fall measured in seconds.

. The area of a triangle is one-half the product of its base
height.

. The circumference of a circular plate is π times its dia-
r.

. The volume of a ball is $4\pi/3$ times the cube of its radius.

. The volume of a tin can equals the area of its circular base
s its height.

. The area of tin in a length of stove-pipe equals its circum-
ce times its length.

. The number of man-hours required for a job equals the
ber of men times the number of days times the number of
s of work per day.

. The average mark equals the sum of the marks in all sub-
divided by the number of subjects.

. To convert a temperature expressed in degrees Fahrenheit
grees Centigrade, subtract 32 from the number of Fahrenheit
ees and multiply the remainder by 5/9.

. "The distance traversed equals the average rate of speed
s the time." Explain what must be understood in using this
ment.

(2) ***Use tabulated data.*** Method (1) requires that we know a rule. How can rules be found? * * * Sometimes their discovery presents a problem of great

difficulty. Many suspected relationships have not been reduced to rule or formula. However, if we ha table which shows the values taken by one quantit given values of the other, we may be able to guess at relation.

Example 1. Consider an aircraft flying in a straight line steady ground-speed of 150 m.p.h. Make a table showing distance covered at the end of 1 hour, 2 hours, 3 hours, etc.

Time elapsed in hours .	1	2	3	4	5	6	
Distance traversed in miles	150	300	450	600	750	900	1

Study this table. Then ask yourself: What is the relat between the distance, d, and the time, t? * * *

If $d = (?)t$, what is (?) ? * * * You see t the unknown number must be 150, so that

$$d = 150t.$$

Here then is a formula which gives the distance covered i known time, at least so far as our table goes.

A word of caution is necessary here. In accepting this formula, we assuming that *numbers not appearing in the table obey the same rule as those appear*. This process of generalizing may give rise to errors. It has said that "jumping to conclusions" is a very popular indoor sport. Are justified in assuming that our formula will be true for numbers not in the ta Will it hold if the aircraft changes its speed or direction within the hour averages out according to our table?

Scientists and others often construct tables using the results of exp ments. If the tabulated data suggest a rule or formula, they must exer caution in applying the rule to numbers not in the table. For our purp common sense is usually a sufficient check. The nature of the problem o tells us how far we can go in applying the formula. Thus, so long as aircraft flies at the same ground-speed in a straight line, the formula $d =$ is true.

mple 2. Lay a rule marked in inches beside one marked in timetres. By placing the rulers properly, you can read off the ber of centimetres in 1 inch, 2 inches, 3 inches, etc. Tabulate r results. * * *

You should have a table like the one below.

mber of in. (i)	1	2	3	4	5	6
mber of cm. (c)	2.5	5.1	7.6	10.1	12.7	15.2

dy the table carefully. Then try to complete the formula necting i and c.

$$c = (?)\, i.$$

You find that no one coefficient of i will give a true formula the table. The unknown number is about 2.5. It is not tly 2.5 for, when $i = 6$, $c = 15.2$ and not 2.5×6 or 15.0. s the formula

$$c = 2.5i$$

approximate **formula giving the relation between inches** centimetres.

As in the first example, the use of this formula for numbers in the table involves an assumption. Do you think that bers not in the table follow the same approximate rule as e which appear? * * * You probably agree the formula gives almost correct results for any value of i.

Perhaps you have already asked yourself: How could this approximate ula be made more accurate? Study the errors which arise for various s of i. For example, we have noted that for $i = 6$, the error is 0.2 or t

$$\frac{0.2}{15.2} \times 100\% \text{ or } 1.3\%$$

e tabular value. If $i = 35$, the formula gives $c = 2.5 \times 35 = 87.5$, eas a careful reading from a metre bar shows that 35 in. equal about cm. Thus the formula gives rise to an error of about

$$\frac{1.4}{88.9} \times 100\% \text{ or } 1.6\%$$

of a direct reading.

Consider the single careful reading that gives us that 35 in. equal ab
88.9 cm. From this, we might conclude that

$$1 \text{ in.} = \frac{88.9}{35} \text{ cm.} = 2.54 \text{ cm.}$$

and deduce the formula

$$c = 2.54\,i.$$

Try this new formula for various values of i, beginning with $i = 1, 2$
etc. Round off the values for c to two figures and compare them with th
in the table. Is the new formula better than the old? * * * W
appears to be the effect of using one careful reading over a wide interval (35 i
* * * *To get a formula with fairly high accuracy, you should m*
the intervals between readings as great as possible. One good reading ov
35-inch interval yields a better formula than the six readings in the t
taken at 1-inch intervals.

Of course, some tables do not lead to formulas. A great d
of the tabulated data collected by statisticians and others len
itself to no such compact form as that of a formula. In t
exercises that follow, all but two of the tables give rise to e
formulas.

EXERCISES

Here are some tables showing relations between pairs
numbers. Where possible, write a formula for the relationsh

1. The Cost of Coal

Amount purchased in tons (n)	1	2	3	4	5	6	7
Total cost in dollars (c)	15	30	45	60	75	90	10

Do you think that your formula holds for numbers not in t
table?

ł. Ages of Mother and Daughter

ther's age in years (m)	21	24	28	32	36	40	44
ighter's age in ears (d)	1	4	8	12	16	20	24

. Some Five-hour Motor Trips

erage rate in m.p.h. (r)	20	25	30	35	40	45
tance traversed in iiles (d)	100	125	150	175	200	225

s your formula hold for average rates not in the table?

. *P* and *Q*

P	2	3	4	5	6	7	8
Q	1	3	6	17	91	118	130

s Q appear to increase or decrease as P increases?

. Feet and Inches

mber of feet (f)	1	2	3	5	8	10	15
mber of inches (i)	12	24	36	60	96	120	180

s the formula hold for numbers not in the table?

6. Wages and Savings

John's weekly wages in dollars (*w*)	15	15	15	15	20
John's weekly savings in dollars (*s*)	0	2	1	6	0

Does it appear that John saves more when he earns m
Would you say that he is saving according to a plan?

7. The Story of m and n

m	2	3	4	5	6	7	8
n	4	9	16	25	36	49	64

8. Quarter Inches and Millimeters

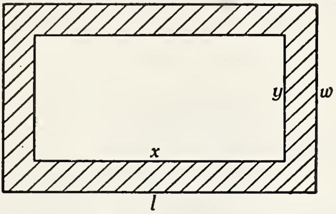
Number of 1/4 in. (*q*) . . .	1	2	3	4	5
Number of mm. (*m*)	6.5	13	19	25.5	31.5

Is your formula exact or approximate for the table? Doe
apply to numbers not in the table? Can you suggest a met
of obtaining a more accurate formula for the relation betw
q and m?

(3) ***Use a geometric figure.*** You have already
practice in this method of making formulas in Cha
II. Formulas for perimeters and areas of squares
rectangles can easily be seen from figures. The s
is true for areas and perimeters of figures made u
squares, rectangles, and circles.

Example 1. The figure shows a plan of a garden surroun
by a cinder path. Look at the figure and the dimensions i

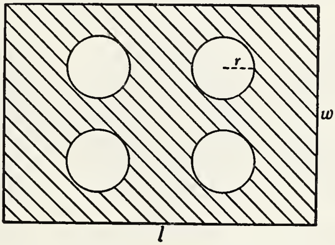
d thereon, and then write a formula for the area of the cinder h (shaded).



The area of the shaded part is given by

$$A = lw - xy.$$

mple 2. The top of a stove with the covers off looks like :



he stove is l units long, w units wide, and has openings of us r, write a formula for (a) the area of the shaded region, he length of the boundary of the shaded region. After you tried this, compare your formulas with those below.

* * *

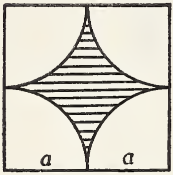
The area of the shaded region is given by

$A = lw - 4\pi r^2$, and its boundary is
$b = 2(l + w) + 8\pi r.$

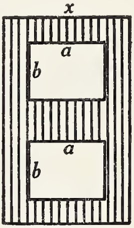
here are some similar problems to test your knowledge of simple, basic formulas.

EXERCISES

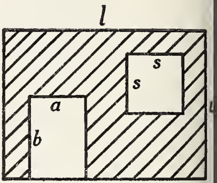
Make formulas for the areas and boundaries of the sha parts of the following figures. (The length of the boundary i include both inner and outer boundaries where they exist.)



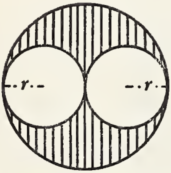
1. Square less 4 bites.



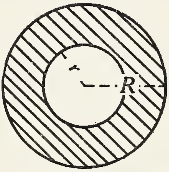
2. Barn door.



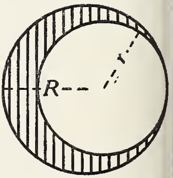
3. Front of Jack's house.



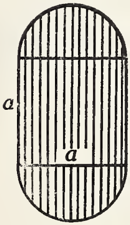
4. Bullseye with 2 hits.



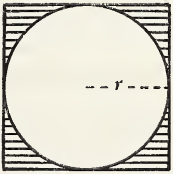
5. Doughnut from above.



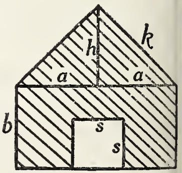
6. New moon.



7. Table top with flaps.



8. Square platter.



9. Doghouse.

Using Formulas

We have been considering three ways of making formulas. us find out some more about using them.

In Chapter III, you learned something about substitution. s plays an important part in using formulas. If a numerical ue of a letter is known, we can replace the letter by its value. page 118 the formula $V = \pi r^2 h$ was examined. We noted that ny two of the letters stand for known numbers, then we can the number which the third letter stands for. Now consider formula $V = lwh$, in which there are four numbers repre-ed by letters. How many of the four numbers need be wn in order to fix the values of all four? * * * If 5 and $w = 4$, can you be sure of the values of V and h? If 5, $w = 4$, and $h = 2$, can you be sure of the value of V? at conclusion can you draw about a formula like $K = x^2yzh$?

* * *

Usually, if all the letters of a formula except one stand for known numbers, then the number which the remaining letter stands for can be found.

Sometimes there are difficulties in finding the one unknown mber, but in our formulas this is relatively easy.

EXERCISES

For each of the following formulas, find the value of the nown number. Take $\pi = 22/7$.

1. $V = \pi r^2 h$; $r = 7$, $h = 10$.
2. $A = \pi(R^2 - r^2)$; $R = 4$, $r = 3$.
3. $d = 16t^2$; $t = 5$.
4. $F = \frac{9}{5}C + 32$; $C = 100$.
5. $C = \frac{5}{9}(F - 32)$; $F = 212$.

6. $S = \frac{n}{6}(n + 1)(2n + 1)$; $n = 20$.

7. $K = x^2yzh$; $x = 2, y = 5, z = 3, h = 10$.

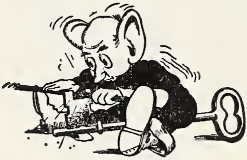
8. $V = 4\pi r^3/3$; $r = 21$.

9. $A = \frac{h}{2}(a + b)$; $h = 10, a = 16, b = 5$.

10. $y = 100/x^2$; $x = 2\frac{1}{2}$.

11. $l = 66/2\pi x$; $x = 7$.

Re-arranging Formulas



Did you notice how conveniently the foregoing formu
were arranged? The unknown letter in every case was the
side of the equation. This set-up makes substitution easy. W
are we to do if the unknown letter is *not* the left member of
formula? Perhaps you can suggest an answer. Work the
lowing example in two ways, first, without changing the formu
and then by arranging the formula in a more convenient fo
After you have given the problem a trial, study the soluti
below.

Exercise. If the area of a circular sheet of tin is 154 sq. in., f
its radius.

* * *

Method 1. If $A = \pi r^2$,
then $154 = (22/7)r^2$.
Therefore $154 \times 7/22 = r^2$ (Why?)
or $49 = r^2$, whence $r = 7$.
Hence, the radius is 7 inches.

tice that we substituted first and then got the unknown r on side of the equation.

ethod 2. It is sometimes better to re-arrange the formula *before* substituting so that the unknown letter is the left member of the formula. Thus,

if $A = \pi r^2$, then $r^2 = A/\pi$. (Why?)

Then $r = \sqrt{A/\pi}$. (Why?)

Hence, for the given circle,

$$r = \sqrt{154 \times 7/22} = \sqrt{49} = 7.$$

The symbol $\sqrt{}$ means *the square root of*. Thus $\sqrt{4}$ means *the square root of* 4, which is 2. The square root of 4 is the number that, when multiplied by itself, gives 4. Similarly, $\sqrt{9} = 3$, $\sqrt{49} = 7$, $\sqrt{x^2} = x$, and so on.

The formula $A = \pi r^2$ is convenient when r is given and A is be found. Likewise the formula $r = \sqrt{A/\pi}$ is convenient finding r when A is known. Sometimes we say that A is the ***bject*** of the first formula and r, the *subject* of the second. e process of passing from one formula to the other is then led ***changing the subject of the formula.*** It is also cusnary to say that ***the first formula is solved for A, and second is solved for r.***

You should note that although the two formulas look different y are really equivalent. We can pass from one to the other using our basic principles for equations (see page 155). As ther example, take $V = \pi r^2 h$.

have $h = V/\pi r^2$ (Principle 4, page 155).

o $r^2 = V/\pi h$ (Why?)

l $r = \sqrt{V/\pi h}$ (Why?).

e formula has been solved for h and r by thinking of it as an ation and applying the basic principles.

> *. . . the letter . . . enables one . . . to paraphrase any statement into a number of equivalent forms. It is this power of transformation that* ***lifts algebra above the level of a convenient shorthand.*** —Dantzig.

Here are two examples for you to try.

Example 1. Solve the formula $V = lwh$ for h.

Example 2. Solve the formula $K = a + bx^2$ for x.

* * *

Compare your solutions with these.

Solution 1. If $V = lwh$, then $h = V/lw$ (Why?).

Solution 2. If $K = a + bx^2$,

then $K - a = bx^2$ (Why?),

and $(K - a)/b = x^2$ (Why?).

Thus $x = \sqrt{(K - a)/b}$ (Why?).

EXERCISES

In each of the following exercises, solve the formula for t unknown letter before substituting. (Take $\pi = 22/7$.)

1. The volume of a box is given by $V = lwh$. If $V = 420$, $w = 1$ $h = 5$, find l.
2. $F = ma$. If $F = 980$ and $m = 5$, find a.
3. $d = 16t^2$. Find t if $d = 400$.
4. $C = pn$. Find p if $C = 1250$ and $n = 25$.
5. $K = x^2yzh$. Find z if $K = 360$, $x = 3$, $y = 4$, $h = 5$.
6. The area of tin in a length of stovepipe is given by $A = 2\pi r$ If the area is 440 sq. in. and the height is 20 in., find t radius.
7. The area of a ball with radius r is given by the formula

$$\boxed{A = 4\pi r^2.}$$

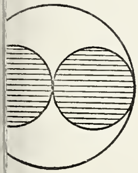
What is the radius of the largest ball that could be cover with 616 sq. in. of tinfoil?

8. If $A = \dfrac{h}{2}(a+b)$ and $A = 60$, $a = 12$, $b = 8$, find h.

Interpreting Formulas

Did you ever stand outside the window of a bake-shop and nire the cookies? They exemplify the circle in one of its most active forms. Mr. Batter's "Cookie Cupboard" exhibited ne appetizing specimens at 15 cents a dozen and some larger s of the same kind at 40 cents a dozen. George advised his her to purchase the larger ones: "They are just as thick as small cookies and have twice the diameter," said George. you think that George gave his mother good advice?

This problem of purchasing cookies involves the question: v does the area of a circle grow as its radius increases? If the ius is doubled, is the area doubled? Study the figure. Then try to decide whether the larger cookies were a bargain at 40 cents a dozen. If you are still not sure, be patient and read on.



Life is full of situations in which it is important for us to know how changes in one thing bring about changes in another. "To be is to be related."[1] Whenever the measure of one quantity is related to, or depends upon, the measure of another, questions about growing mount of change are likely to arise. Is it twice as risky to p from a height of 10 feet as from a height of 5 feet? Does ake half as long to stop a car travelling at 30 m.p.h. as it s to stop one travelling at 60 m.p.h.? More or less? You ke a plasticine ball in your modelling class. How much maal will be needed to make a ball with twice the diameter. ce as much? Four times as much? Eight times as much?

Formulas answer such questions. A formula expresses a tion between the measures of quantities; it tells, for exam- how the area of a circle depends on the length of its radius. can study this dependence by substituting numbers in the nula and noting how changes in one number produce changes in another. This is laborious. As we become more familiar w formulas, we learn to read important information from the f mula itself.

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[1] Keyser, C. J., *Mole Philosophy and Other Essays* (E. P. Dutton & Co., 1922).

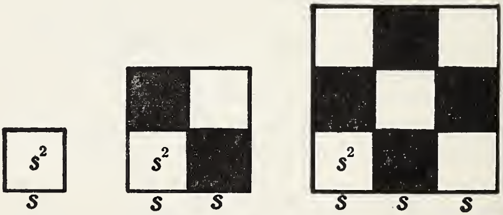
Give yourself a trial. Example 1 on page 176 suggests th for an automobile travelling at a steady rate, r, the dista covered in time, t, is given by the formula

$$d = rt.$$

Study the formula. Then, without using a pencil, try to ansv the following questions. How does the distance change wh (*a*) the time remains unchanged and the speed is doubled? the time remains unchanged and the speed is halved? (*c*) t rate of speed remains unchanged and the time is doubled? the speed is halved and the time is doubled?

If you can't see how to answer these questions orally, try find the answers by substituting different numbers in $d =$ You can compare your results with the answers given later this section.

Ways of Growing. A careful inspection of a checkerbo may help us see how changes in one number affect another wh depends on it. The figure shows some squares such as make



part of the design of a checkerboard. You recall at once t if the side of a square is s units, then the perimeter and ar respectively, are given by

$$p = 4s \qquad \text{and} \qquad A = s^2.$$

Let us study perimeters first. The figures show that if the of the square is doubled, the perimeter becomes $8s$, or twice original length. Also, we see that the largest square in the re, having sides three times those of the smallest, has a peri- er $12s$, or three times the perimeter of the smallest square. can tabulate our results, thus:

ngth of side	s	$2s$	$3s$	$4s$	$5s$	$6s$	$7s$	$8s$
ngth of Perimeter . .	$4s$	$8s$	$12s$	$16s$	$20s$	$24s$	$28s$	$32s$

he side of a square is increased ten-fold, what is the effect on perimeter?

Now look at the areas of the squares of the checkerboard. en the side of a square is doubled, what is the effect on its ? What is the effect when the length of its side is tripled? following table, obtained directly from the checkerboard, s all the answers.

ngth of side	s	$2s$	$3s$	$4s$	$5s$	$6s$	$7s$	$8s$
ea of square	s^2	$4s^2$	$9s^2$	$16s^2$	$25s^2$	$36s^2$	$49s^2$	$64s^2$

Note that when the side of a square is doubled, its perimeter oubled. But its area increases four-fold! When the side is led, the perimeter is tripled, while the area increases nine- ! Suggest why the area grows at a faster rate than the meter.

* * *

The reason is this. The formula for the perimeter is $p = 4s$, ch contains only ***one*** factor s. We say that the term $4s$ olves s ***to the first power.*** Since only ***one*** factor s occurs, nges in s affect the perimeter only ***once.*** If s is doubled, doubled. If s is halved, p is halved, and so on.

Now look at the formula for the area: $A = s^2$. This for- a contains the factor s ***twice.*** The term s^2 is said to involve

s to the second power. Since ***two*** factors s occur, changes s affect the area ***twice***, once for each factor. Thus if s doubled, A is doubled twice or increased four-fold. If s tripled, A is tripled twice or increased nine-fold. You cou put it this way:

When s increases to $4s$,

then $s^2 = s \times s$ increases to $4s \times 4s = 16s^2$.

Here is an investigation for you to make by yourself. T object is to find out how the ***third power*** affects growing. Y know that the volume of a cube is given by $V = s^3$. Well, g a supply of sugar cubes and find out the effect on the volu of a cube of doubling the length of its edge. When the edge tripled, what is the effect on the volume?

* * *

You should be able to show to your satisfaction that wh the edge is increased two-fold, the volume is 8 (or 2^3) times large. Similarly, when the edge is increased three-fold, t volume becomes 3^3 or 27 times as large. That is rapid growi indeed!

EXERCISES

1. *A ring.* The length of a circular wire ring is given by $c=2$ that is, the length depends on the first power of r. What the change in the length when the radius is (*a*) double (*b*) halved, (*c*) increased 5-fold, (*d*) quartered?

2. *A cookie.* The area of the top of a circular cookie is giv by $A = \pi r^2$. What is the effect on the area when the radi is (*a*) doubled, (*b*) halved, (*c*) tripled, (*d*) reduced to one-thi of its original value?

3. *A ball.* The volume of a spherical ball is given by $V=\frac{4}{3}\pi$ that is, the volume depends on the third power of r. Wh is the effect on the volume of a ball when the radius is (doubled, (*b*) halved, (*c*) tripled?

We can now answer the questions raised about $d = rt$. This mula tells us that the distance covered depends on the *first* wer only of both the rate and the time. It follows that, if the ıe remains unchanged, the distance is doubled when the rate doubled and halved when the rate is halved. If the rate ıains unchanged, the distance is doubled when the time is ıbled. Halving the rate and doubling the time produces no ınge in the distance, since it is also halved and then doubled.

Growing inversely. Let us now consider a formula with a ıominator. We can study the effect of a denominator on wing by examining the following two tables. The first shows distances covered in two hours for different rates of speed; second shows the times taken to cover a distance of 360 miles different rates of speed. Study both tables carefully.

Some two-hour trips. $(d = 2r)$

istance in miles (d)	40	80	120	144	180	240	360
verage rate in m.p.h. (r)	20	40	60	72	90	120	180

Some 360-mile trips. $(t = 360/r)$

ime in hours (t)	18	9	6	5	4	3	2
verage rate in m.p.h. (r)	20	40	60	72	90	120	180

A glance at the first table shows that when r increases, d reases, and when r decreases, d decreases. But look at the ond table. There you see that increases in r bring about reases in t, and decreases in r bring about increases in t. In first table, d and r increase and decrease together; r is said to affect d ***directly***. In the second table, when r gets larg t gets smaller; r is said to affect t ***inversely***.

The second table will give you information about the eff of r in the denominator of the formula $t = 360/r$. How d doubling r affect t? What happens to the value of a fracti when its denominator is doubled? You see that the value the fraction is halved. You can also see from the table th when r is tripled, t becomes one-third as great; and when becomes one-third as large, t is tripled. In short, multiplyin by n has the effect of dividing t by n. That is, if r is multipli by 2, 3, $\frac{1}{4}$, then t is multiplied by $\frac{1}{2}$, $\frac{1}{3}$, 4, respectively.

In order to work the following exercises, you must recall t effects produced by first, second, and third powers as well as t inverse effect produced by a first power in the denominator.

EXERCISES

1. *The Total Cost Formula.* If n articles are purchased at each, the total cost C is given by $C = pn$. What is the eff on C when (a) p is doubled? (b) n is tripled? (c) p is doubl and n is tripled? (d) n is halved?
2. *The Falling Body Formula.* If a body is released to fall fre under gravity, in t seconds it falls a distance d, where $d = 1$ (ignoring the effect of the air). How is d affected if (a) t tripled? (b) t is halved? (c) t is doubled?
3. *A Radio Formula.* If W is the wave-length in meters and the number of kilocycles, a formula used in radio work gi

$$W = \frac{300{,}000}{k}.$$

What is the effect on W when (a) k is doubled? (b) k is triple (c) k is quartered?
4. *The Gas Formula.* The pressure of a gas on the walls of t containing vessel is given, under certain conditions, $P = c/V$, where V is its volume. How is the pressure affect by (a) increasing the volume 5-fold? (b) halving the volum (c) quartering the volume? (d) tripling the volume?

A Coal Bin Formula. If the dimensions of a bin, in feet, are , w, and h, then the capacity in tons is approximately

$$T = 3lwh/94.$$

Vhat change is produced in T by (*a*) doubling l and doubling v? (*b*) halving w and increasing h fourfold? (*c*) halving h ınd w and doubling l?

An Ocean Liner Formula. The cost, C, of building a liner is ;iven approximately by $C = ks^3$, where k is a number deɔending on the type of ship and s is its top speed. What :hange is produced in the cost by (*a*) doubling the desired ›peed? (*b*) by tripling the speed? (*c*) by halving the speed?

An Aeroplane Formula. The air pressure in pounds per sq. 't. on the wings of an aeroplane travelling at v m.p.h. is given ɔy $p = 0.0005v^2$. What is the effect on p when (*a*) v is douɔled? (*b*) v becomes one-third its original value? (*c*) v is ncreased to 3/2 times its first value?

The Ball Formula. We have noted that the volume of a ball s $V = 4\pi r^3/3$. What change occurs in V when (*a*) r is triɔled? (*b*) r is quartered? (*c*) r is doubled? (*d*) r is halved?

A Postscript on Formulas

Many interesting and amusing facts come from formulas. u know something of their usefulness in finding areas, periters, and volumes. Did it ever occur to you that some forlas are of assistance in the buying of food? The Ball Formula this page throws some interesting light on the results of buying lersized oranges or those with too thick skins. Look again at Tin Can Formula, $V = \pi r^2h$. Would you prefer to buy a can 'ruit that is 1/2 an inch higher or one that is 1/2 an inch wider n a given can, provided the prices and quality were the same? e formula will tell you the answer. The late E. R. Hedrick, ımous educator, once said that "every tall can in every grocery re is a monument to the women who 'do not need to know any ebra' ".

You should also be aware of the importance of formula physics, chemistry, radio, electricity, medicine, nursing, me anics, nutrition, and engineering. Of course, you will hav accept the truth of such formulas on faith for the present. following examples are offered in order to broaden your vi Read them carefully. In the review exercises at the end of chapter, you will be given an opportunity to use some interest formulas which scientists and others have made.

The Think-of-a-number Game. Everyone has at one t or another been waylaid by an individual who hopes to show magic powers. He may approach you with a look of myst and issue the following commands. (1) Think of a num (2) Multiply it by 5. (3) Add 6 to the result. (4) Multi the sum by 4. (5) Add 9 to the product. (6) Multiply sum by 5.

You are an agreeable fellow so you do all this. Your qu tioner urges you to be accurate and then report the final re to him. Whereupon he goes into a short trance and then t you the number with which you began. But you need not unduly impressed. Let us make a formula of the final resul the 6 steps. If we denote by n the number you begin with, th the steps give the following results. (1) n. (2) $5n$. (3) $5n +$ (4) $20n + 24$. (5) $20n + 33$. (6) $100n + 165$.

The formula shows that if we subtract 165 from the fi result and then divide by 100, we get the original number. the trick wasn't so difficult. Now you can be the magician a try it yourself.

If you find such number games amusing, you can get m information about them in *Mathematical Recreations and Ess* by W. W. Rouse Ball (Macmillan) or in *Riddles in Mathema* by E. P. Northrop (Van Nostrand).

Knowing When to Stop. The automobile is definitely h to stay. The question now seems to be whether or not supply of pedestrians will last. Accidents are all too comm ny drivers are unaware of the effect that their speed has on distance required for stopping the car. Suppose that a car good brakes is travelling at r miles per hour on a dry pave- nt. Then the shortest distance in feet in which the car can is given approximately by the formula

$$d = 0.045r^2 + 1.1r.$$

Use this formula to show that at 30 m.p.h., $d = 73.5$ feet, le at 60 m.p.h., $d = 228$ feet. Can you suggest why doubling uses more change in d than doubling?

Watching your Weight. Here is an interesting use of a for- a in health education. For people over 5 feet in height, the nal weight, in pounds, should be approximately that given the formula

$$w = 5.5(h - 60) + 110,$$

re h is the number of inches in the height. For example, if r height is 64 inches, then your weight, in pounds, under nal conditions should be about

$$\begin{aligned} w &= 5.5(64 - 60) + 110 \\ &= 5.5 \times 4 \qquad + 110 \\ &= 132. \end{aligned}$$

For those under 5 feet in height, there is a similar formula:

$$w = 110 - 5.5(60 - h).$$

ut how much should a person 54 in. tall weigh normally?

A Suggestion

Before trying the problems in the review exercise, make a list of all the formulas which you have met to date. Then, as you encounter new formulas, add them to your list.

EXERCISES ON CHAPTER V

A

1. Write the following rule as a formula: The expected t mileage is 6545 times the diameter of the tire in feet. Use yo formula to find the expected mileage of a tire whose diame is 2.4 feet.

2. Sometimes contractors estimate the number of bricks quired for a wall by using the formula $n = 22lwh$, where l, w, a h stand for the number of feet in the length, width, and heig respectively. What is the effect on n of doubling both w and How many bricks would be needed (approximately) for a wall feet long, 1 foot wide, and 20 feet high?

3. Use the following table to make a formula connectin and w.

Number of days (n).	1	2	3	4	5
Total wages in dollars (w).........	6.50	13	19.50	26.00	32.50

4. John is 5 feet 6 inches tall. Use the formula on page 1 to find how much he should weigh normally.

5. Sylvia is 4 feet 8 inches tall. Use the formula on page 1 to find how much she should weigh normally.

6. Mr. Jenkins has 132 feet of fencing material which plans to put around a circular plot in his garden. What the radius of the largest plot he could fence? (Take $\pi = 22/$

7. Some storage tanks are made in the form of spheres, that their areas are given by $A = 4\pi r^2$. How does doubli the radius affect the amount of material required for the tan Find the area of a tank with radius 21.0 feet.

8. Solve the formula $P = dz - a$ for z.

9. Use the aeroplane formula on page 193 to find the pressu on the wings of an aircraft travelling at 200 m.p.h.

10. Use the following table to find a formula connecting x an

Values of x	5	6	7	8	9	10
Values of y	7.5	8.5	9.5	10.5	11.5	12.5

1. Use the falling body formula on page 192 to find the dis- ce which a body falls in 20 seconds under gravity.

2. Here are two formulas of interest to farmers. If a bin feet long, w feet wide and h feet high, it will hold about /2.5 bushels of corn (on the cob), or about $lwh/1.25$ bushels vheat. Find, in bushels, the amount of corn and the amount vheat that could be stored in a bin 25 feet long, 15 feet wide, 10 feet high.

3. Use the radio formula on page 192 to find the wave-length neters sent out by station CFIT which uses a frequency of kilocycles.

4. If shingles are laid with 4 inches "to the weather", the owing rule is useful: The number of shingles required equals times the number of square feet in the area to be covered. this statement into a formula. How does tripling the area ct the number of shingles required? About how many igles are needed to cover a roof whose area is 720 square feet?

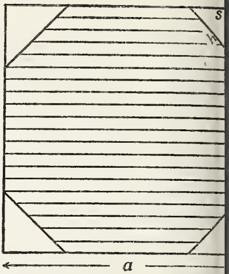
5. The load for a rope c inches in circumference should not eed L pounds where $L = 100c^2$. What size of rope would be ded to carry a load of 2500 pounds?

B

6. Using the following table, find a formula showing how epends on w.

Weight of letter in oz. (w) . .	1	2	3	4	5	6
Cost of postage in cents (c) .	4	6	8	10	12	14

7. Make formulas for the area and the boundary of the ded region in the figure at the top of the next page.



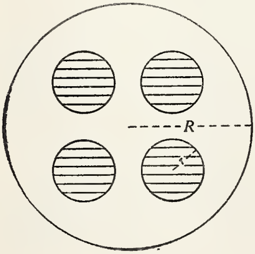
18. Find the radius of a circle whose area is 616 sq. in.

19. The cost of operating an electric appliance such as a toaster is given by $C = Wtr/1000$, where W is the number of watts required, t the time of use in hours, and r the cost of electricity per kilowatt-hour. How much does it cost to operate a 200-watt bulb for 10 hours in a town where electricity costs one cent per kilowatt-hour?

20. The volume of a tank such as those mentioned in Ex
is given by $V = 4\pi r^3/3$. How does doubling the radius aff
the capacity of the tank? State an advantage of storing ga
line in large tanks rather than small ones.

21. The edge of a cubical cardboard carton is x inches. Wr
a formula for (*a*) the area of the surface of the box when clos
(*b*) the volume of the box. If x is doubled, what change
produced (*c*) in the area, (*d*) in the volume? Is it cheaper
pack certain goods in large boxes (provided that the cardboa
doesn't give way)?

22. The horsepower rating of an automobile is given
$H = 2Nd^2/5$, where N is the number of cylinders and d is t
bore of the cylinders in inches. H
is H affected by (*a*) doubling
(*b*) doubling d? (*c*) halving d? Wh
bore would be needed in a 6-cylind
car to give 60 horsepower?



23. The shaded areas in the figu
have been punched out. Find t
area of the part remaining.

24. Use the formula on page 1
to find the number of feet requir
to stop a car with good brakes trav
ling at 50 m.p.h. on a dry pavemen

5. If an automobile travelling at r m.p.h. hits a solid object, orce of impact is the same as that with which it would strike ground after falling h feet where $h = 0.0336r^2$. When a car elling at 60 m.p.h. collides with a wall, the force of the blow uivalent to that resulting from a drop of how many feet?

. Here is a table with information about taxi fares. What ula connects q and c?

mber of $\frac{1}{4}$ miles (q)	1	2	3	4	5	6	7	8
e in cents (c).....	15	20	25	30	35	40	45	50

. A pine beam l feet long, w inches wide and d inches deep support safely a load of L pounds, where $L = 180wd^2/l$. t is the effect on the load when (*a*) l is doubled? (*b*) l is ed? (*c*) d is doubled and w is tripled? (*d*) w is doubled l is reduced to one-third its former value? (Note: the load t be distributed along the beam.)

. Use the formula of Ex. 27 to find the depth needed so that am 12 feet long, and 4 inches wide can support safely a ibuted load of 3840 lb.

C

. Find the relation between t and r by studying the follow- able.

Time in hours (t)...	1	2	4	5	8	10
Rate in m.p.h. (r)...	200	100	50	40	25	20

. A tennis ball is a spherical shell, or a sphere inside of h is a concentric spherical hole. Make a formula for the me of the material in a spherical shell. If the outer radius shell is 5 in. and the inner radius 4 in., find the volume of its part.

31. A length of tile is a cylinder from which an inner cylin has been removed. Make a formula for the volume of the s part of a tile.

32. It has been stated that the energy yielded up when a m of m units changes into energy is given by the formula $E =$ where c is the velocity of light. Can you suggest why sn amounts of matter may yield very large amounts of energy?

33. Investigate the effect that the *fourth* power has on gr ing. If $d = x^4$, what is the effect on d when (a) x is doubl (b) tripled? (c) halved? The energy stored in a flywheel eq kr^4, where r is the radius of the wheel. If the radius beco 3/2 of its former value, how is the amount of energy affected?

34. Sometimes it is necessary to put as many wires as poss within a large hollow cable. The approximate number, n, of w of diameter d which may be fitted inside a cable of in diameter D is given by

$$n = 0.907\left(\frac{D}{d} - 0.94\right)^2 + 3.7.$$

How many wires 0.25 in. in diameter can be fitted into a c which has an inner diameter of 3 in.?

35. Steel rails such as those used on railroads and for gir expand as the temperature rises. A rail l feet long when hea through t Fahrenheit degrees will increase in length by e where $e = 0.000006\ lt$. How much should be allowed for expansion of a 30-foot rail if the difference between winter summer temperatures may be 120°? How much should b lowed for the expansion of a 1000-foot steel bridge?

RECREATIONS

1. *Another Number Game.* This game is simpler than tha page 194. Make a formula to show how the following steps alw lead to the answer 3. Here are the steps: 1. Think of a num 2. Double it. 3. Add 6 to the result. 4. Take half of the s ubtract from your result the number you thought of first. ur answer is 3.

Make up some similar "thought-reading" exercises.

2. *A Puzzle about a Ring.* Suppose we have a wire ring ch fits about the earth snugly at the equator. We wish to add ugh wire to the ring so that it will circle the earth just 6 feet ve the equator. Do you think that the addition of 40 feet vire to the ring would be enough?

CHAPTER VI

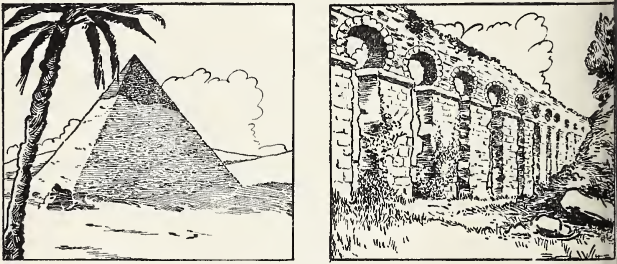
INTEGERS

"O Mount Vesuvius! here we are in arithmetic!"

W. S. Gilb

A Glance at Numbers

Why did men begin to study numbers centuries ago? answer this question think of the Phoenician traders who sai their vessels over the Mediterranean and Red Seas and the Indi Ocean to barter their wares for the produce of India. In th trading they had continually to ask and to answer the questi 'how long?', 'how far?', 'how much?', and 'how many?'. Th are questions which require numerical calculations and numeri answers. Again, think of the priestly architects who plann the temples and the pyramids of Egypt or of the army enginee who built the military roads and the aqueducts of Imperial Rom



Ancient masterpieces of engineering

In those days as well as in our own an architect had to have 'head for figures'. It was the ***uses*** which they found for nu bers which made them necessary to the ancient traders a engineers.

There is another reason. Many people studied numbers, as ple still do, without a single thought of their usefulness, ause they liked to do so. We are told that Pythagoras and followers were so delighted by the things they found out about nbers that they would sacrifice an ox to celebrate some disery. They were undoubtedly as much excited by what you soon learn to call the Pythagorean theorem as Columbus by the discovery of America. Indeed, who is not interested umber puzzles? Here is one for you to try. It is a game rranging four 4's, with arithmetical symbols, +, ×, etc. example, $4/4 + 4/4 = 2$. Now try to make an arrangement ch gives 3, 8, 9, 12, 15, 16, 17, 20.

Whole Numbers

Arithmetic may be said to have had its beginning when men ched the point where, instead of 2 tigers, 2 spears, 2 men, they an to pick out the idea which belongs to all pairs of objects. s is the number 2. In the same way they began to think of , 5, . . . without reference to specific things. These numbers,

$$1, \quad 2, \quad 3, \quad 4, \quad . \quad . \quad . \, ,$$

ch are used for counting, we now call ***whole numbers*** or ***egers***. The number 1 is referred to as ***unity*** or the ***unit eger***.

To assist them in counting, early people used convenient ects as counters, as, indeed, many people still do. The most ural counters for anyone to use are his ten fingers, a fact of ch we are reminded when we call the symbols 0, 1, 2, . . ., 9 ***its*** (Latin, *digitus*, finger). From this natural set of nters arose our practice of counting by tens and of writing nbers with attention to the place values of the digits. For mple, you already know that 3576 means 3 thousands, 5 dreds, 7 tens and 6 units, each of the digits 3, 5, 7 having a ***ce value*** ten times as great as it would if it were one place her to the right. This is the reason for calling our system umeration a ***decimal*** system (Lat. *decem*, ten).

EXERCISES

1. Read the numbers: 26,547; 302,068; 1,540,704.

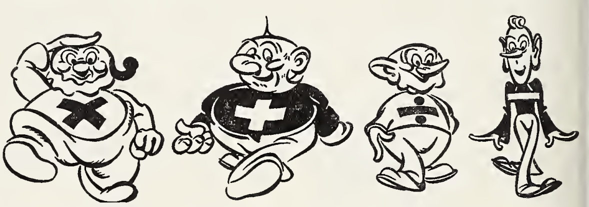
2. Give the names of the place values of the digits in number, from units to billions, where a billion means a thousa million.

3. In the number 333,333 the fifth digit from the right h how many times the value of the second digit from the right?

4. Make a list of ten different uses of numbers in everyd life.

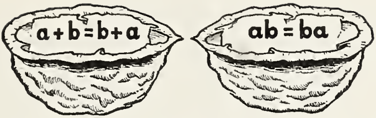
The Integers Go to Work

Let us recall our old friends addition, subtraction, mul plication, and division. They are the ***four fundamental op ations*** of arithmetic. When two or more of these operati enter into a computation does the order of the steps matt Let us see.



First let us take addition. It is clear that in a sum such $3 + 4 + 8 + 12$ the order makes no difference. In fact, y are accustomed to adding a column of figures from the bott and from the top in order that each answer shall be a check the other. A sum is not changed by changing the order of terms or, to put it in a nutshell, $a + b = b + a$.

Take next a product, such as $3 \times 4 \times 5$. Work this first multiplying 3 by 4, then by first multiplying 3 by 5, a then by first multiplying 4 by 5. Your experience with this a similar examples will make you ready to believe that a produ is not changed by changing the order in which the numbers a multiplied, or, in another nutshell, $a \times b = b \times a$ or $ab =$



Are there any operations in which the order does make a erence? An example will give us the answer. In working 5 + 2 × 8 − 6, what will you do first? We may use parenses, as in algebra, to indicate which operations are to be perned before the others. One way to do it is 5 + (2 × 8) − 6, ich gives 15; another way is (5 + 2) × (8 − 6), which gives A good rule is to use parentheses whenever there is any ıbt about the order to be followed. It is customary, however, en the order is not indicated, to assume that a multiplication a division is to be performed ***before*** addition or subtraction. s agreement has already been mentioned on page 137. Accordly, 5 + 2 × 8 − 6 is assumed to mean 5 + (2 × 8) − 6.

Some Tips for Good Computing

Aim at accuracy. When a book-keeper or an engineer makes omputation in arithmetic, he knows the serious consequences

5×9
8=642
ACCURACY

that may result from an incorrect answer. It is while you in school that you have the chance of forming good habit computing. Let your first aim be accuracy. Make each s with as much care as if the safety of the pilot in an aeropl depended on it. It is worth noting that accuracy is someti interfered with by working too slowly or by working too rapi This suggests a second aim—a reasonable degree of speed.

ACCURACY FIRST, THEN MODERATE SPEED

Estimate your answer. Does a floor space 65 feet by 27 contain more than 1000 square feet, or as much as 3000? rough mental calculation (take 60 × 30, for example) soon sh that the answer is nearer to 2000 than to 1000 or 3000. forming the habit of making just such estimates of your answ you will avoid some of the more glaring mistakes which might otherwise make. Of course, an estimate is not the ans

ESTIMATE YOUR ANSWER

but only a check on the answer. The important thing in estimate is the first digit and its place value.

Two short cuts in multiplication. What is 25 times 64? times 118? If you see a short way to do these then pass on the next paragraph. * * * Since 25 is ¼ of 100, hence times 64 is ¼ of 6400, or 1600. To multiply a number by append two zeros at the right of the number and divide the number by 4.

You will now be able to invent a similar rule for multiply a number by 125. Remember that 125 = 1/8 of 1000 and th state the rule. The following examples show how the w appears in finding two products:

(1) 346 × 25; $4\overline{)34600}$ 8650 (2) 257 × 125; $8\overline{)257000}$ 32125

EXERCISES

Answer these orally

1. In $7 + 5 - 6 + 4 - 2$, does the order of making the itions and subtractions affect the answer?

2. Find the values of $8 + 3 \times 4$, $2 \times 5 + 8 \div 4$.

3. In $3 \times 4 \div 2$, does the order of the steps affect the answer?

4. In $8 \div 4 \times 2$, show that the order of the steps affects the ver. Indicate by brackets the two meanings.

5. Make mental estimates of the answers to the following: 141 + 151 + 161; (*b*) 22×88; (*c*) 5 times 495; (*d*) 18×19 2; (*e*) 324×90; (*f*) $836 \times 48 \div 23$; (*g*) $84256 \div 12$.

---

n these, check your answers for accuracy and note your time. n a score of 100 take 10 off for each error and 3 off for each ute over 6.

6. Find the sum of the numbers from 18 to 23 inclusive.

7. From the sum of 378 and 199 subtract their difference.

8. The larger of two numbers is 29 and their difference is 3. l their product.

9. Compute by short methods: (*a*) 3478×25; (*b*) $864 \times$; (*c*) 624×125; (*d*) 125^2.

10. Find the distance between the first and last of 44 tele-
h poles which are in line at intervals of 150 feet.

11. A Sunderland flying boat carries 2034 gallons of gasoline. consumes 113 gallons an hour (*a*) how much remains after urs' flight, (*b*) what is its flying range in hours?

Factors

Multiplication does a somersault

In the multiplication tables you learned the products various pairs of integers. Let us work the multiplication ta backwards. What two integers give the product 42, 64, 99? N we need a new name. In example 30 = 5 × 6 the inte 5 and 6 are called ***factors*** of Each of the numbers 5 and 6 vides into 30 exactly, that is w out remainder. *One integer factor of another if it divides actly into that other.* You see t that 5 and 6 are not the only tors of 30. Other factors are 2 and 15. Also 1 and 30 are fac but this fact is trivial since ev integer is exactly divisible by it and by unity. A number exa divisible by 2 is called an ***even*** number; a number not exa divisible by 2 is called ***odd***.

EXERCISES

(*for oral answers*)

1. Give three factors of 12, four factors of 18.

2. What is the largest factor of 36 other than 36?

3. What is the smallest factor of 45 other than 1?

4. Give all the factors of 60.

5. When you begin with 1 and count by 2's what kin numbers do you use?

6. When you count by 3's what is the character (odd or ev of the numbers you use?

Prime Numbers and Composite Numbers

The number 7 has no factors other than itself and unity. r this reason it is called a ***prime*** number. Likewise 2, 3, 5, l 11 are prime numbers and you can easily add to this list. egers which are not prime, like 4, 6, 8, and 9 are called ***nposite***. Now try some numbers for yourself. You find, example, that 31 is prime since its only factors are 1 and 31, 51 is composite since 51 = 17 × 3. Again, 30 = 5 × 6 = × 2 × 3. These numbers, 2, 3, and 5 are called the ***prime*** ***tors*** of 30. Similarly, the prime factors of 36 are 2, 2, 3, l 3.

The study of prime and composite numbers is full of fasci- ing mathematical riddles to which men have not yet found out the answers. Notice in what an irregular way the prime nbers are distributed among the integers. Between 90 and the only prime number is 97 whereas between 100 and 110 re are four primes, 101, 103, 107, and 109! One fact which been found out is that no matter how large a number you ne there are prime numbers which are larger. If you write vn 1 followed by 100 zeros you have an immensely large nber. It was named a *googol* by a nine-year-old boy and the ne has stuck! It is sure that there are prime numbers larger n a googol but no one has yet been able to name one. Would like to see the largest number which anyone knows to be ne? Here it is. Do not try to remember it!

170,141,183,460,469,231,731,687,303,715,884,105,727.

EXERCISES

1. Is any even number prime? Explain.

2. What is the smallest composite odd number?

3. What prime factors are contained in all integers which end ? Explain.

4. Write the prime numbers between 1 and 50.

5. Write the composite numbers between 50 and 60.

6. Express as the product of prime factors: 18, 24, 33, 56, 96.

7. In the formula $N = n^2 - n + 1$, give n the integral val from 1 to 10 inclusive. How many of the resulting numbers are prime and how many are composite?

8. Write each of the even numbers from 2 to 20 as the s of two prime numbers.

(The conjecture that every even number is the sum of t primes is known as Goldbach's theorem. Although it has b verified for all numbers up to 10000 no one has yet given a gene proof.)

Multiples

The fact that 5 is a factor of 30 is also expressed by say that 30 is a multiple of 5. *Any integer is a multiple of any of factors.* The successive multiples of 3 are 3, 6, 9, 12 and so you merely begin with 3 and count by 3's. This means t when you learned the 3-times multiplication table you w merely learning the successive multiples of 3. You can m similar statements for multiples of any other number.

EXERCISES

(for oral answers)

1. Give the first eight multiples of 4 and those of 6.

2. 50 = $2\frac{1}{2}$ times 20. Does this make 50 a multiple of

Explain.

Read the following sentences with blanks filled in.

3. Any even number is a multiple of —— .

4. 15, 20, 25 and 30 are all multiples of —— .

5. 21 is a multiple of —— and also of —— .

6. If a is a multiple of b then b is a —— of a.

Common Factors and Common Multiples

We often say of a friend of mine who is also a friend of yours t he is a common friend of us both, or that we have a friend in nmon. This use of the word common is applied to factors and o to multiples.

Common Factors. Let us write the factors (omitting unity) 6 and those of 8.

Factors of 6: 2, 3, 6.
Factors of 8: 2, 4, 8.

e number 2 is a common factor of 6 and 8. It is the only nmon factor, other than 1 (We shall usually not mention the tor 1). Now try 24 and 30.

Factors of 24: 2, 3, 4, 6, 8, 12, 24.
Factors of 30: 2, 3, 5, 6, 10, 15, 30.

ee factors are common to 24 and 30, namely 2, 3 and 6. The atest of these common factors is 6; the other common factors factors of it.

Common Multiples. Is there any number which is a multiple and also of 8? How many such multiples are there? Is there mmon multiple which is greater than all the others? Is there ommon multiple which is less than all the others? If the wers to these questions are obvious to you, you may omit the t two paragraphs.

Let us write the successive multiples of 6 and those of 8.

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, . . .
Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, . . .

s clear that these lists of multiples are endless. A number no greatest multiple.

You see also that 6 and 8 have common multiples: 24, 48, 96 and so on. There is no largest common multiple; ,000,000 is a common multiple and it is only an infant com-ed with others which you can write if you try. On the other d, there is no common multiple less than 24; all the other mon multiples are, in fact, multiples of it.

EXERCISES

(*Answer orally*)

For each of the following groups of numbers, give all common factors. Also pick out the *greatest common factor.*

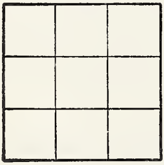
1. 15 and 20
2. 16 and 24
3. 30 and 40
4. 24 and 26
5. 36 and 48
6. 42 and 56
7. 4, 6 and 8
8. 6, 12 and 15
9. 20, 30 and 4
10. 30, 36 and 42
11. 32, 44 and 60
12. 15, 300 and

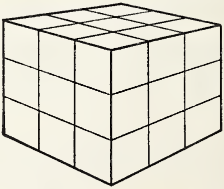
For each of the following groups of numbers give two com multiples. Also pick out the *least common multiple.*

13. 7 and 9
14. 5 and 15
15. 8 and 10
16. 12 and 16
17. 15 and 25
18. 32 and 48
19. 22 and 33
20. 30 and 45
21. 24 and 60
22. 2, 3 and 4
23. 9, 12 and 18
24. 6, 9 and 15

Powers of Integers

You will recall from Chapter III that a^2 means the produc two a's, a^3 the product of three a's and so on for other ***pow*** It is easy to see why a^2 came to be called the ***square of a*** si if the side of a square is a units long, its area is a^2 square un Also if the edge of a cubical block is a units long, the volum the block is a^3 cubic units, which explains why a^3 is termed





cube of a. Other powers, such as a^4, a^5, . . . cannot be illustra by figures in this way.

When a is an integer, a^2 is also an integer which is calle *perfect square.* The perfect square integers, in order, are: $1^2=$

= 4, $3^2 = 9$, $4^2 = 16$, and so on. Now take the perfect cubes. u get 1, 8, 27, 64, and so on. In this way we may continue h fourth and still higher powers of the integers.

Working with powers often yields some surprising results. us compute 2^{10}. You find the product of five 2's to be 32 l therefore the product of ten 2's is 32×32 or 1024. So = 1024. Again 2^{20} is the product of 20 2's or 1024×1024 ich gives $2^{20} = 1{,}048{,}576$. This large number of over a million vritten very concisely as a power.

A more striking example of the conciseness of the power ation is in the representation of the immense number which s named the *googol*. This number, 1 followed by 100 zeros, 0^{100}.

To illustrate further the usefulness of the power notation for ge numbers, consider the following problem. How many grains vheat are contained in a year's yield of Canada's wheat crop? us suppose the yield is 500 million bushels, which can be ressed as 5×10^8 bushels. In our table of weights we learn t 1 pound avoirdupois is equal to 7000 grains. Now 1 grain s originally supposed to be the weight of a grain of dry wheat en from the centre of the ear. It follows that 1 bushel of wheat tains about 60×7000 or 420,000 grains. This is 42×10^4 ins. The 500 million bushel crop would then contain 5×10^8 42×10^4 grains. This number is 210×10^{12} or 21×10^{13}. at is, the number of grains lies between 2×10^{14} and 3×10^{14}. ile this is a huge number, it falls far short of a *googol*. In fact, could soon convince yourself that a *googol* is larger than the nber of grains of wheat that have been grown in all the earth ce the ice age!

EXERCISES

1. Give the values of: (*a*) the square of 50, (*b*) the cube of 4, the fourth power of 3, (*d*) the twentieth power of 1.

2. Find the values of: (*a*) 2^7, (*b*) 5^4, (*c*) 9^5, (*d*) 100^3.

3. Write the squares of the integers from 1^2 to 20^2.

4. Write the cubes of the integers from 1^3 to 10^3.

5. Calculate (*a*) $2^3 \times 3^4 \times 4^2$, (*b*) $2^3 - 3 \times 2$, (*c*) $6^4 - 5 \times$ (*d*) $10^3 - 9 \times 10^2$.

6. Find the value of 2^{2^2}. (This is taken to mean $2^{(2^2)}$). N try 3^{3^3}. Without working out its value estimate the number digits in it. (Use the fact that 3^9 is close to 20,000.)

7. Here are four ways of arranging three 2's: 222, 22^2, 2^{22}, List these numbers in order of magnitude.

8. By analogy with the numbers in the preceding exerci arrange four 2's in as many ways as you can. Then list t resulting numbers in order of magnitude.

The Power Reverses Gears

The ***square root*** of the number a^2 is a. In other words, t square root of one number is a second number whose square equal to the first one. Since $25 = 5^2$ then the square root 25 is 5. This is written $\sqrt{25} = 5$. You can now supply t values of $\sqrt{49}$, $\sqrt{64}$, $\sqrt{100}$, $\sqrt{144}$.

Similarly, the ***cube root*** of a^3 is a. Now make for yours a definition of the cube root of a number. Since $125 = 5^3$ he the cube root of 125 is 5. This is written $\sqrt[3]{125} = 5$. N supply values for $\sqrt[3]{1}$, $\sqrt[3]{8}$, $\sqrt[3]{64}$, $\sqrt[3]{1000}$.

What do you understand by the fourth root of a numb Give the values of $\sqrt[4]{16}$, $\sqrt[4]{625}$, $\sqrt[4]{x^4}$.

Although squaring an integer always gives another integ it is not the same with taking square roots. The square r of an integer is generally not an integer and in such cases otl methods of calculation are called for. However, you can es mate roots of numbers when the roots are not integers. It clear, for example, that $\sqrt{80}$ is slightly less than 9. $\sqrt{110}$ between 10 and 11, and $\sqrt{600}$ is between 24 and 25. To ta one more example, $\sqrt[3]{200}$ is between 5 and 6, since 200 is betwe 125 (5^3) and 216 (6^3).

For your convenience a table of the squares and the squa roots of the integers from 1 to 100 is given on page 392.

EXERCISES

(Try these by inspection.)

1. Name the numbers whose square roots are 3, 4, 5, 6, 7, 8, 10.

2. Give the values of $\sqrt{169}$, $\sqrt{256}$, $\sqrt{289}$, $\sqrt{400}$, $\sqrt{3600}$, $\overline{1{,}000{,}000}$.

3. Give the values of $\sqrt[3]{216}$, $\sqrt[3]{729}$, $\sqrt[3]{8000}$, $\sqrt[3]{27{,}000{,}000}$.

4. Give the values of $\sqrt[5]{32}$, $\sqrt[4]{81}$, $\sqrt[100]{1}$, $\sqrt[6]{1{,}000{,}000}$.

5. Find between what two consecutive integers each of the lowing roots lies: (*a*) $\sqrt{50}$, (*b*) $\sqrt{75}$, (*c*) $\sqrt{90}$, (*d*) $\sqrt{150}$, $\sqrt{220}$, (*f*) $\sqrt{420}$, (*g*) $\sqrt[3]{16}$, (*h*) $\sqrt[3]{30}$, (*i*) $\sqrt[3]{65}$, (*j*) $\sqrt[3]{124}$, $\sqrt[3]{300}$, (*l*) $\sqrt[3]{1020}$.

REVIEW EXERCISES

A

1. What are (*a*) even integers, (*b*) odd integers?

2. What is (*a*) a factor of an integer, (*b*) a multiple of an eger?

3. Find the prime factors of (*a*) 60, (*b*) 56, (*c*) 80, (*d*) 330, 108, (*f*) 700.

4. Give the common factors of the following groups of mbers and pick out the greatest common factor of each group.

(*a*) 20 and 50 (*b*) 12 and 15 (*c*) 22 and 33
(*d*) 28 and 36 (*e*) 6, 9 and 15 (*f*) 8, 12 and 20
(*g*) 50, 75 and 100 (*h*) 18, 30 and 42 (*i*) 24, 36 and 60

5. Give two common multiples of the numbers in each of the lowing groups. For each group pick out the least common ltiple.

(*a*) 3 and 7 (*b*) 4 and 6 (*c*) 22 and 33
(*d*) 8 and 36 (*e*) 12 and 20 (*f*) 4, 8 and 12
(*g*) 10, 15 and 20 (*h*) 3, 9 and 15 (*i*) 5, 7 and 21

6. What is the smallest sum of money which I can pay co pletely in nickels, in dimes, or in quarters?

7. What is the shortest length that I can measure evenly wi a 6-inch rod, or with an 8-inch rod, or with a 9-inch rod?

8. In the following list name the integers which are (*a*) prim (*b*) composite, (*c*) odd, (*d*) multiples of 3, (*e*) perfect squar (*f*) perfect cubes.

6, 7, 8, 9, 15, 16, 17, 24, 25, 27, 29.

9. Read the following sentence with the proper word in t blank space:

If m is a factor of n, then n is a ——— of m.

10. Choose the proper word, *factor* or *multiple*, for each the blank spaces in the following: An integer is a ——— of i square; the square of an integer is a ——— of the integer.

11. Estimate each of the following by giving the near number in which all the digits except the first are zeros: (72 × 41, (*b*) 453 × 52, (*c*) 896 × 12, (*d*) 653 × 8, (*e*) 536 ÷ (*f*) 6375 ÷ 15, (*g*) 9247 ÷ 80, (*h*) 51044 ÷ 7.

B

12. What integer between 1000 and 1100 is a perfect fif power?

13. Is any integer between 300 and 600 a perfect four power?

14. Show that any integer whose last two digits are 00 exactly divisible by 4 and that one whose last three digits are 0 is exactly divisible by 8.

15. Suggest a rule for multiplying by 5 by division. Use yo rule to calculate 6847 × 5. Do you find the rule worth whi

16. Suggest a rule for multiplying by $12\frac{1}{2}$ by division. U your rule to calculate 872 × $12\frac{1}{2}$.

17. The number 3472 can be written 3400 + 72. Use th method of writing a number to show that an integer is divisi by 4 if the number formed by its last two digits is divisible by

18. Tell by inspection which of the following numbers are visible by 4:

704, 536, 758, 590, 3364.

19. Observe that $67 \times 99 = 67 \times (100 - 1) = 6700 - 67 =$ 33. Use a similar method to find by subtraction (*a*) 83×99,) 26×99, (*c*) 42×990, (*d*) 16×98.

20. Here is a rule for telling leap years. A leap year is one hose number is (*a*) a multiple of 4 but not of 100 *or* (*b*) a multiple of 400. Name the leap years from 1950 to 1970. Which the century years, 1900 to 2500, are leap years?

21. If you were taking part in a *counting marathon* you might ve to stop on account of fatigue or hunger. Would you ever ve to stop because of running out of numbers? Is there any *t* integer in the sequence 1, 2, 3, . . . ?

C

22. What integer is a perfect *n*th power for all integral values *n*?

23. By breaking up the numbers into their prime factors, find e greatest common factor (sometimes called the *highest common ctor*, or H. C. F.) of: (*a*) 270, 396 and 558; (*b*) 315, 420 and 252;) 180, 630, 660 and 924.

24. By breaking up the numbers into their prime factors, find e least common multiple (L. C. M.) of: (*a*) 132, 210 and 308;) 207, 217 and 483; (*c*) 24, 68, 117 and 221.

25. Two integers are said to be prime to each other if their ly common factor is 1. Give an example of two integers ither of which is prime but which are prime to each other. hat is the H. C. F. of such integers? How would you find eir L. C. M.?

26. Show that the L. C. M. of *two* numbers equals the product the two numbers divided by their H. C. F. By means of amples, find out whether this is true for *three* numbers.

27. If it were given that the H. C. F. of two numbers is eq to their L. C. M., what conclusion would you draw regarding t numbers?

RECREATIONS

1. *Twelve-finger arithmetic.* If human beings had 12 finge instead of 10 they would probably have learned to count by 1 instead of by 10's. What then would have been the meaning the numbers 17, 23, 136?

2. *The sieve of Eratosthenes.* The ancient Greek mathem tician Eratosthenes devised a process for sifting out the pri numbers. Writing the integers in order,

$$1\ \ 2\ \ 3\ \ \cancel{4}\ \ 5\ \ \cancel{6}\ \ 7\ \ \cancel{8}\ \ \cancel{9}\ \ \cancel{10}\ \ 11\ \ldots,$$

he crossed out every second number after 2, then every thi number after 3, every fifth number after 5, etc. Are the numbe crossed out prime or composite? What is the character of t remaining numbers? Use the sieve of Eratosthenes to find t prime numbers less than 200.

3. By a process similar to that described in the precedin exercise, sift out all the integers between 1 and 50 which are n multiples of 2 or of 3.

4. *A multiplication game.* Notice the form of the produ you get by multiplying 37 by 3. Next, compare the value 37 × 6 with that of 37 × 3. Now write down without mult plication the products 37 × 6, 37 × 9, 37 × 12, 37 × 15, 37 × 1 37 × 21, 37 × 24, and 37 × 27.

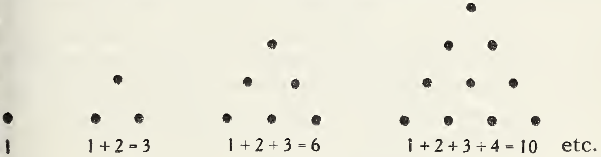
5. *A magic square.* In the following block of numbers

8	1	6
3	5	7
4	9	2

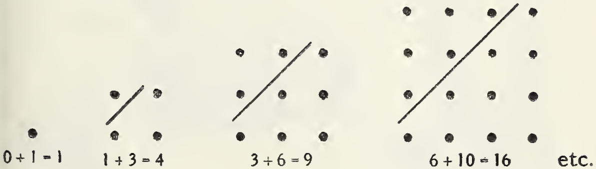
d up the three digits in each horizontal row, then add those in h vertical column, and finally those in each of the two diagls. Now can you tell why this array is called a *magic square*? y to build up another magic square with three rows. If you curious about these squares, ask your librarian for a book led *Mathematical Recreations and Essays* by W. W. R. Ball.

6. *Triangular numbers.* We have already mentioned the eek scholar Pythagoras and you will learn more about him er. He founded a secret mathematical school or lodge about e centuries B.C.

The Pythagoreans called the numbers 1, $1 + 2$, $1 + 2 + 3$, $+ 2 + 3 + 4$, etc. triangular numbers. The reason for this ne you will see from the following illustration



If any two adjacent triangular numbers are added together, sum is a square number.



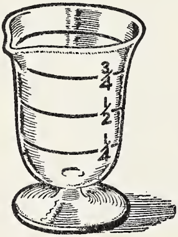
Write the first ten triangular numbers and verify for them relation between square and triangular numbers.

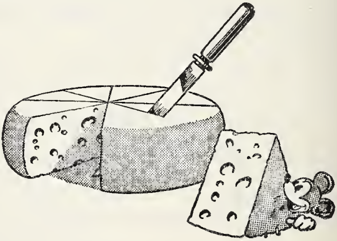
CHAPTER VII

FRACTIONS AND DECIMALS

> "*Two husbands have managed to acquire three wives. That's two-thirds of a husband to each wife.*"
>
> W. S. Gilbert.

In their household science classes, girls become accustom to recipes involving three-quarters of a cup of flour or third of a cup of sugar; in wood-working shops, boys ma measurements such as six and five-eighths inches or three-s teenths of an inch. In order to handle these measures with ea you must be able to work with ***fractions***. Civilized peo have always needed fractions in order to record lengths, volum weights or other measures when these could not be expressed whole numbers of units. We break our whole numbers in parts called fractions (just as we call a break in a bone a fractur





When we divide the number 1 into 5 equal parts, you kn that each of these parts is called one-fifth, which is written $\frac{1}{5}$ sometimes 1/5. Three of these parts together make the fr tion $\frac{3}{5}$. Since three of the parts are 3 times as much as one pa it follows that

$$\tfrac{3}{5} = \tfrac{1}{5} \times 3.$$

ce 5 fifths make up the whole unit, we have $\frac{5}{5} = 1$. Again, s greater than 1 as are also $\frac{7}{4}$, $\frac{19}{16}$, and so on.

So much for particular fractions. Let us now condense the tter into statements which apply to all fractions. For this rpose we recall the algebraic device of using letters to stand any numbers. You have already found that letters used in s way are convenient mathematical slaves which do the work many words. So we may speak of the fraction a/b, where nd b are integers. The ***denominator*** b indicates the num- r of equal parts into which the unit has been divided and the ***imerator*** a tells the number of these parts which make up fraction a/b.

The numerator and denominator of a fraction are called its ***rms***. If the numerator is less than the denominator then fraction has a value less than unity and is called a ***proper iction***. So 3/5, 5/8, 16/25 are examples of proper fracns. If the numerator is greater than the denominator, the ction is greater than unity and is called an ***improper fracon***. Examples of improper fractions are 4/3, 9/8, 16/5,—you n extend this list for yourself. If the numerator and denomitor of a fraction are equal, as in 5/5 or a/a, the fraction stands 1. A whole number can be written in the *form* of a fraction using the denominator 1; so $5 = 5/1$ and $a = a/1$.

If a is less than b, a/b is a proper fraction;
if a is greater than b, a/b is an improper fraction;
$a/a = 1; \qquad a/1 = a.$

EXERCISES

(*for oral answers*)

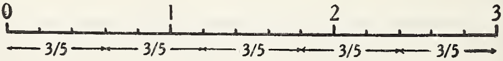
1. Name the numerator and the denominator of each of the llowing fractions: three-eighths, five-twelfths, sixteen-thoundths, 15/32, 35/40, 1/64, 12/1.

2. Of the following fractions pick out those which are pro and those which are improper: 7/8, 8/7, 15/2, 99/100, 16/ 31/29, 101/99.

3. A standard concrete mixture is formed by mixing one p of cement with 2 parts of sand and 4 of gravel. What fract of the mixture is each ingredient?

How Fractions Behave The Basic Principles

If a line 1 inch long is divided into 5 equal parts, then 3 these parts together make up 3/5 of an inch. Now take a l



3 inches long and let each inch be divided into fifths. The figu shows that the 3-inch line contains 5 of the 3/5 inch segmen In other words, 3/5 of 1 inch is 1/5 of 3 inches. Since we mig equally well have used pounds or quarts for illustration inste of inches, what we have here is really a property of number That is

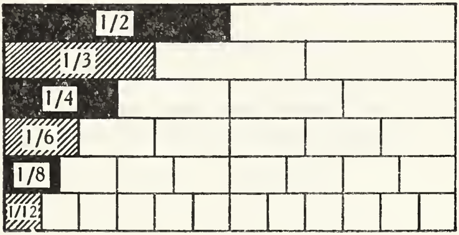
$$3/5 = 1/5 \text{ of } 3 \qquad \text{or} \qquad 3/5 = 3 \times 1/5.$$

We can put the matter in another way. When a 3-inch li is divided into 5 equal parts, one of these parts is 3/5 of an inc Dropping the inches, we see that 3/5 is one of the parts obtain when 3 is divided into 5 equal parts. This can be abbreviat by saying that 3/5 is 3 divided by 5, or 3/5 = 3 ÷ 5. Now repe the reasoning in this paragraph, using another fraction, say 2/ or 4/7, and making the appropriate figure for yourself. You s that this really applies to all fractions. We therefore call in o slaves a and b to condense the results into a formula.

First basic principle of fractions

$$a/b = a \div b$$

A second important principle can be deduced from the figure which a unit length is divided into halves, thirds, quarters, ths, eighths, and twelfths. You have only to look closely at e dividing lines to see that one-half is the same as 2 fourths or sixths or 4 eighths or 6 twelfths. This is more easily expressed symbols:



$$\tfrac{1}{2} = \tfrac{2}{4} = \tfrac{3}{6} = \tfrac{4}{8} = \tfrac{6}{12}.$$

nother glance shows that

$$\tfrac{2}{3} = \tfrac{4}{6} = \tfrac{8}{12}.$$

milarly you could make or imagine further divisions of the unit hich would show that

$\tfrac{1}{2} = \tfrac{16}{32} = \tfrac{50}{100}$ and also that $\tfrac{2}{3} = \tfrac{18}{27} = \tfrac{44}{66}$ and so on.

e conclude that if the numerator and denominator of a fraction e both multiplied by the same integer the value of the fraction not changed.

Let us follow this reasoning a step further. We see that 21 = 2/7, since 6/21 is obtained from 2/7 by multiplying umerator and denominator by 3. This means that if the numator and denominator of a fraction contain a common factor is factor can be divided out. We find, for example, that

$$\tfrac{36}{84} = \tfrac{18}{42} = \tfrac{9}{21} = \tfrac{3}{7}.$$

nce 3 and 7 have no common factor except 1, the fraction 3/7 said to be in its ***lowest terms***. Any fraction is reduced to s lowest terms by dividing into the numerator and denominator their highest common factor. Since the results just obtain apply to any fraction a/b, we are ready to state another ba principle.

Second basic principle of fractions

***A fraction is not changed in value by multiplying, or b dividing, its numerator and denominator by the sam number, so long as this number is not* 0.**

In symbols, $\frac{a}{b} = \frac{ac}{bc}$.

EXERCISES

Copy the following fractions into your workbook and fill the missing terms

1. $\frac{4}{9} = \frac{\quad}{27} = \frac{\quad}{72} = \frac{20}{\quad} = \frac{240}{\quad}$

2. $\frac{3}{16} = \frac{\quad}{32} = \frac{\quad}{96} = \frac{12}{\quad} = \frac{21}{\quad}$

Reduce to their lowest terms:

3. $\frac{8}{18}$ **4.** $\frac{12}{30}$ **5.** $\frac{9}{36}$ **6.** $\frac{18}{45}$ **7.** $\frac{63}{70}$ **8.** $\frac{45}{60}$ **9.** $\frac{2}{28}$

10. $\frac{175}{225}$ **11.** $\frac{112}{128}$ **12.** $\frac{91}{143}$

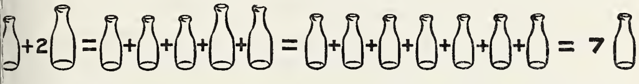
We Put the Fractions to Work

We shall now call in our four friends, whom we labelled −, × and ÷ and introduce them to the fractions. First, en *addition* and *subtraction.*

By referring to the figure, page 223, you see at once that

$$\tfrac{1}{6} + \tfrac{3}{6} = \tfrac{4}{6} \qquad \text{and} \qquad \tfrac{3}{8} + \tfrac{5}{8} = \tfrac{8}{8}.$$

each of these sums, the fractions have the same denominator they are of the same ***denomination.*** To add 1/6 to 3/6 is ke adding 1 quart to 3 quarts. It is different when we add pints to 2 quarts. Here we should reduce both quantities to



e same denomination, pints, so that 3 pints + 2 quarts = 3 nts + 4 pints = 7 pints. So, with 2/3 + 5/6, we reduce the st fraction to the denomination *sixths* by multiplying its terms 2, obtaining

$$\tfrac{2}{3} + \tfrac{5}{6} = \tfrac{4}{6} + \tfrac{5}{6} = \tfrac{9}{6} = \tfrac{3}{2}.$$

nilarly,

$$\tfrac{1}{6} + \tfrac{1}{4} = \tfrac{2}{12} + \tfrac{3}{12} = \tfrac{5}{12};$$

so,

$$\tfrac{5}{9} - \tfrac{1}{6} = \tfrac{10}{18} - \tfrac{3}{18} = \tfrac{7}{18}.$$

ie fractions 1/6 and 1/4 were changed into fractions with the mmon denominator 12, 5/9 and 1/6 into fractions with the mmon denominator 18. This is the operation of ***reducing a common denominator.*** We could have chosen larger mmon denominators, for example,

$$\tfrac{1}{6} + \tfrac{1}{4} = \tfrac{4}{24} + \tfrac{6}{24} = \tfrac{10}{24}.$$

makes the result simpler to use the least of the common nominators, that is, the least common multiple of the denomtors. Now take the example

$$\tfrac{3}{8} + \tfrac{5}{12} - \tfrac{2}{3}.$$

e least common denominator is 24. Hence we express each ction in 24ths. Then

$$\tfrac{3}{8} + \tfrac{5}{12} - \tfrac{2}{3} = \tfrac{9}{24} + \tfrac{10}{24} - \tfrac{16}{24}$$
$$= \frac{9 + 10 - 16}{24} = \tfrac{3}{24} = \tfrac{1}{8}.$$

With practice, you will probably prefer to leave out the seco step and proceed as in the following example:

$$\frac{7}{15} - \frac{5}{12} + \frac{3}{20} = \frac{28 - 25 + 9}{60} = \frac{12}{60} = \frac{1}{5}.$$

Rule for adding or subtracting fractions

1. *Find the least common denominator.*
2. *Replace each fraction by an equal fraction having the chosen common denominator.*
3. *Combine the numerators as indicated by the signs + or − to obtain the numerator of the answer.*

You will recall that a fraction greater than 1 is called i proper. For example, the improper fraction 9/4 can be writt

$$\frac{9}{4} = \frac{8}{4} + \frac{1}{4} = 2 + \frac{1}{4}$$

This is the same reasoning as we use when we say that 9 quarts 2 gallons + 1 quart. The number 2 + 1/4 is written $2\frac{1}{4}$ a is called a ***mixed number***. By dividing the denominator in the numerator, any improper fraction can be written as a mix number. Now discover for yourself how to change a mix number into an improper fraction. You see that $\frac{23}{6} = 3$ $5\frac{3}{8} = \frac{43}{8}$, and so on.

EXERCISES

(*The first 20 may be answered orally*)

Find the values of:

1. $\frac{4}{9} + \frac{5}{9}$
2. $\frac{1}{2} + \frac{1}{4}$
3. $\frac{3}{10} + \frac{2}{5}$
4. $\frac{7}{12} - \frac{1}{3}$
5. $\frac{1}{2} + \frac{1}{3}$
6. $\frac{2}{3} - \frac{1}{4}$
7. $\frac{1}{7} + \frac{1}{8}$
8. $\frac{2}{3} - \frac{3}{5}$
9. $\frac{3}{4} + \frac{5}{6}$

Express as mixed numbers:

10. $\frac{17}{3}$ **11.** $\frac{26}{7}$ **12.** $\frac{31}{10}$ **13.** $\frac{54}{11}$ **14.** $\frac{76}{15}$ **15.** $\frac{47}{16}$

press as improper fractions:

16. $6\frac{2}{3}$ 17. $5\frac{4}{9}$ 18. $11\frac{3}{4}$ 19. $3\frac{7}{12}$ 20. $16\frac{2}{9}$

21. If the tail of a fish measures 1/10 of its total length and body 3/4, what fraction of its length is the head?

22. John spends 3/8 of his day in sleep, 1/9 at meals, 1/3 at rk and the rest at play. What fraction of the day does he y?

23. If I budget 1/4 of my income for rent, 1/10 for clothing, 1/3 for food, what will be left for other purposes?

24. Mr. Case planned a 4-day journey so as to cover 1/4 of distance each day. The second day, on account of tire uble, he covered only 1/7 of the distance. What fraction of journey should he cover the third day so as to get back on edule?

d the values of:

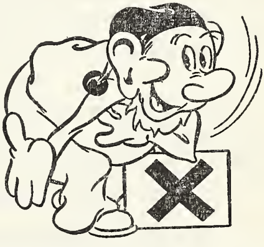
$\frac{6}{35} - \frac{8}{49}$ 26. $\frac{25}{54} - \frac{19}{42}$ 27. $\frac{3}{56} + \frac{5}{72}$

$\frac{32}{75} - \frac{37}{90}$ 29. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ 30. $\frac{1}{5} - \frac{1}{6} + \frac{1}{15}$

$\frac{2}{3} + \frac{5}{6} - \frac{11}{12}$ 32. $\frac{5}{9} - \frac{4}{15} + \frac{7}{10}$ 33. $\frac{1}{6} - \frac{1}{7} + \frac{1}{8}$

$\frac{5}{24} - \frac{19}{42} + \frac{11}{36}$ 35. $\frac{4}{5} - \frac{3}{4} + \frac{2}{3} - \frac{1}{2}$ 36. $\frac{4}{15} - \frac{3}{16} + \frac{7}{24} - \frac{11}{40}$

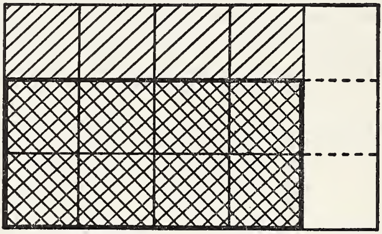
Multiplication



It is clear that the value of 2 es 3/5 is 6/5. This is an example fact which you see to be true for fractions: *the product of a fraction in integer is obtained by multiply- the numerator by the integer and ing the denominator unchanged.* s long statement can be condensed a formula by another use of our hful slaves a, b and c; if these letters denote integers, then

$$\frac{a}{b} \times c = \frac{ac}{b}.$$

How shall we multiply two fractions? Let us see wheth we can work out a reasonable meaning for $\frac{1}{2} \times \frac{1}{2}$ or $\frac{2}{3} \times \frac{4}{5}$. Fir notice that we can illustrate the product 2 × 3 by thinking 2 bundles, each of 3 sticks, or, to abbreviate, 2 of 3. Now let agree that $\frac{1}{2} \times \frac{1}{2}$ shall mean $\frac{1}{2}$ of $\frac{1}{2}$ and that $\frac{2}{3} \times \frac{4}{5}$ shall me $\frac{2}{3}$ of $\frac{4}{5}$. If this is agreed, then the meaning begins to appe Anyone who has ever cut up an apple knows that $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$ a that $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$. To get $\frac{2}{3}$ of $\frac{4}{5}$ of an apple would take too mu skill to draw so we shall take a rectangle instead. By means o vertical lines the whole rectangle is divided into fifths; let take the four strips on the left to make up $\frac{4}{5}$ of the whole. N by means of 2 horizontal lines we divide this latter rectangle in thirds. The rectangle with the heavy boundary is $\frac{2}{3}$ of $\frac{4}{5}$ of t whole. Now count up the small blocks. The whole rectan has 15 and the rectangle with the heavy border has 8. It follo then that $\frac{2}{3}$ of $\frac{4}{5}$ is $\frac{8}{15}$, or that



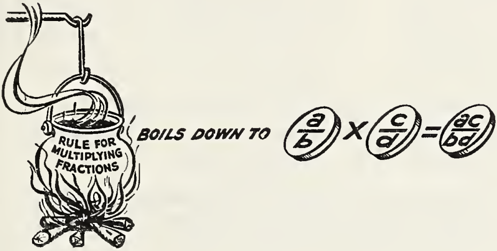
$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}.$$

Now dissect a rectangle for yourself to show that $\frac{3}{4}$ of $\frac{5}{7}$ is The reasoning in these examples leads us to make the followi rule.

Rule for multiplying fractions

1. *Multiply the numerators.*
2. *Multiply the denominators.*
3. *Divide the product of the numerators by that of the denominators.*

Again let us call on our slaves, the letters, to put this defi- ion into a concise form. It all boils down to this: $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$ d here is something to remember it by:



Short cuts in multiplying fractions. Consider this example:

$\frac{4}{9}\times\frac{3}{8}=\frac{12}{72}$ (rule for multiplying fractions)

$=\frac{1}{6}$ (second basic principle; dividing 12 from numerator and denominator).

ce the factors 4 and 3 are eventually divided out of both terms the product, they could have been divided out *before* the ctions were multiplied. This saves work and the multipli- ion now appears:

$$\frac{4}{9}\times\frac{3}{8}=\frac{\overset{1}{\cancel{4}}\times\overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}}\times\underset{2}{\cancel{8}}}=\frac{1}{6}.$$

take another example, we have

$$\frac{9}{2}\times\frac{8}{3}\times\frac{7}{4}=\frac{\overset{3}{\cancel{9}}\times\overset{1}{\cancel{8}}\times 7}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{3}}\times\underset{1}{\cancel{4}}}=\frac{21}{1}=21.$$

EXERCISES

(The first 15 may be answered orally)

Find the values of:

1. $\frac{1}{7} \times 3$

2. $4 \times \frac{1}{9}$

3. $6 \times \frac{1}{9}$

4. $8 \times \frac{1}{6}$

5. $\frac{5}{8} \times 12$

6. $\frac{1}{4} \times \frac{1}{5}$

7. $\frac{3}{8} \times \frac{5}{7}$

8. $\frac{5}{6} \times \frac{2}{3}$

9. $\frac{3}{4} \times \frac{8}{9}$

Find the cost of:

10. $5\frac{1}{2}$ yards of muslin at 20 cents a yard;

11. 250 feet of lumber at $40 a thousand;

12. 30 inches of tape at 18¢ a yard;

13. 3 quarts of vinegar at 80¢ a gallon;

14. $2\frac{3}{4}$ lb. of cheese at 16¢ a pound;

15. $3\frac{1}{2}$ yards of cotton at 18¢ a yard.

Find the values of:

16. $\frac{9}{16} \times \frac{8}{15}$

17. $3\frac{1}{7} \times 5$

18. $2\frac{1}{3} \times 3\frac{1}{4}$

19. $5\frac{2}{5} \times 4\frac{4}{9}$

20. $\frac{10}{4} \times \frac{3}{5} \times \frac{2}{9}$

21. $\frac{7}{8} \times \frac{6}{11} \times$

22. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$

23. $(\frac{1}{2} + \frac{1}{3}) \times (\frac{1}{3} + \frac{1}{4})$

24. $\frac{35}{42} \times \frac{24}{55} \times$

25. Find the cost of a $5\frac{3}{4}$–pound roast of beef at $21\frac{1}{2}$¢ a pou

26. Find the cost of flooring for a room $9\frac{1}{2}$ feet by $8\frac{3}{4}$ feet 16¢ a square foot.

27. Find the volume of a rectangular bin $3\frac{1}{2}$ feet wide, $5\frac{1}{3}$ f long and $4\frac{5}{6}$ feet high.

28. A rectangle is divided into smaller rectangles by drawi 4 lines equally spaced parallel to two sides and 6 lines equa spaced parallel to the other two sides. A block of 8 of th small rectangles is what fraction of the whole area?

29. A child builds, with cubical building blocks, a rectangu solid, 10 blocks long, 6 blocks wide, and 5 blocks high. W fraction of all the blocks are in the outside layers? (Make a gu first, then calculate.)

Division

Take some integers first. From the fact t $3 \times 4 = 12$, we deduce that $12 \div 3 = 4$. ilarly, if $a \times b = c$ then $c \div a = b$. This rue when the letters denote integers. Let now agree that this shall be adopted as a ciple when the letters denote integers or ctions. Now try it out. We have

$$\tfrac{4}{5} \times \tfrac{2}{3} = \tfrac{8}{15}.$$

ce $\frac{8}{15} \div \frac{2}{3}$ should be $\frac{4}{5}$. Now you notice that

$$\tfrac{8}{15} \times \tfrac{3}{2} = \tfrac{4}{5}.$$

other words, $\frac{8}{15} \div \frac{2}{3}$ has the same value as $\frac{8}{15} \times \frac{3}{2}$. ***To ide by 2/3 we multiply by 3/2.*** Is this a general rule? us see whether our slaves will answer this question for us. know that

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}.$$

ce

$$\frac{ac}{bd} \div \frac{c}{d} = \frac{a}{b}.$$

$$\frac{ac}{bd} \times \frac{d}{c} = \frac{a}{b}.$$

ollows that to divide a fraction by c/d is the same as multing it by d/c. Since c/d may stand for any fraction, we are ly to state a general rule. Before doing so, however, we shall oduce a new term.

If the numerator and denominator of the fraction 2/3 play frog, we get 3/2; the two fractions are called ***reciprocals.*** ilarly, 2 and $\frac{1}{2}$, 1/10 and 10, 7/4 and 4/7, a/b and b/a are s of reciprocal numbers. We can now state the definition ivision:

To divide a number by a fraction,
multiply the number by the reciprocal of the fraction.

This rule, like that for multiplication, can be boiled down in a formula. Here it is:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}.$$

EXERCISES

(The first 16 may be answered orally)

1. Give the reciprocals of: $\frac{1}{8}$, 21, $\frac{5}{6}$, $3\frac{1}{2}$, $8\frac{2}{5}$.

Give the values of:

2. $\frac{4}{7} \div 2$
3. $\frac{9}{16} \div 3$
4. $\frac{5}{8} \div 2$
5. $\frac{8}{9} \div 6$
6. $\frac{5}{16} \div 10$
7. $\frac{12}{13} \div 18$
8. $5 \div \frac{1}{2}$
9. $6 \div \frac{2}{3}$
10. $15 \div \frac{5}{8}$
11. $\frac{1}{2} \div \frac{1}{3}$
12. $\frac{2}{5} \div \frac{1}{2}$
13. $\frac{3}{4} \div \frac{3}{5}$
14. $\frac{6}{7} \div \frac{2}{7}$
15. $\frac{5}{8} \div \frac{3}{4}$
16. $\frac{8}{15} \div \frac{4}{5}$

17. How many marbles can be bought for 20c if they are a dozen?

18. How many $2\frac{7}{8}$ inch floor boards would be required cover a width of 5 feet?

19. In a planing mill a pile of boards is $6\frac{1}{2}$ feet high. If ea board is $\frac{13}{16}$ of an inch thick, how many are in the pile?

20. A cubic foot holds $6\frac{1}{4}$ gallons. How many gallons contained in a reservoir which is $2\frac{1}{2}$ feet long, 9 inches wide, a $1\frac{1}{2}$ feet deep?

21. A U.S. gallon is $\frac{4}{5}$ of a Canadian gallon. How ma Canadian gallons are equivalent to $6\frac{1}{2}$ U.S. gallons?

Complex Fractions

Our original definition of a fraction requires that the nu erator and denominator be integers. We can now extend t definition in such a way that our two basic principles shall ap even when the terms are not integers. Let us agree, for examp t the double-decker fraction $\frac{\frac{2}{3}}{\frac{3}{4}}$ shall mean $\frac{2}{3} \div \frac{3}{4}$ which is al to $\frac{2}{3} \times \frac{4}{3}$ or $\frac{8}{9}$. You see that this is in accord with our first ic principle of fractions; it is also in accord with our second ciple, since, if the numerator and denominator of $\frac{\frac{2}{3}}{\frac{3}{4}}$ are both tiplied by 12, we get

$$\frac{\frac{2}{3} \times 12}{\frac{3}{4} \times 12} = \frac{8}{9}.$$

xtending the meaning of the word fraction we shall allow the erator and denominator to be integers, fractions, or any bination of integers and fractions. Such a ***complex* ction** is usually handled by reducing it to a ***common* ction**, that is to one whose terms are integers.

The actual work of making this reduction is done most quickly means of the second basic principle. Take, for example, $\frac{\frac{5}{8}}{\frac{3}{20}}$. t notice that the L. C. M. of the denominators 8 and 20 is 40. n multiply the numerator and denominator of the complex tion by 40. This gives

$$\frac{\frac{5}{8}}{\frac{3}{20}} = \frac{\frac{5}{8} \times 40}{\frac{3}{20} \times 40} = \frac{25}{6}.$$

ractice, you may omit the middle step, so that the solution es only one step.

Notice that the same method works for more complicated tions such as

$$\frac{\frac{2}{3} - \frac{3}{5}}{\frac{5}{6} + \frac{1}{2}}.$$

ental calculation shows that the L. C. M. of the four de- inators 3, 5, 6 and 2 is 30. Now multiply the numerator denominator of the complex fraction by 30. This gives

$$\frac{\frac{2}{3} - \frac{3}{5}}{\frac{5}{6} + \frac{1}{2}} = \frac{20 - 18}{25 + 15} = \frac{2}{40} = \frac{1}{20}.$$

Here is a final word of caution in writing a complex fracti
Be careful to draw the line separating the numerator from
denominator distinctly so as to avoid any confusion as to
meaning of the fraction. To see how confusion may arise, no
that

$$\frac{5}{\frac{2}{3}} = \frac{15}{2}, \quad \text{whereas} \quad \frac{\frac{5}{2}}{3} = \frac{5}{6}.$$

EXERCISES

(The first 12 may be answered orally)

Express as common fractions:

1. $\dfrac{7}{\frac{2}{7}}$ **2.** $\dfrac{\frac{7}{2}}{7}$ **3.** $\dfrac{\frac{6}{11}}{3}$ **4.** $\dfrac{6}{\frac{11}{3}}$

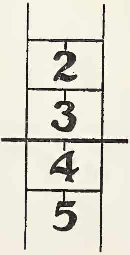
5. $\dfrac{\frac{1}{2}}{\frac{1}{3}}$ **6.** $\dfrac{\frac{2}{5}}{\frac{4}{5}}$ **7.** $\dfrac{\frac{3}{8}}{\frac{1}{4}}$ **8.** $\dfrac{\frac{4}{3}}{\frac{4}{9}}$

9. $\dfrac{\frac{3}{7}}{\frac{2}{3}}$ **10.** $\dfrac{\frac{4}{9}}{\frac{5}{6}}$ **11.** $\dfrac{\frac{5}{8}}{\frac{3}{7}}$ **12.** $\dfrac{\frac{15}{16}}{\frac{3}{4}}$

13. $\dfrac{3\frac{1}{2}}{2\frac{2}{3}}$ **14.** $\dfrac{5\frac{1}{4}}{2\frac{1}{8}}$ **15.** $\dfrac{3\frac{1}{7}}{2\frac{4}{9}}$ **16.** $\dfrac{\frac{3}{4} - \frac{5}{8}}{3}$

17. $\dfrac{\frac{3}{4}}{\frac{1}{2} + \frac{1}{3}}$ **18.** $\dfrac{2 - \frac{5}{3}}{3 + \frac{1}{4}}$ **19.** $\dfrac{\frac{2}{3} + \frac{3}{4}}{\frac{1}{2} + \frac{4}{5}}$ **20.** $\dfrac{\frac{4}{9} - \frac{3}{7}}{\frac{6}{7} + \frac{5}{3}}$

21. Here is a little game for a class contest. The numb
2, 3, 4 and 5 are hung on the four rungs of a ladder to form
complex fraction



n the same four numbers are hung in different positions on four rungs in all possible ways. Now here is your problem: the values of the 24 fractions which they can make.

Decimal Fractions

You have learned that our system of numeration is bound vith the number 10 and is therefore called a decimal system. whole number, the value of a digit depends on its position e number, being increased ten-fold when moved one place to left and decreased ten-fold when moved one place to the t. You are accustomed to name the places of the digits as s, tens, hundreds, and so on.

You also know that we can extend the decimal system, placing ts to the right of the units place. The first place to the right e units place represents tenths of the unit, the second place dredths, and so on. Thus, in the number which stands for ns, 6 units, 5 tenths, and 4 hundredths, the order of the digits ld be 3654. When the number is written in this way, how- , there is a difficulty. Which is the units digit? The device vhich we resolve this difficulty is the simple one of placing a ***imal point*** immediately after the units digit. The num- in our example is then written 36.54. The most familiar of the decimal point is in separating dollars from cents in oting a sum of money; $3.25 means $(3 + \frac{2}{10} + \frac{5}{100})$ dollars.

Changing a Decimal Fraction to a Common Fraction. In an ger the decimal point is not written, but is understood at right of the number. A number with digits after the mal point, such as 2.75, is a ***decimal fraction***. The ber stands for

$$2 + 7 \text{ tenths} + 5 \text{ hundredths},$$

is

$$2 + \tfrac{7}{10} + \tfrac{5}{100},$$

ch is the same as $2\frac{75}{100}$ or $2\frac{3}{4}$. In this way a decimal fraction sily converted into a common fraction. As another example,

$$0.48 = \tfrac{48}{100} = \tfrac{12}{25}.$$

EXERCISES

Read the values of the following as common fractions in th lowest terms:

1. 0.1
2. 0.2
3. 0.3
4. 0.4
5. 0.5
6. 0.6
7. 0.7
8. 0.8
9. 0.9
10. 0.25
11. 0.45
12. 0.75
13. 0.95
14. 0.16
15. 0.88
16. 0.125
17. 0.02
18. 0.05
19. 0.08
20. 0.075
21. 0.016
22. 0.004
23. 0.0001
24. 0.0025
25. 0.002

Decimals at Work

Addition and Subtraction. For such a sum as

$$32.54 + 6.483,$$

the addition is done in the same way as with integers, with obvious precaution that units are added to units, tenths to tent and so on. This means that when a column of figures is to added the decimal points are placed in a vertical line. Her the work for the example just given and also for one in s traction.

$$\text{Addition:} \quad \begin{array}{r} 32.54 \\ 6.483 \\ \hline 39.023 \end{array} \qquad \text{Subtraction:} \quad \begin{array}{r} 783.57 \\ 94.826 \\ \hline 688.744 \end{array}$$

EXERCISES

Find the value of :

1. $23.47 + 6.28 + 37.5$
2. $42.84 + 3.4 + 6.02$
3. $58.64 - 49.7$
4. $0.002016 + 0.072$
5. \$4.65 − 88¢
6. $385.6 - 79.28 + 61.74$
7. From the sum of 38.46 and 5.274 subtract 18.8.
8. To the difference between 5.48 and 1.973 add 0.5824.
9. From the sum of 3.407 and 0.5209 subtract the sum 1.983 and 0.079.
10. Diminish 7.049 by 6.45 and increase the difference by

Multiplication. Consider the product

$$3.24 \times 5.6.$$

:he first number is multiplied by 100 and the second by 10 result is 324×56. This latter product is therefore 1000 es the required product. Hence the required product is ained by multiplying the two numbers as if the decimal nts were not present, then placing the decimal point in the duct so that three digits follow it. With this example to ıll the practice you have had in multiplying decimals, you uld be able to formulate a general rule for placing the imal point in the product of two numbers.

Division. Take the example

$$93.42 \div 34.6$$

our first basic principle of fractions this can be written $\frac{93.42}{34.6}$ vhich the dividend becomes the numerator and the divisor the ıominator. Now by our second basic principle we may mul- y the denominator by any number provided we multiply the nerator by the same number. Let us multiply dividend and isor by 10, thus making the divisor a whole number. The ision now appears as follows:

$$\mathbf{93}.42 \div 34.6 \qquad\qquad \begin{array}{r} 2.7 \\ 346\overline{)934.2} \\ \underline{692} \\ 2422 \\ \underline{2422} \end{array}$$

tice that the quotient is written above the dividend in such vay that ***the decimal point is directly above the deci- ıl point in the dividend.***

For a second example, let us change the positions of the imal points.

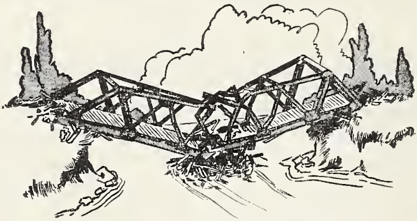
ke $0.0009342 \div 0.0346.$

re we move the decimal point in both divisor and dividend four places to the right, which amounts to multiplying both 10,000. The division then appears as follows:

$$0.0009342 \div 0.0346 \qquad \begin{array}{r@{\,}l} & 0.027 \\ \cline{2-2} 346) & 9.342 \\ & 692 \\ \cline{2-2} & 2422 \\ & 2422 \\ \cline{2-2} \end{array}$$

Now put into words a rule for placing the decimal point the quotient of two decimals.

You have now probably learned that, in playing the game decimals, the thing to watch is the decimal point.



Beneath this wreckage lies Cyrus Joint.
Someone omitted a decimal point.

Always estimate your answer.

You have been given this advice before. An estimate is use here in furnishing a check on the position of the decimal poi It is clear, for example, that the product

$$35.64 \times 7.82$$

is greater than 100 and less than 1000, so that the decimal poi comes after the third digit. Again, in

$$857.3 \div 15.6$$

we see, by estimating, that the decimal point follows the seco figure of the quotient.

EXERCISES

l. State the rule for placing the decimal point in the product vo numbers.

. Without carrying out the operations, make a mental calcu- n of the number of digits before the decimal point in the ver to each of the following:

(*a*) 37.2 × 5.14 (*b*) 73.9 × 0.68
(*c*) 62.54 ÷ 41.32 (*d*) 546.85 ÷ 8.7.

. Without carrying out the operations, make a mental cal- tion of the position of the first significant digit in the answer ach of the following:

(*a*) 7.54 × 0.0046 (*b*) 0.47 × 0.023
(*c*) 5.24 ÷ 43.1 (*d*) 0.006 ÷ 0.003.

l the values of:

54.76 × 3.5 **5.** 0.0638 × 0.0074 **6.** $(0.1)^{10}$
$(0.9)^4$ **8.** $(1.03)^3$ **9.** 23.732 ÷ 3.4
21.333 ÷ 0.39 **11.** 0.1404 ÷ 0.54 **12.** 0.002016 ÷ 0.072
4.3625 × 0.068 **14.** 9.462 ÷ 0.83 **15.** 34.0726 × 4.58
0.064665 ÷ 0.0000135.

Changing Fractions to Decimals

ince the fraction a/b means $a \div b$ (by our first basic prin-), it is possible to obtain the decimal equivalent of a fraction

> *To change a common fraction to a decimal, divide the denominator into the numerator.*

arrying out a division. It is sometimes an aid to insert zeros e right of the numerator.

s,

$$2/5 = 2.0 \div 5 = 0.4 \quad \text{and} \quad 1/8 = 1.000 \div 8 = 0.125.$$

ome fractions are so frequently used that their decimal valents are worth keeping in mind. These are:

$\frac{1}{2} = 0.5 \quad \frac{1}{5} = 0.2 \quad \frac{1}{8} = 0.125$

$\frac{1}{4} = 0.25 \quad \frac{2}{5} = 0.4 \quad \frac{3}{8} = 0.375$

$\frac{3}{4} = 0.75 \quad \frac{3}{5} = 0.6 \quad \frac{5}{8} = 0.625$

$\frac{4}{5} = 0.8 \quad \frac{7}{8} = 0.875$

Recurring Decimals

When we convert the fraction 1/3 into a decimal by divi we obtain

$$\frac{1}{3} = 0.3333\ldots$$

in which the division does not terminate. This result differs f those in the foregoing table. What causes this difference? have seen that

$$\frac{1}{4} = 0.25$$

which is a ***terminating*** decimal, the reason being th divides exactly into 100. On the other hand 3 does not di exactly into 10 or 100 or 1000 or any higher power of 10. clear now that *a fraction in its lowest terms will convert in terminating decimal only if the denominator is a factor of power of 10.* This will be true only if the prime factors o denominator are all 2's and 5's, since no other prime numb a factor of 10 or of any of its powers. Hence a fraction ch at random is not likely to lead to a terminating decimal. find that

$$\frac{1}{6} = 0.1666\ldots \qquad \frac{1}{9} = 0.1111\ldots$$

$$\frac{1}{12} = 0.08333\ldots \qquad \frac{2}{11} = 0.181818\ldots$$

In each of these decimals certain digits recur so that the c acter of the whole sequence of digits is known without fur division. Is this a characteristic of all non-terminating deci which are obtained from common fractions?

To answer this question consider the fraction 1/7. We by division

$$7\overline{)1.0000000\ldots}$$
$$0.1428571\ldots$$

w observe two things:

(1) at no stage of this division can we have a remainder ater than 6, so that not more than six different remainders possible;

(2) The sixth remainder is 1 and hence the next step consists dividing 7 into 10. But this is the same as the step with which division began. Hence the digits of the quotient must now eat in groups of 6. That is,

$$\tfrac{1}{7} = 0.142857142857\ldots$$

e digits *142857* make up the ***period*** of the ***recurring*** (or ***eating***) decimal.

The reasoning in this example shows that any fraction leads her to a terminating or to a recurring decimal. In any recurring imal it is important to recognize the period of the repeating its. These are often marked off by dots over the first and t digits of the period, according to the notation

$$\tfrac{1}{3} = 0.\dot{3}, \qquad \tfrac{2}{11} = 0.\dot{1}\dot{8}, \qquad \tfrac{1}{7} = 0.\dot{1}4285\dot{7}.$$

EXERCISES

For each of the following fractions (*a*) state whether it leads a terminating or a repeating decimal, (*b*) convert it into imal form.

1. $\frac{3}{20}$ 2. $\frac{4}{9}$ 3. $\frac{7}{12}$ 4. $\frac{5}{16}$ 5. $\frac{3}{7}$ 6. $\frac{2}{11}$ 7. $\frac{7}{15}$ 8. $\frac{96}{125}$

9. Convert into decimal fractions: $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$. What do observe regarding the periods of these fractions?

Approximate Computation with Decimals

In industrial plants it is the general practice to record the asures of machine parts, weights, etc. in decimals. Such mbers are always approximate and care must be given to the mber of significant figures in these measures. Recall from

Chapter 2 how you determine the number of significant digit the product of two *approximate* numbers. If, for exam both numbers have three significant digits, you keep three di in the product; if one number has two significant digits and other more than two, you keep two digits in the product.

It is not hard to see that the same rule works in reverse, t is for division. In the two following examples the divide and divisors are approximate, all being given to three figu except the second divisor, which is to two figures. Digits wh may be in error by as much as half a unit are underlined. Y see that in the first problem the quotient is not known beyo three figures and in the second, not beyond two figures.

$$85.\underline{4} \div 3.4\underline{6} \qquad \begin{array}{r} 24.\underline{7} \\ 34\underline{6})\overline{85\underline{40.0}} \\ 69\underline{2} \\ \hline 16\underline{20} \\ 138\underline{4} \\ \hline 2\underline{360} \end{array} \qquad \Bigg| \qquad 0.72\underline{3} \div 5.\underline{2} \qquad \begin{array}{r} 0.1\underline{4} \\ 5\underline{2})\overline{7.2\underline{3}} \\ 5\,\underline{2} \\ \hline 2\,\underline{03} \end{array}$$

The number of digits retained in the quotient is t same as the number of significant digits in the less a curate of the two given numbers.

EXERCISES

1. If a measured length is found to be 3 tenths of an in correct to the hundredth part of an inch, how should it recorded?

2. If a weight is found to be 5.2 grams, correct to a centigra how should it be recorded?

3. Each of the following numbers is the measure of a leng in metres. How many significant figures does each contain?

(*a*) 3.02 (*b*) 0.025 (*c*) 0.050 (*d*) 0.005 (*e*) 0.500

4. In each of the following products of approximate numbers, ide how many significant figures should be retained in the wer and find them:

(*a*) 8.04×5.16 (*b*) 0.52×4.37 (*c*) 0.064×0.38.

5. In each of the quotients of the following approximate nbers, decide how many significant figures should be retained l find them:

(*a*) $7.83 \div 0.294$ (*b*) $57.2 \div 73$ (*c*) $0.37 \div 0.247$.

REVIEW EXERCISES

A

(*These may be answered orally*)

1. Reduce the following fractions to their lowest terms:

(*a*) $\frac{18}{60}$ (*b*) $\frac{14}{63}$ (*c*) $\frac{35}{56}$ (*d*) $\frac{55}{80}$ (*e*) $\frac{48}{108}$ (*f*) $\frac{55}{121}$.

2. Copy the following fractions into your workbook and fill he missing terms:

$$\frac{7}{12} = \frac{\quad}{156} = \frac{84}{\quad} = \frac{\quad}{300} = \frac{0.007}{\quad} = \frac{4.9}{\quad} = \frac{\quad}{10.8}.$$

3. Reduce to decimal form:

(*a*) $\frac{23}{50}$ (*b*) $\frac{11}{8}$ (*c*) $\frac{113}{250}$ (*d*) $\frac{17}{125}$ (*e*) $\frac{302}{625}$.

4. (*a*) What number is equal to its reciprocal? (*b*) What nber is equal to 4 times its reciprocal?

5. Give the squares of:

a) 0.9 (*b*) 0.7 (*c*) 1.2 (*d*) 2.5 (*e*) 0.11 (*f*) 1.01
g) 0.03 (*h*) 0.005 (*i*) $1\frac{1}{2}$ (*j*) $2\frac{1}{2}$ (*k*) $3\frac{1}{3}$ (*l*) $1\frac{2}{5}$
m) $2\frac{2}{3}$.

6. Give the cubes of:

a) 0.1 (*b*) 0.3 (*c*) 0.5 (*d*) 0.02 (*e*) 0.004 (*f*) 1.1
g) $1\frac{1}{2}$ (*h*) $1\frac{1}{3}$.

7. Without making the calculations tell the position of decimal point in the answer to each of the following:

(*a*) 624.7×35.72 (*b*) 25.8×0.43 (*c*) 0.348×0.0
(*d*) $75.8 \div 3.62$ (*e*) $5.4 \div 65$ (*f*) $0.124 \div 0.0$

8. Which of the following fractions will give terminating which repeating decimals?

$$(a)\ \frac{21}{50} \quad (b)\ \frac{32}{75} \quad (c)\ \frac{108}{125} \quad (d)\ \frac{25}{56} \quad (e)\ \frac{256}{300} \quad (f)\ \frac{243}{512}.$$

9. What numbers are increased by squaring and what nu bers are decreased?

10. Four-fourths exceeds three-fourths by what fractio part of the latter? Five-sevenths exceeds three-sevenths what fractional part?

11. A motorist sees a highway sign *Picnic table 1000 feet ahe* What approximate change will be produced in his mileage readi in this distance?

B

12. Find the value of:

(*a*) $5\frac{1}{3} + 6\frac{2}{5} - 4\frac{1}{2}$ (*b*) $7\frac{5}{6} - 3\frac{2}{3} - 2\frac{1}{4}$ (*c*) $9\frac{3}{8} - 4\frac{3}{4} + 2\frac{1}{2}$
(*d*) $10\frac{4}{9} + 8\frac{7}{12} + 6\frac{7}{18}$.

13. Reduce to their lowest terms:

$$(a)\ \frac{65}{91} \quad (b)\ \frac{51}{119} \quad (c)\ \frac{185}{296} \quad (d)\ \frac{253}{529} \quad (e)\ \frac{779}{943}.$$

14. Find the value of:

$$(a)\ 4\tfrac{1}{11} \times 4\tfrac{8}{9} - 5\tfrac{3}{7} \times 1\tfrac{2}{19} \qquad (b)\ \frac{3\frac{2}{3}}{5\frac{5}{6}} + \frac{4\frac{3}{4}}{6\frac{1}{8}}$$

$$(c)\ 2\tfrac{2}{35} \times \tfrac{49}{132} - 1\tfrac{23}{33} \div 3\tfrac{17}{21}.$$

15. 55 c.c. of oil weighing 0.830 gm. per c.c. are mixed wi 45 c.c. of gasoline weighing 0.543 gm. per c.c. Find the weig per c.c. of the mixture.

16. If the distance of Mars from the sun is 1.524 astronomical ts (correct to 4 figures) and the astronomical unit is 92,900,000 es (correct to 3 figures) find the distance in miles of Mars from sun and round off the measure to the proper number of digits.

7. Because of the shortage of raw materials the diameter of tch sticks in Japan was reduced from 0.072 to 0.06 of an inch. a match stick is 2 inches long how much wood was saved in manufacture of 1,000,000 matches?

8. The following numbers are approximate, correct to the nber of digits given. Carry out the operations and round off answers as required.

(*a*) 3.725×48.7 (*b*) 0.576×0.0495 (*c*) 0.1001×0.101

(*d*) 64000×3800, each correct to 2 figures.

9. The following numbers are approximate. In each problem ry out the division keeping only as many digits as there are he number with the fewer digits.

(*a*) $637.4 \div 82.3$

(*b*) $56200 \div 3950$, each to 3 significant digits

(*c*) $0.072 \div 0.344$ (*d*) $0.1111 \div 0.101$.

C

20. Add the same constant, say 3, to the numerator and to the nominator of the fraction 4/9. Is the new fraction greater smaller than the old? Now try the fraction 7/3. What is result? Do you suspect a general rule? Try to prove the e by using letters.

21. If m and n are positive numbers what is the effect on the ction m/n of (*a*) multiplying m and n by the same number, 0, (*b*) increasing m while n remains constant, (*c*) decreasing while m remains constant, (*d*) increasing m and n by the same ount, (*e*) decreasing m and n by the same amount, less than smaller of m and n?

22. The number 1.23454321 is the square of a certain number. ess the number and verify your guess.

23. When the fraction 1/17 is converted into a decimal, w is the greatest number of digits there could be in the peri Convert the fraction and count the digits in the period.

24. Suppose we wish to convert the recurring decimal 0.5 to a common fraction. Denote the fraction by D. Then

$$\begin{array}{lrcl} & D & = & 0.52363636\ldots \\ \text{Then} & 10000D & = & 5236.3636\ldots \\ \text{and} & 100D & = & 52.3636\ldots \\ \hline \text{Subtraction gives} & 9900D & = & 5184 \\ \text{whence} & D & = & \dfrac{5184}{9900} = \dfrac{144}{275}. \end{array}$$

Now convert the following decimals to common fractions:

(*a*) $0.42\dot{3}$ (*b*) $0.2\dot{1}\dot{5}$ (*c*) $0.45\dot{1}\dot{8}$.

CHAPTER VIII

SIGNED NUMBERS

"Her age upon the date
Of his birth was minus eight,
If she's seventeen and he is five and twenty."
W. S. Gilbert.

Let's Talk About the Weather

Mark Twain has said that everyone talks about the weather t no one ever does anything about it. Canadians are no excepn. In the heat of summer, we complain of temperatures of) in the shade". In winter, a cold wave of "20 below" sends to the news reports to see how other communities have fared. nada is a large country and often the temperatures vary widely m place to place. Here is a typical newspaper weather report.

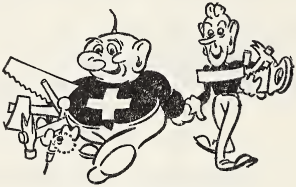
Here and There on the Weather Map

High temperatures yesterday

Vancouver	+43	White River	−25
Calgary	+49	Toronto	+12
Edmonton	+40	Montreal	− 4
Winnipeg	+21	Moncton	− 8
Port Arthur	0	Halifax	+15

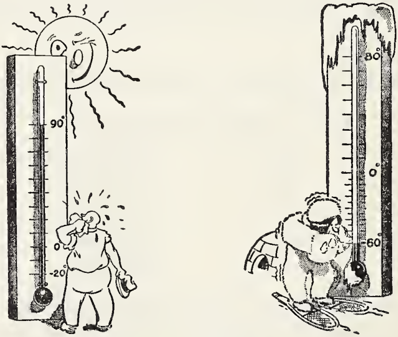
Observe the numbers in this report. They denote the tematures in degrees Fahrenheit. You will observe that some nbers are preceded by a plus sign and some by a minus sign. some weather reports, plus signs are omitted and under- od.)

Now this is a new use for our old friends + and −. We have d the symbols + and − previously to indicate that we are to add or subtract. The new use is quite different. The +43 af Vancouver means that the temperature was 43 degrees ***abo*** zero. Similarly, the −8 after Moncton means that the temp ature was 8 degrees ***below*** zero.



Starting on a new job.

You see now that these numbers with signs attached are us to denote the distance of the mercury surface ***above*** or ***bel*** the zero mark on the thermometer scale. *The numbers not o* *tell us* ***how far****, but they tell us* ***in which direction.*** This an example of another common and very useful service of nu bers in everyday life.



Perishing by degrees.

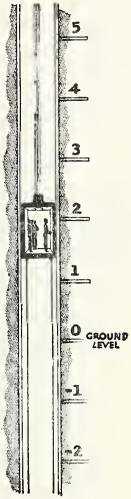
Up and Down

“Nothing is permanent but change”. When a quantity is anging in value, we often want to know ***two*** things:

(1) the ***amount*** of the change, and

(2) the ***direction***, or ***sense***, of the change.

th facts can be indicated by using numbers th signs attached. As in some weather reports, e plus sign is sometimes omitted and taken for anted. Therefore, *if no sign precedes a number, lus sign is understood.* For example, 10 means 10 and 0.5 means +0.5, etc.



EXERCISES

1. In checking his accounts, J. D. Smart incates a withdrawal of five dollars by −5. What uld +5 indicate? Show how to indicate the llowing by using numbers with signs attached.

(*a*) A withdrawal of $20.

(*b*) A deposit of $8.

(*c*) A withdrawal of $*b*.

(*d*) A deposit of $15 and a withdrawal of $18.

2. During a football game, a record is kept of the yards ined or lost on each play. If a gain of 8 yards is denoted in e record by 8, how might a loss of 8 yards be indicated?

How would you record each of the following?

(*a*) A loss of four and one-half yards.

(*b*) A gain of twelve yards.

(*c*) A penalty of ten yards.

(*d*) A gain of eighteen yards.

(*e*) No gain or loss.

3. In the following table, cost prices and selling prices of rtain articles are shown. Use numbers with attached signs to present the gain or loss on each article. (Prices are in dollars.)

Article	A	B	C	D	E	F	G	H
Cost Price	20.00	8.75	30.00	5.25	43.00	12.25	10.80	3.5
Selling Price	24.50	9.50	22.00	7.25	41.50	10.00	14.00	3.0
Gain	+4.50							

4. Sea-level is conveniently taken as zero in measuring heigh and depths on earth. Here are some interesting heights an depths on our planet. Use numbers with signs to represent the measures in feet.

(*a*) Mount Everest rises 29,002 feet above sea-level.

(*b*) The surface of the Dead Sea is 1290 feet below sea-leve

(*c*) Mindanao Deep in the Pacific Ocean has a depth 35,400 feet.

(*d*) The greatest height reached by a manned balloon is 72,39 feet above sea-level.

(*e*) Mount Robson rises 12,972 feet above sea-level.

(*f*) Dr. Beebe reached a depth of 3028 feet below the surfac of the Atlantic in a bathysphere.

5. Suppose the temperature in a room is 70° Fahrenheit. this temperature is denoted by +70, what would the tempe ature be after

(*a*) a fall of 20 Fahrenheit degrees?

(*b*) a rise of 15 Fahrenheit degrees?

(*c*) a fall of 80 Fahrenheit degrees?

(*d*) a fall of 70 Fahrenheit degrees?

(*e*) a rise of 1 Fahrenheit degree?

(*f*) a fall of 71 Fahrenheit degrees?

(*g*) a rise of 8 Fahrenheit degrees?

6. John C. Citizen keeps account of his income and expenses dollars. How could he use numbers with signs for this purse? Using your method, how would he represent the followg items?

(*a*) Salary received	$200.00
(*b*) Clothes purchased	$18.50
(*c*) Rent and food	$62.50
(*d*) Interest on bank deposit	$22.30
(*e*) Fuel	$15.00

Some New Meanings

You have seen two old friends in a new role. The first role of e signs + and − was to tell us to add or subtract. Here is the w role:

> *The signs + and − are attached to numbers to denote* ***opposite senses or directions.***

Some typical uses of + and − are tabulated below.

++++ PLUS ++++	−−−− MINUS −−−−
Above zero	Below zero
Rise in price	Fall in price
Gain	Loss
Above sea-level	Below sea-level
Income	Expenses
Movement right	Movement left
Movement up	Movement down

Of course, the choice of which direction is to be called + and hich called − is entirely in your hands. For example, above ro might well be designated − and below zero designated +. ce the directions are chosen, however, we must be consistent in our use of the signs. Describe the use of + and − in the tab on page 18.

You can easily see how many situations lend themselves this new use of plus and minus.

The new numbers with signs attached are called ***signed numbers*** *or* ***directed numbers.***

Thus -3, $+1.8$, $-\frac{3}{4}$, $+2x^2$, $-4.2a^2$, etc. are signed numbe or directed numbers. *Those numbers with the plus signs* (li $+1.8$, $+2x^2$, $+7$) *are called* ***positive*** *numbers.* We read + as "plus seven". It has been mentioned before that positi numbers often have the plus signs omitted and understood.

Numbers with a minus sign attached (like -3, $-\frac{3}{4}$, -4.2 *are called* ***negative*** *numbers.* We read -3 as "minus three

The numerical value *or* ***absolute value*** *of a signed nu ber is its value when the sign is not considered.* For exampl the numerical value of -5 is 5, and the numerical value of bo $+\frac{3}{4}$ and $-\frac{3}{4}$ is $\frac{3}{4}$. The absolute value or numerical value of signed number tells us the ***amount*** of a quantity witho specifying its ***direction***.

EXERCISES FOR ORAL ANSWERS

Which of the following numbers are positive and which neg tive? State the numerical value of each number.

$+5$, $-3/5$, -1.5, 8, $9/7$, -1005, $-5\frac{2}{3}$, $+47$, 47,
-3.2, 61, 0, -11, 765.2, $-\frac{6}{7}$, a^2, $-a^2$, $-a$.

Back and Forth

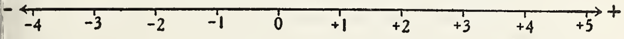
A signed number, used for the measure of a quantity, tel

(1) *the amount of the quantity,*

(2) *its sense or direction.*

For this reason, the points on a straight line can be used to repr sent signed numbers.

Select some point on a line to represent the number 0. Then stances measured to the right from 0 can be represented by sitive numbers and distances to the left from 0 can be repre- nted by negative numbers. Thus, the



int one unit to the right of 0 is labelled +1, and the point one it to the left of 0 is labelled −1. The number +2.7 represents point 2.7 units to the ***right*** of 0. What point does −2.6 rep- sent? How would you locate points which correspond to e following numbers?

+0.5, $-1\frac{3}{4}$, −7/5, +82, −100, −3.2, 2, +7.9, 0, 3456.

In this way, you can get a number for every point on the line. so, you can get a point for every signed number. The ***nu- erical value*** of the number tells you ***how many units*** its int is from 0. The ***sign*** of the number tells you whether point lies to the ***right or left*** of 0. This arrangement of ints on a line with their signed numbers affixed is called ***he number scale"***. You can learn a lot about signed mbers by studying it. Look carefully at the number scale ove, and then try to answer these questions.

1. Is a signed number greater or smaller than those to the right of it on the number scale?
2. Is a signed number greater or smaller than those to the left of it on the number scale?
3. How do negative numbers compare in value with 0?
4. How do positive numbers compare in value with 0?

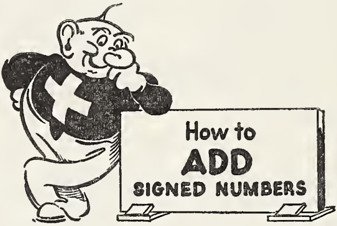
EXERCISES FOR ORAL ANSWERS

Which number in each of the following pairs is the greater?

+7, +9; −3, −4; $2\frac{5}{6}$, −5; 0, 6; 0, −40;
17, −90; +4.1, +4.2; −3.7, −3.6.

A number is less than those to the right of it and greater than those to the left of it on the number scale. Thus:

***Negative numbers are less than* 0.**
***Positive numbers are greater than* 0.**



You expect, of course, that signed numbers can be adde subtracted, multiplied, and divided. They can. Our task is no to find out how it is done.

Take, for example, addition. Naturally, we should like have such a rule for the addition of signed numbers as would gi results that check with our everyday experiences. We want o answers to agree with common sense. We know that a gain $5 (+5) followed by a loss of $3 (−3) produces a net gain $2 (+2). We therefore expect that

$$(+5) + (-3) = +2.$$

Again, a gain of $5 (+5) followed by a loss of $7 (−7) produces net loss of $2 (−2). Hence

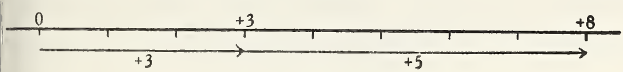
$$(+5) + (-7) = -2.$$

Make up some more examples of the same kind for yoursel Then try to make up a rule for adding signed numbers.

The number scale is very helpful in suggesting the proper wa to add signed numbers. As is customary, let us agree that

(1) $+4$ denotes a movement of 4 units to the right, and
(2) -4 denotes a movement of 4 units to the left, and so on.

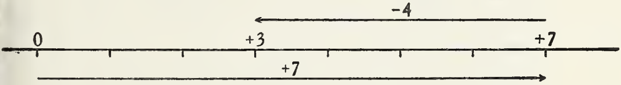
uppose that we wish to add $+3$ and $+5$. Let a point start at 0



nd move right 3 units, and then move right 5 more units arving at $+8$. This suggests that

$$(+3) + (+5) = +8, \qquad \text{or} \qquad \begin{array}{r} +3 \\ \underline{+5} \\ +8 \end{array}$$

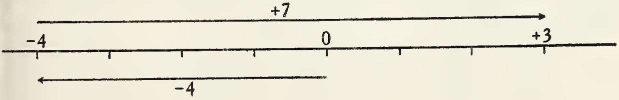
ow let us add $+7$ and -4 using the number scale. Let a point art at 0 and move 7 units to the right. Then let the point



ove 4 units to the left arriving at $+3$. This suggests that

$$(+7) + (-4) = +3, \qquad \text{or} \qquad \begin{array}{r} +7 \\ \underline{-4} \\ +3 \end{array}$$

otice that you get the same answer by letting the point first ove left 4 units and then move right 7 units.



$$(-4) + (+7) = +3, \qquad \text{or} \qquad \begin{array}{r} -4 \\ +7 \\ \hline +3 \end{array}$$

This means that the ***order*** in which we add the two numbers do not affect our answer. Investigate other pairs of numbers f yourself, and you will soon be convinced that

$$a + b = b + a,$$

where a and b stand for any signed numbers. In other word the Nutshell Rule of page 205 holds for signed numbers.

If you use the number scale as above, you can easily show th

$$(-5) + (+3) = -2,$$

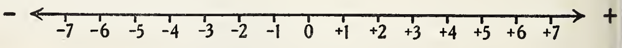
and

$$(-4) + (-2) = -6.$$

> *In arithmetic, addition always produces an increase. To add a number to another gives a larger number.* ***This is not always true with signed numbers.*** *If to a given number you add a negative number, the sum is* ***less*** *than the first number. So we must no longer think of addition as necessarily producing an increase.*

EXERCISES

Here is a number scale:



Use the number scale in the manner illustrated above to find th following sums.

$$\begin{array}{rrrrrrrr} +2 & -3 & -5 & -6 & +3 & -3 & +7 & +5 \\ \underline{+5} & \underline{-4} & \underline{+7} & \underline{+5} & \underline{+2} & \underline{-1} & \underline{-4} & \underline{-9} \end{array}$$

ow study your results carefully. Try to discover for yourself ule for adding signed numbers

(1) with ***like*** signs, and
(2) with ***unlike*** signs.

* * *

ter you have stated your rules, compare them with those on ge 271.

Some Examples to Study.

xample 1. $(-3\frac{1}{4}) + (-2\frac{1}{2}) = -5\frac{3}{4}.$

e signs are like, so we add the numerical values ($3\frac{1}{4}$ and $2\frac{1}{2}$) d affix the common sign.

xample 2. $(-2.8) + (+4.6) = +1.8.$

e signs are unlike, so we first find the difference of the numer- l values (4.6 and 2.8) getting 1.8, and then affix the sign of e number with the greater numerical value.

xample 3. $(-30)+(+25)+(-12)+(+8) = -9.$

o the sum of the first two numbers, (-5), we add e third number. This gives -17. Then add $+8$ -17 getting the final sum -9. The vertical rangement of the work is shown at the right.

$$\begin{array}{r} -30 \\ +25 \\ -12 \\ \underline{+8} \\ -9 \end{array}$$

EXERCISES

erform the following indicated additions.

1. $\begin{array}{r} +6 \\ \underline{+2} \end{array}$	**2.** $\begin{array}{r} +6 \\ \underline{-2} \end{array}$	**3.** $\begin{array}{r} +8 \\ \underline{-9} \end{array}$	**4.** $\begin{array}{r} -6 \\ \underline{0} \end{array}$	**5.** $\begin{array}{r} +7 \\ \underline{0} \end{array}$
6. $\begin{array}{r} -3 \\ \underline{-1} \end{array}$	**7.** $\begin{array}{r} -5 \\ \underline{+5} \end{array}$	**8.** $\begin{array}{r} -2.5 \\ \underline{+.5} \end{array}$	**9.** $\begin{array}{r} +4\frac{1}{4} \\ \underline{+2\frac{3}{4}} \end{array}$	**10.** $\begin{array}{r} -6.4 \\ \underline{-3.7} \end{array}$
11. $\begin{array}{r} -34 \\ \underline{+23} \end{array}$	**12.** $\begin{array}{r} +60 \\ \underline{-35} \end{array}$	**13.** $\begin{array}{r} +56 \\ \underline{+44} \end{array}$	**14.** $\begin{array}{r} +9.9 \\ \underline{-7.5} \end{array}$	**15.** $\begin{array}{r} \frac{1}{2} \\ \underline{-\frac{1}{3}} \end{array}$

$$\begin{array}{lrlrlrlrlr}
\textbf{16.} & -8\frac{3}{8} & \textbf{17.} & -0.6 & \textbf{18.} & +4.7 & \textbf{19.} & -2a & \textbf{20.} & -2a \\
 & \underline{+5\frac{1}{4}} & & \underline{-0.4} & & \underline{-4.7} & & \underline{+\ a} & & \underline{-\ a}
\end{array}$$

$$\begin{array}{lr}
\textbf{21.} & +2a \\
 & \underline{-\ a}
\end{array}$$

22. $(+9)+(+4)=\quad.$

23. $(+15)+(-9)=\quad.$

24. $(-8)+(-7)=\quad.$

25. $(-70)+(30)=\quad.$

26. $(-4.5)+(-1.5)=\quad.$

27. $(+6.2)+(+3.4)=\quad.$

28. $(+7.9)+(-1.4)=\quad.$

29. $(0.45)+(-0.45)=\quad.$

30. $(+3.1)+(-3.2)=\quad.$

$$\begin{array}{lrlrlrlrrlr}
\textbf{31.} & +6 & \textbf{32.} & -22 & \textbf{33.} & 4.5 & \textbf{34.} & 45 & & \textbf{35.} & -0. \\
 & -8 & & -11 & & 6.6 & & 22 & & & -2. \\
 & +3 & & +6 & & -3.2 & & -18 & & & -5. \\
 & +7 & & -9 & & -7.9 & & 10 & & & 11. \\
 & -10 & & +30 & & -4.2 & & -45 & 77 & & 4. \\
 & \underline{+12} & & \underline{-4} & & \underline{+4.2} & & \underline{-27} & 40 & & \underline{-9.} \\
 & & & & & & & & 13 & &
\end{array}$$

36. $(+6)+(-9)+(-5)+(-10)=\quad.$

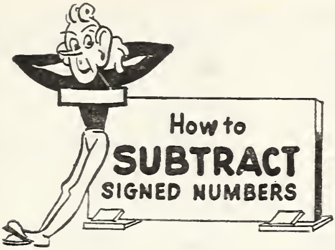
37. $(+18)+(-23)+(+7)+(-31)=\quad.$

38. $(-11)+(+19)+(-30)+(+65)=\quad.$

39. $(-9.2)+(+3.4)+(8.8)+(-4.7)=\quad.$

40. Add $+10$ to each of the following:
$+5,\ -5,\ +13,\ -18,\ -10,\ +40,\ -40,\ -3,\ -25,\ 0.$

41. Add -5 to each of the following:
$+5,\ -5,\ +10,\ +35,\ -20,\ -1,\ +17,\ +9,\ -3.2,\ +6.1,$



In order to arrive at a suitable method of subtracting signed mbers, you should re-read the discussion on subtraction on ge 124. Note that ***subtraction can always be regarded the opposite of addition.*** For example,

$$3a - a = 2a \quad \text{because} \quad a + 2a = 3a.$$

find $5m - 3m$, we ask $3m + ? = 5m$. That is, the difference tween two numbers is the number which, ***added*** to the sec-d number, produces the first.

Similarly, in order to find the value of

$$(+5) - (+2),$$

ask $(+2) + ? = +5.$

is gives $(+5)-(+2) = +3,$ or $\begin{array}{r} +5 \\ \underline{+2} \\ +3 \end{array}$

To find $(+5) - (-2),$

ask $(-2) + ? = +5.$

his gives $(+5) - (-2) = +7,$ or $\begin{array}{r} +5 \\ \underline{-2} \\ +7 \end{array}$

Study the following subtractions.

1. $\begin{array}{r} -5 \\ \underline{-2} \\ -3 \end{array}$ Ask: $(-2) + ? = -5.$

2. $\begin{array}{r} +9 \\ \underline{-4} \\ +13 \end{array}$ Ask: $(-4)+\,?=+9.$

3. $\begin{array}{r} +7 \\ \underline{+3} \\ +4 \end{array}$ Ask: $(+3)+\,?=+7.$

4. $\begin{array}{r} -12 \\ \underline{+10} \\ -22 \end{array}$ Ask: $(+10)+\,?=-12.$

Notice that if we change the problem

$$\begin{array}{rcr} -5 & \xrightarrow{\hspace{4em}} & -5 \\ \underline{-2} & \text{mentally into} & \underline{+2} \\ \text{Subtract: } -3 & & \text{Add: } -3 \end{array}$$

the answers are the same. In other words, subtracting −2 giv the same result as adding +2. Try this scheme on the oth three examples above. What rule does this suggest for sub traction? * * * Compare your rule with the one on pag 271. LEARN THIS RULE. It tells us what we mean b subtraction.

This new kind of subtraction may seem strange to you. In arithmetic, when you subtracted or "took away" one number from another, you always ***decreased*** *the second number. If you took 5 from 12, you had less than 12 left.* ***Not necessarily so with signed numbers.***

If you take away a $5 debt from a man, you leave him better off than before. In fact, to subtract a negative number will produce an ***increase.***

It is best that you should regard this new kind of subtraction not as a "taking away". From now on, ***to subtract a number we mentally change its sign and add.***

If you feel sure that you have understood and memorized e rule, try the following exercises.

EXERCISES

erform the following subtractions:

$$\begin{array}{llllllllll}
\textbf{1.} & \begin{array}{r}+6\\+2\\\hline\end{array} & \textbf{2.} & \begin{array}{r}+6\\-2\\\hline\end{array} & \textbf{3.} & \begin{array}{r}+8\\-9\\\hline\end{array} & \textbf{4.} & \begin{array}{r}-6\\0\\\hline\end{array} & \textbf{5.} & \begin{array}{r}+7\\0\\\hline\end{array}
\end{array}$$

$$\begin{array}{llllllllll}
\textbf{6.} & \begin{array}{r}-3\\-1\\\hline\end{array} & \textbf{7.} & \begin{array}{r}-5\\+5\\\hline\end{array} & \textbf{8.} & \begin{array}{r}-2.5\\+0.5\\\hline\end{array} & \textbf{9.} & \begin{array}{r}+4\frac{1}{4}\\-2\frac{3}{4}\\\hline\end{array} & \textbf{10.} & \begin{array}{r}-6.4\\-3.7\\\hline\end{array}
\end{array}$$

$$\begin{array}{llllllllll}
\textbf{11.} & \begin{array}{r}-34\\+23\\\hline\end{array} & \textbf{12.} & \begin{array}{r}+60\\-35\\\hline\end{array} & \textbf{13.} & \begin{array}{r}+56\\+44\\\hline\end{array} & \textbf{14.} & \begin{array}{r}+9.9\\-7.5\\\hline\end{array} & \textbf{15.} & \begin{array}{r}+1/2\\-1/3\\\hline\end{array}
\end{array}$$

$$\begin{array}{llllllllll}
\textbf{16.} & \begin{array}{r}-8\frac{3}{8}\\+5\frac{1}{4}\\\hline\end{array} & \textbf{17.} & \begin{array}{r}-0.6\\-0.4\\\hline\end{array} & \textbf{18.} & \begin{array}{r}+4.7\\-4.7\\\hline\end{array} & \textbf{19.} & \begin{array}{r}-2a\\+a\\\hline\end{array} & \textbf{20.} & \begin{array}{r}-2a\\-a\\\hline\end{array}
\end{array}$$

$$\begin{array}{ll}
\textbf{21.} & \begin{array}{r}+2a\\-a\\\hline\end{array}
\end{array}$$

22. $(+9)-(+4)$.

23. $(+15)-(-9)$.

24. $(-8)-(-7)$.

25. $(-70)-(+30)$.

26. $(-4.5)-(-1.5)$.

27. $(+6.2)-(+3.4)$.

28. $(+7.9)-(-1.4)$.

29. $(+0.45)-(-0.45)$.

30. $(+3.1)-(-3.2)$.

se the subtraction of signed numbers to answer the following:

31. From 10° above zero the temperature drops 12°. Find e resulting temperature.

32. From 3° below zero the temperature drops 7°. Find the sulting temperature.

33. A ship sails south from latitude 35°N to latitude 15°S. rough how many degrees of latitude does the ship sail?

34. What is the result of removing a debt of \$5 from a debt \$9?

35. Give the meaning of (*a*) -100 feet above sea level, (*b*) $-8°$ low zero, (*c*) $-30°$ north latitude, (*d*) a loss of -5 dollars.

A Word About Notation

The symbols $+$ and $-$ have now two uses:

(1) They indicate the operations of ***addition and su traction.***

(2) They are attached to numbers to indicate ***sense direction.*** You probably feel that this double use will cau confusion. Which meaning is intended in a given situation?

One way to avoid confusion is to use parentheses arou signed numbers, where necessary. For example: $(+7)$, $(+3.$ etc. However, an equality like

$$(+4)+(+2)=+6$$

is clumsy, even though it makes clear the roles of the plus si In practice, we can simplify the notation by agreeing that ***if sign precedes a number, the plus sign is understoo*** This allows us to write

$$4+(+2)=6,$$

which is a little simpler.

A further simplification can be made without losing clarit Study the following pairs of equalities.

$$(1)\ \begin{cases} 4 + (+2) = 6 \\ 4 \quad\ \ +2 = 6 \end{cases} \qquad (2)\ \begin{cases} 7 + (-2) = 5 \\ 7 \quad\ \ -2 = 5 \end{cases}$$

The plus sign which indicates the addition of two signed numbers is unnecessary. It is customary to omit it.

Thus $5-8$ means $5+(-8)$,

and $-5-8$ means $-5+(-8)$.

ORAL EXERCISES

Complete the following statements.

1. $6-9=$.

2. $-6-9=$.

3. $-6+9=$.

4. $2-7=$.

5. $-7+5=$.

6. $-5-8=$.

7. $-10+2.5=$.

8. $-15-12=$.

9. $-6.5+10=$

10. $8-13=$.

11. $0-6=$.

12. $-12+20=$.

How to
MULTIPLY
SIGNED NUMBERS

Once again we want to make some rules. These rules are to us how to multiply signed numbers. They should also give ults that fit in with the facts about numbers that we have eady learned. For example, we have

$$3 \times 5 = 5 + 5 + 5 = 15.$$

nce, we certainly want to have

$$(+3)(+5) = (+5) + (+5) + (+5) = +15.$$

is suggests that

$$(+3)(-5) = (-5)+(-5)+(-5) = -15.$$

reover, we should like to have

$$\begin{aligned} (-3)(+5) &= (+5)(-3) \\ &= (-3) + (-3) + (-3) + (-3) + (-3) = -15. \end{aligned}$$

Let us agree to adopt these answers. There is just one other e of product. Take $(-3)(-5)$. What answer should we pt for this in order to be consistent with our earlier work? example with parentheses suggests the answer. We should to have

$$(-3)(7-5) = (-3)\,(2) = -6.$$

ce $(-3)(7-5)$ is the product of a monomial and a binomial, rule on page 133 gives

$$\begin{aligned} (-3)(7-5) &= (-3)(7) + (-3)(-5) \\ &= -21 \quad + (-3)(-5). \end{aligned}$$

To get the desired answer -6, we must agree that

$$(-3)(-5) = +15.$$

You might justify the result in this way: If you relieve a m of 3 separate debts of \$5 each, you leave him \$15 better off.

Of course, we might have used any numbers instead of and 5. The four results give, in general,

$$\boxed{\begin{array}{lcl} (+a)(+b) & = & +ab \\ (+a)(-b) & = & -ab \\ (-a)(+b) & = & -ab \\ (-a)(-b) & = & +ab \end{array}}$$

These answers suggest a method for multiplying signed numbe How do you find the sign of the product? How do you find i numerical value? State a rule for multiplying signed numbe

* * * Compare your statement with the rule given page 271. After you have memorized the rule, try the followi exercises.

EXERCISES

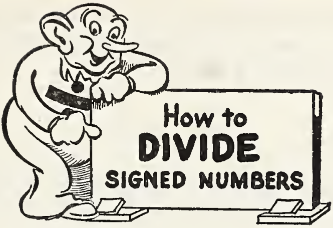
Multiply:

1. $\begin{array}{r}+3\\ \underline{+5}\end{array}$	**2.** $\begin{array}{r}+3\\ \underline{-5}\end{array}$	**3.** $\begin{array}{r}-3\\ \underline{+5}\end{array}$	**4.** $\begin{array}{r}-3\\ \underline{-5}\end{array}$
5. $\begin{array}{r}-5\\ \underline{+0}\end{array}$	**6.** $\begin{array}{r}-12\\ \underline{+\ 4}\end{array}$	**7.** $\begin{array}{r}+9\\ \underline{-5}\end{array}$	**8.** $\begin{array}{r}+7\\ \underline{-0}\end{array}$
9. $\begin{array}{r}-2.5\\ \underline{+\ 2\ }\end{array}$	**10.** $\begin{array}{r}-10\\ \underline{-10}\end{array}$	**11.** $\begin{array}{r}+4.5\\ \underline{-\ 3\ }\end{array}$	**12.** $\begin{array}{r}-8\\ \underline{-3}\end{array}$

13. $(-5)(+6) =$. **14.** $(+7)(-8) =$. **15.** $(-3)(-4) =$

16. $(+1.5)(+2) =$. **17.** $(-5)(+12) =$. **18.** $(+2)(-9) =$

19. $(-5)(-7)(+2) =$. **20.** $(+4)(-3)(-3)(-5) =$.



Once we have our rule for multiplication, it is easy to find the le for division. The quotient 24 ÷ 6 is the number by which s ***multiplied*** to give 24. You can change any problem in vision into a problem in multiplication in this way. That akes it easy.

We have

$$\frac{+24}{+6} = +4, \qquad \text{since } (+6)(+4) = +24;$$

d $$\frac{+24}{-6} = -4, \qquad \text{since } (-6)(-4) = +24;$$

d $$\frac{-24}{+6} = -4, \qquad \text{since } (+6)(-4) = -24;$$

d $$\frac{-24}{-6} = +4, \qquad \text{since } (-6)(+4) = -24.$$

Such examples suggest our rule for the division of signed ımbers. How do you find (1) the sign of the quotient, and) the numerical value of the quotient? State a rule for division. * * * Compare your rule with that given on ıge 271.

After you have learned the rule, study the following examples. Then work the exercises.

xample 1. $$\frac{-3.6}{6} = -0.6.$$

Since the signs of the divisor and dividend are ***unlike***, the si of the quotient is minus. The numerical value of the quotie is obtained by dividing 6 into 3.6.

Example 2. $\frac{-9}{-2} = +4.5.$

Since the signs of the divisor and dividend are ***like***, the sign the quotient is plus. The numerical value of the quotient found by dividing 9 by 2.

EXERCISES

Divide the following.

1. $\frac{+25}{+5}$. 2. $\frac{-25}{+5}$. 3. $\frac{+25}{-5}$.

4. $\frac{-9}{3}$. 5. $\frac{12}{-4}$. 6. $\frac{-6}{-2}$.

7. $\frac{-7}{-2}$. 8. $\frac{-4.2}{6}$. 9. $\frac{2.4}{-4}$.

10. $\frac{(-5)(-6)}{-10}$. 11. $\frac{(+7)(+7)}{-7}$. 12. $\frac{(+8)(-3)}{6}$.

13. $\frac{(-4)(-4)}{-4}$. 14. $\frac{(-12)(+5)}{-3}$. 15. $\frac{5(-9)}{3}$.

16. $\frac{(+10)(-8)}{-16}$. 17. $\frac{(-4)^2}{-8}$. 18. $\frac{(-3)^3}{9}$.

19. $\frac{-4^2}{-8}$. 20. $\frac{-3^3}{9}$. 21. $\frac{(-2)^2(-3)^2}{-12}$.

Do the Rules Work with Literal Numbers?

Up to this point, you have not been asked to work with liter numbers having signed numbers for coefficients. For exampl ne of your exercises has involved the collecting of terms like

$$-3x^2 + 2x^2 - 5x^2 + x^2.$$

ch questions are now within your power. What is the answer this one?

You have at hand all the rules for working with signed mbers. It is merely a matter of combining these with the es already learned for working with literal numbers. (Now a good time to go back and review the rules of operation for ral numbers. Refer to Chapter III.)

Once you have thoroughly learned both sets of rules, try the lowing exercises.

1. Add: $-3xy$, $+5xy$, $-12xy$.
2. Subtract: $15x^2$ from $-4x^2$.
3. Multiply: $8ab$ by $-2a^2$.
4. Divide: $-25x^2y^2$ by $-5xy$.
5. Multiply: $-4m(3k - 5n)$.

After you have tried these five exercises, compare your solu-ns with those which follow.

ample 1.

$$\begin{array}{r} -3xy \\ +5xy \\ -12xy \\ \hline -10xy \end{array}$$

Method: You add the coefficients getting -10. Then multiply this sum by the common factor of the like terms which is xy.

ample 2.

$$\begin{array}{r} -4x^2 \\ 15x^2 \\ \hline -19x^2 \end{array}$$

Method: Subtract the coefficients getting -19. Then multiply this number by the common factor, x^2, of the like terms.

ample 3.

$$\begin{array}{r} 8ab \\ -2a^2 \\ \hline -16a^3b \end{array}$$

Method: The product of the numerical parts is -16, and that of the literal parts is a^3b. Multiply these two products to get the answer.

Example 4. $\frac{-25x^2y^2}{-5xy} = 5xy$ *Method:* Divide factors of the
nominator into factors
the numerator. The
divides into -25 to gi
5. The xy divides in
x^2y^2 to give xy.

Example 5.

$$\begin{array}{r} 3k - 5n \\ -\ 4m \\ \hline -12km + 20mn \end{array}$$

Method: Multiply each term of t
binomial by $-4m$.

Here follows a set of exercises involving literal numbers wi
signed coefficients. Attack them in earnest. It is importa
that you should be able to perform such operations with accura
and speed. When in doubt, recall the meanings of symbols a
our rules for handling them.

EXERCISES

Add:

1. $\begin{array}{r} -3a \\ +2a \\ \hline \end{array}$ **2.** $\begin{array}{r} +5x^2 \\ -3x^2 \\ \hline \end{array}$ **3.** $\begin{array}{r} -7xy \\ -6xy \\ \hline \end{array}$

4. $\begin{array}{r} +9km \\ -12km \\ \hline \end{array}$ **5.** $\begin{array}{r} -18r \\ +20r \\ -12r \\ \hline \end{array}$ **6.** $\begin{array}{r} -1.5a^2b \\ -3.5a^2b \\ +\ 7a^2b \\ \hline \end{array}$

Subtract:

7. $\begin{array}{r} +8n \\ +5n \\ \hline \end{array}$ **8.** $\begin{array}{r} +8n \\ -9n \\ \hline \end{array}$ **9.** $\begin{array}{r} +14m^3 \\ +19m^3 \\ \hline \end{array}$

10. $\begin{array}{r} -xyz \\ -6xyz \\ \hline \end{array}$ **11.** $\begin{array}{r} -17a^2 \\ -11a^2 \\ \hline \end{array}$ **12.** $\begin{array}{r} -4t \\ +3t \\ \hline \end{array}$

Multiply:

13. $(-3b)(+2a)$. **14.** $(+4k)(-5k)$.

15. $(-7x^2)(-7x)$. **16.** $(+4ab^2)(-a^2b)$.

17. $\begin{array}{r} 12ac - 6bc \\ - 4ab \\ \hline \end{array}$ **18.** $\begin{array}{r} -5x + 6y \\ + 3x \\ \hline \end{array}$

ivide:

19. $-24x^2y$ by $8xy$. **20.** $18abc$ by $-9b$.

21. $-36k^3$ by $-4k^2$. **22.** $-3.5m^2n^2$ by $7mn$.

23. $56pq$ by $-14pq$. **24.** $-10c^3d^3$ by $-10cd$.

mplify by collecting like terms

25. $-3x^2 + 2x^2 - 5x^2 + x^2$.

26. $12ab - 8ab - 10ab + 6ab$.

27. $-6b - 7b + 10b - 9b + 18b$.

28. $5x^2y + 8x^2y - 7x^2y - 12x^2y + 5x^2y$.

Some More About Parentheses

You have already learned on page 132 that

$$a(b + c) = ab + ac.$$

ach term within the parentheses is multiplied by the number tside and the products are added. In the same way,

$$-2(a + b) = -2a + (-2b) = -2a - 2b.$$

so $$-4x(x - 2y) = -4x^2 + (+8xy) = -4x^2 + 8xy.$$

Parentheses indicate multiplication.

In examples like $+(2x - 3y)$ and $-(2x - 3y)$, the numbers tside the parentheses are understood to be $+1$ and -1, respectively. Thus

$$+(2x - 3y) = +1(2x - 3y) = 2x - 3y,$$

d $$-(2x - 3y) = -1(2x - 3y) = -2x + 3y.$$

An expression like

$$2x(3x - 2y) - 5x(x + 4y)$$

volves two multiplications. The first is by $2x$ and the second is by $-5x$. The terms of the products are then collected follows:

$$\begin{aligned}2x(3x-2y)-5x(x+4y) &= 6x^2-4xy-5x^2-20xy\\ &= x^2-24xy.\end{aligned}$$

Parentheses can always be removed by performing t indicated multiplications.

EXERCISES

Remove the parentheses and, where possible, collect like term

1. $-6(x+y)$. 2. $-2(m-n)$. 3. $12(a+b)$.
4. $-3a(2a-b)$. 5. $7x(4x-y)$. 6. $-k(m+n)$.
7. $-(7-3b)$. 8. $-(a+b)$. 9. $-y(6y+7)$.
10. $3x(2-4y)-x(9+3y)$. 11. $5a(3a+b)-4a(3a-b)$.
12. $-k(m+n)+5k(3m-n)-7k(2m-3n)$.
13. $10r(3r^2-5)+7r(-r^2-1)$.
14. $12x(5x-3y)-5x(12x-7y)$.
15. $-6a(b+c)+10a(b-c)$.

Summing Up

You had better stop now and recall the facts about sign numbers. Here they are. Study them carefully before y attempt the graded exercises which follow.

1. *When the measures of a quantity involve the idea of sen or* **direction** *(such as* **up** *or* **down**, **right** *or* **left**, **abo zero** *or* **below zero**, *etc.), signed numbers are used f these measures.*

2. *Signed numbers are of two kinds,* **positive** *and* **negativ** *Besides these there is the number 0 which is neither po tive nor negative.*

3. *The* **numerical** *or* **absolute** *value of a signed numb is its value without the sign.*

4.

Rule for Adding Signed Numbers

(a) *If the numbers have* **like** *signs, add their numerical values and attach the common sign.*

(b) *If the numbers have* **unlike** *signs, find the difference of their numerical values, and attach the sign belonging to the number with the greater numerical value.*

5.

Rule for Subtracting Signed Numbers

Mentally change the sign of the number being subtracted and add the resulting number.

6.

Rule for Multiplying Signed Numbers

If the numbers have **like** *signs, their product is positive. If the numbers have* **unlike** *signs, their product is negative.*

In either case, the numerical value of the product is found by multiplying the numerical values of the numbers.

7.

Rule for Dividing Signed Numbers

If the numbers have **like** *signs, their quotient is positive. If the numbers have* **unlike** *signs, their quotient is negative.*

In either case, the numerical value of the quotient is found by dividing the numerical values of the numbers.

8. *Literal numbers with signed coefficients are dealt with l the literal numbers of Chapter III. It is only a matter using the foregoing rules in dealing with the coefficients*

9. *Parentheses indicate multiplication. They can be remo by performing the multiplication.*

Now that you have learned about + and − signs, you w appreciate the joke of the boy who stated that a + changes a − in passing through 0 because the cross-piece gets knock off on the way through.

REVIEW EXERCISES

A—Spade Work

Time Test No. 1 (7 minutes)

Add: **1.** $\begin{array}{r} -14 \\ \underline{\quad 9} \end{array}$ **2.** $\begin{array}{r} -7a \\ \underline{-5a} \end{array}$ **3.** $\begin{array}{r} 6z^2 \\ -4z^2 \\ \underline{z^2} \end{array}$

Subtract: **4.** $\begin{array}{r} -24 \\ \underline{-18} \end{array}$ **5.** $\begin{array}{r} 75 \\ \underline{-25} \end{array}$ **6.** $\begin{array}{r} -8b^2c \\ \underline{4b^2c} \end{array}$

Multiply: **7.** $\begin{array}{r} -4x^2y \\ \underline{7xy^2} \end{array}$ **8.** $\begin{array}{r} -m \\ \underline{-2m^2} \end{array}$ **9.** $\begin{array}{r} 8a-3b \\ \underline{-a} \end{array}$

Divide: **10.** $\dfrac{-35xyz}{-7xy}$ **11.** $\dfrac{-14t^3}{7t}$ **12.** $\dfrac{(6ab)(-2bc}{4ac}$

Remove parentheses and collect like terms:

13. $-6(x-7)+4(5x-9)$. **14.** $3a(2b+a)-4b(2a-3b)$.

Time Test No. 2 (12 minutes)

15. Add: $\begin{array}{r} -9b^2 \\ 3b^2 \\ 5b^2 \\ \underline{-4b^2} \end{array}$ **16.** Add: $\begin{array}{r} 8a \\ -2a \\ 4a \\ -7a \\ \underline{-3a} \end{array}$

17. Subtract: $\begin{array}{r} -22mn \\ 7mn \\ \hline \end{array}$ **18.** Subtract: $\begin{array}{r} 17x^2y^2z^2 \\ -4x^2y^2z^2 \\ \hline \end{array}$

llect like terms:

19. $2x^2-7y^2+11xy-4y^2-3xy-5x^2$.

20. $12a-17b+8a-20a+20b$.

21. $ab-5bc-4ca-8ab+6bc-ca$.

22. $5m^2+7m-14-4m^2-7m+11$.

nplify:

23. $\dfrac{(3x)(-5a)(-2b)}{-10ax}$. **24.** $\dfrac{(-a^2)(5ab)(-3b^2)}{a^3b^3}$.

25. $(5x^2-7x+4)-(3x^2+3x-9)$.

26. $3a(7a-b)-5a(4a+3b)$.

27. $2(x^3-5x^2+8x-3)-x(2x^2+10x)$.

28. $7p(q^2+6r)+9p(2q^2-5r)$.

B—Uses for Signed Numbers

29. If the year 1492 A.D. is represented by +1492, what does 55 represent? Using this notation, we can write that Archi-des lived from the year −287 to the year −212. How old s he when he died?

30. Using the notation of Ex. 29, we can write that Octavius esar lived from the year −63 to the year +14. How old was when he died?

31. Use the system of Ex. 1 page 249 to represent the follow-: (*a*) a deposit of $55, (*b*) a withdrawal of $24, (*c*) a withdrawal $32. Add the three signed numbers to find the net result of three transactions.

32. Use the method of Ex. 2 on page 249 to record the follow- gains and losses of the High School football team: (*a*) a loss 7 yards, (*b*) a gain of 18 yards, (*c*) a penalty of 5 yards, (*c*) a n of 8 yards. Find the sum of the signed numbers and report net result of the four items.

33. If the downward pull of gravity on a submerged log indicated as −250 pounds, how would you indicate an upwar force of 350 pounds due to the buoyancy of the water? What the sum of the two forces?

34. An aircraft carrier is proceeding due north at +25 knot What does −25 knots represent? If an aircraft travels alo the flight deck at +80 knots, how fast is the aircraft travelli relative to the water? If an aircraft moves along the deck a −80 knots, what is its speed relative to the water?

35. Use signed numbers to represent the following movemen of an elevator: (*a*) 18 floors up, (*b*) 20 floors down, (*c*) 12 floo down, (*d*) 17 floors up. Find the sum of the numbers and repo the position of the elevator with respect to its starting point.

36. If +5 stands for a gain of $5, how would you represent loss of $5? Use this method to record the following results business transactions: (*a*) a gain of $500, (*b*) a loss of $1200, (*c*) gain of $1000, (*d*) a gain of $700, (*e*) a loss of $900. Add the signe numbers and report the net result of the transactions.

37. At noon on Monday, the thermometer reads +20°C. U the method of Ex. 5 on page 250 to record the temperature aft (*a*) a fall of 15 Centigrade degrees; (*b*) a fall of 25 Centigra degrees; (*c*) a rise of 10 Centigrade degrees.

38. J. P. Agent has credits of $100, $400, and $80, and deb of $60 and $150. Use signed numbers to stand for these deb and credits and find their sum. What is J. P.'s net financi position?

39. As Fido the Dog walks east at 5 m.p.h. a flea named Fr crawls due west on Fido's back at 0.5 m.p.h. Use signed numbe to represent these speeds. Add the signed numbers and thus fi Fred's speed relative to the ground.

40. Casey and Goldstein are opponents in an election. U signed numbers to record Casey's results: in Ward One, he has majority of 312; in Ward Two, a minority of 250; in Ward Thr a minority of 13; in Ward Four (Little Galway), a majority

62. Find the sum of the signed numbers, and report the result the election.

RECREATIONS

1. *A Frog's Problem.* A certain frog, who wishes to remain onymous, was trying to jump out of a well 30 feet deep. Every ne the frog jumped 3 feet upwards (+3), he fell back downrds 2 feet (−2). What was the net result of one jump? How ny jumps would it take for the frog to get out of the well? e careful!)

2. Find the sum of the numbers

$$1 - 2 + 3 - 4 + 5 - 6 + . \quad . \quad . \quad \text{to } -100.$$

3. Consider the numbers

$$1 - 1 + 1 - 1 + 1 - 1 + \quad . \quad . \quad .$$

hat is the sum of all these numbers if the expression contains) 684 ones? (*b*) 7321 ones?

CHAPTER IX

GRAPHS

"For knowledge is of things we see."

Tennyson.

Do you learn more through your ears or through your eyes?

Which do you remember more easily, the names of people their faces?

Do advertisers rely more on printed descriptions or on pi tures and diagrams?

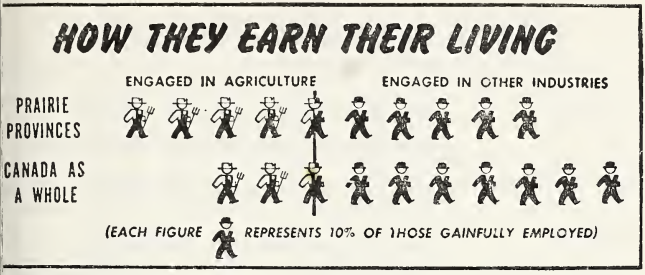
Do you carry away a more vivid memory of a lecture or a movie?

* * *

Here are a sentence and a picture graph which teach the san lesson. Which do you think is more effective?

> *In the Prairie provinces* 45% *of the people gainfully employed are engaged in agriculture, whereas in Canada as a whole the percentage engaged in agriculture is* 25%.

Many people have to prepare information regarding su things as population, manufactured goods, farm products, i



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rts and exports, motor accidents, weather reports, and many her things. Moreover, we all have to read and interpret some this information. There are two ways in which such data can concisely presented. One of these is by means of tabulated ts of figures. The other is by means of charts or diagrams ich show at a glance the facts which are conveyed by the bles. Such charts are called ***graphs***. You must have seen any graphs of various kinds in the course of your reading. It uld be a useful exercise for you to make a collection of some teresting graphs which you can clip from newspapers or agazines.

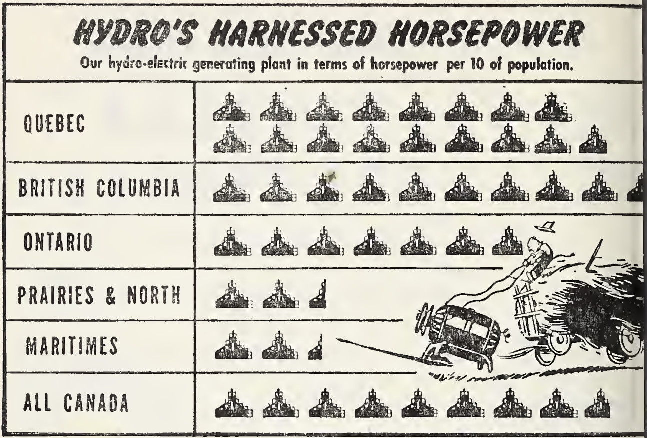
Pictographs

In a pictograph, such as the one given above, information is esented by means of picture symbols. On the next page are o more pictographs.

Some advice on reading and making graphs. After you have oked carefully at these graphs, read the following items and see w many of the facts you have observed.

(1) The quantity which is being graphed (horsepower or pulation) is arrived at by counting the symbols.

(2) There is a vertical line at the left from which the symbols gin.



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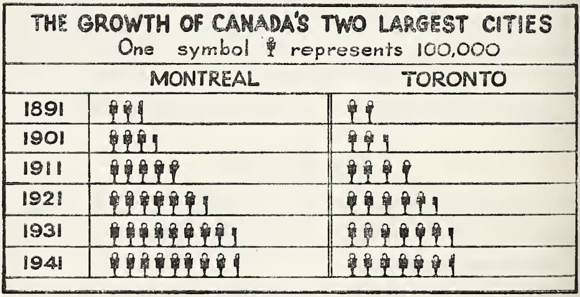


(3) The symbols in each line are equally spaced.

(4) Fractional symbols are used at the end of a line whe necessary.

(5) An explanation of the symbols is given at the top of th graph.

Pictographs are only approximate. In the statistics from which e population graph was made, the figures for 1891 were: Mont- al, 254,278 and Toronto, 181,215. Since it is impossible to present these numbers accurately by means of our chosen sym- ls, they are first rounded off. The numbers rounded to two gnificant digits are 250,000 and 180,000. Hence, if one symbol the graph represents 100,000, these numbers should be repre- nted by $2\frac{1}{2}$ symbols and $1\frac{4}{5}$ symbols respectively. These frac- nal symbols can be estimated only approximately. It follows at pictographs can seldom be used to make accurate compar- ns of data.

The advantage of a pictograph is in the striking way in which e data are presented. The disadvantages are (1) the time and re needed for their construction, and (2) the lack of exactness th which the facts are portrayed.

EXERCISES

1. Read from the graph the amount of power generated by dro-electric plants for each 10 of the population in (*a*) Quebec,) Ontario, (*c*) all Canada.

2. (*a*) Give the approximate population of (i) Montreal in 01, (ii) Toronto in 1941. (*b*) State the increase in population (i) Montreal, (ii) Toronto, from 1891 to 1941.

3. After first rounding off the numbers to two digits, repre- nt by a pictograph the production of aircraft in the United ingdom from 1940 to 1943.

For a picture symbol use a skeleton aeroplane of your own vising.

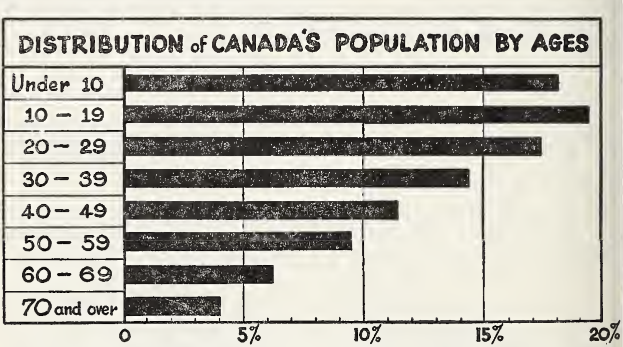
1940 — 15,049 1941 — 20,093
1942 — 23,671 1943 — 26,263

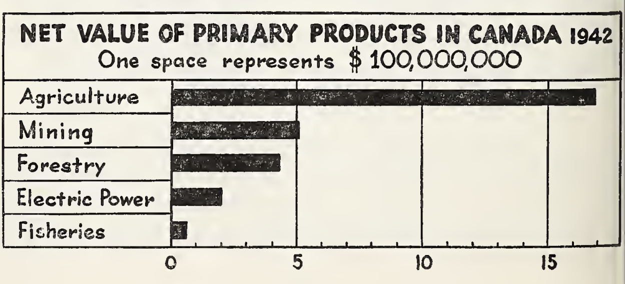
4. Represent by a pictograph Canada's wheat production as ven by the following figures. You may represent a hundred illion bushels by a symbol such as a sheaf or a bushel basket a bag of wheat. The number after each year gives the yield millions of bushels.

1930 — 421
1931 — 321
1932 — 443
1933 — 282
1934 — 276
1935 — 282
1936 — 219
1937 — 180
1938 — 360
1939 — 521
1940 — 540
1941 — 315
1942 — 557
1943 — 294
1944 — 448

Bar Graphs

A glance at the two graphs which follow will show you wh they are called ***bar graphs***. They are almost as easy to re as pictographs and you will admit that they are easier to dra People engaged in making such graphs sometimes call the *organ-pipe charts* because of the resemblance of the bars to orga pipes when they are held vertical.





Now test yourself on reading information from these graphs.

Which of the given age groups contains more people than any her? What percentage of the population are 70 years and ver? Are there more people in Canada under 30 years of age more over 30?

What was the approximate value in 1942 of Canadian forestry oducts?, of electric power?, of the products of the mines?

Let us examine now how these graphs are constructed. What you observe regarding (*a*) the widths of the bars, (*b*) the acing of the bars, (*c*) the position of the left-hand ends of the rs? What is the purpose of the scale of numbers along the rizontal line at the bottom of each graph?

The illustration on p. 282, with bars drawn vertically, is of terest to motorists. It shows the relation between the speed a car and the length of life of its tires. The importance of is graph is emphasized in the note penned by the late President oosevelt.

The following exercises will give you further practice in read- g these graphs and in making others for yourself. Before lecting the scale on which to measure the lengths of the bars, bserve the spread of the numbers which you have to represent. hen use as large a scale as will allow you to represent the data n your page conveniently. Does the scale need to begin at 0?

Here is a chance for you to develop some craftsmanship in aking good graphs.

> *A well-drawn graph speaks for itself. It is mathematics that has "come alive".*

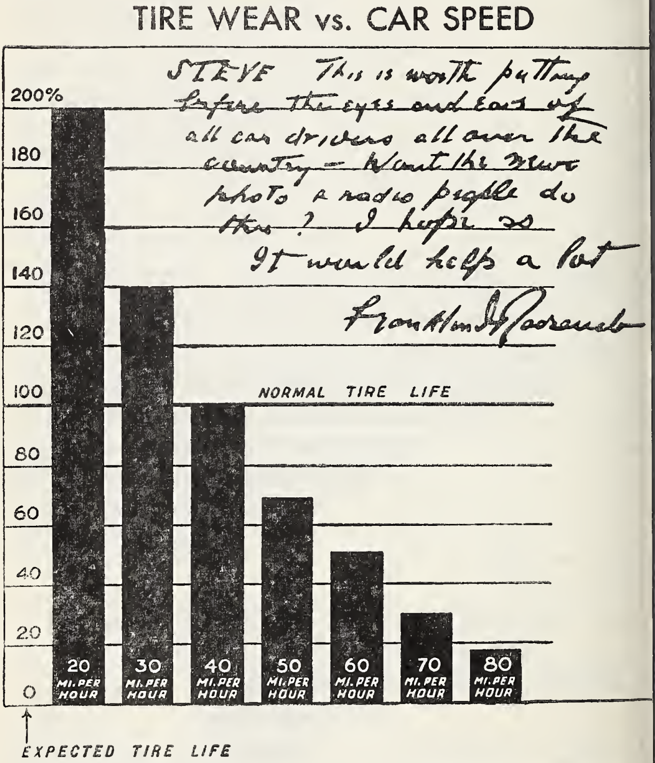
EXERCISES

1. (Fig. p. 280) State the percentage of the Canadian opulation (*a*) between 40 and 50, (*b*) under 20, (*c*) 60 and over.

2. By how many times does the value of agricultural pro- ucts exceed that of forestry products, according to the graph

given on p. 280? (*b*) Is it true that the value of agricultur products is greater than that of the other four combined?

3. (Fig. below) Mr. Long drives his car at an averag speed of 30 miles per hour and Mr. Swift drives at 60 mile



The memorandum beside the graph shows what President Roosevelt thought of its importance

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er hour. If the mileage of the two cars is the same, how many ts of tires will Mr. Swift need for one set of Mr. Long's tires?

4. From the following data construct a bar graph to show the ngths of the earth's five longest rivers.

Amazon, 3800 miles; Congo, 3000 miles: Mississippi, 4200 miles; Nile, 3500 miles; Yangtze, 3400 miles.

5. In a publication on the natural resources of Quebec, the llowing figures were given for the average number of hours of nshine in Montreal, for a 10-year period. Make a bar graph represent this information.

Jan.,	76;	Feb.,	103;	Mar.,	146;	Apr.,	174;
May,	205;	June,	217;	July,	238;	Aug.,	219;
Sept.,	172;	Oct.,	122;	Nov.,	69;	Dec.,	60.

6. Make a bar graph to show the diameters of the planets as iven by the following figures, in miles:

Mercury,	3030;	Venus,	7700;	Earth,	7920;
Mars,	4230;	Jupiter,	86,500;	Saturn,	73,000;
Uranus,	31,900;	Neptune,	34,800;	Pluto,	4000(?).

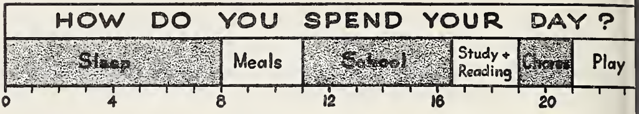
Rectangular and Circular Distribution Graphs

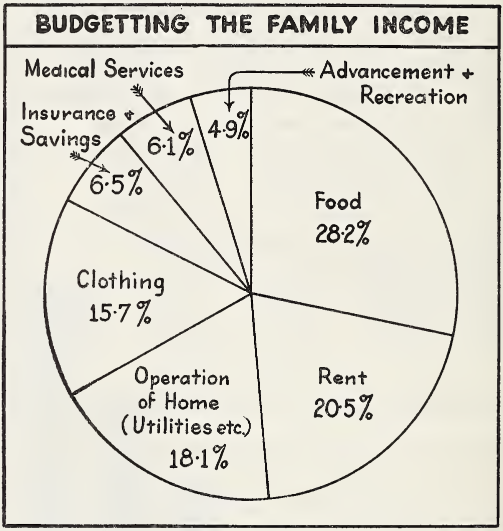
These are only big names for another very simple kind of aph. If the meaning of the two following graphs is quite vious to you, you will not need to spend much time on the escription which follows.

In each of these graphs a certain whole (a 24-hour day in the rst, a family income in the second) is considered to be made up parts and the object is to show how much each part contributes to the whole. The rectangle and circle graphs are parcularly fitting for this purpose.

In the rectangle graph on page 284 the scale at the bottom rresponds to the 24 hours of the day, so that the number of urs in each division is easily read off.

The construction of a circle graph requires a little more calcution. Consider, for example, the family-budget graph on ge 284.





You read that 28.2% of the budget is spent for food. To repr
sent this on the graph requires a sector which is 28.2% of th
area of the whole circle. The angle of the sector is therefo
$\frac{282}{1000}$ of 360°, which is approximately 102°. This angle is the
laid off with a protractor. The other angles are found similarl
and so the graph is constructed.

On account of its shape the circle graph is sometimes referre
to as a ***sector chart*** or as a ***pie chart***.

EXERCISES

1. From the rectangular graph on page 284 find how many the 24 hours are spent (*a*) in sleep; (*b*) at school; (*c*) in study d reading; (*d*) at play.

2. How do you spend *your* day? Show your division of the y by a rectangular graph.

3. (*a*) If a family income is spent according to the circle aph, state the fraction of it used for (i) rent; (ii) insurance and vings; (iii) clothing.

(*b*) How would you expect the sectors of such a graph in me of war to differ from the corresponding sectors in time of ace?

4. *Where do school accidents happen?* The following figures e taken from the 1945 edition of *Accident Facts*, published in hicago. Represent each set of figures on a rectangular graph.

Accidents in school buildings: Gym., 36%; Hall and Stairs, 20%; Class Rooms, 15%; Shops, 13%; Other, 16%.

Accidents in school grounds: Unorganized Activities, 45%; Football, 18%; Baseball, 11%; Other Organized Games, 16%; Apparatus, 10%.

5. Represent on a circle graph the distribution of the male opulation of Canada by occupations, as follows:

Agriculture, 31.7%; Manufacturing, 17.1%; Trade, 8.1%; Transportation, 7.6%; Fishing, trapping and logging, 3.9%; Mining, 2.1%; Construction, 6.0%; Service, 9.4%; Clerical, 5.4%; Others, 8.7%.

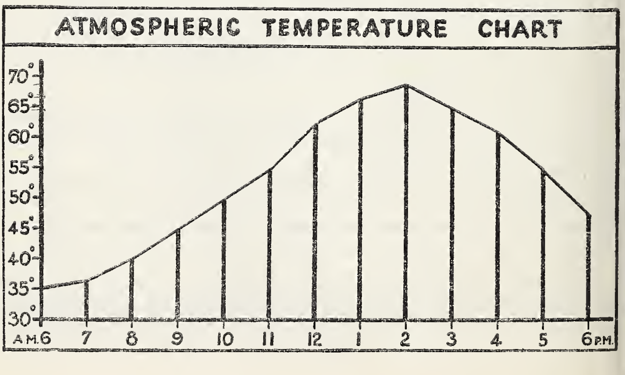
6. In order to express the force of the wind, aviators make se of the *Beaufort Scale of Wind Force*, in which a single number assigned to represent a given range of wind velocity, as given the following table. Construct a rectangular graph to show ind velocities up to 80 miles an hour, divide it into sections and bel these with the proper Beaufort numbers. (Wind velocities e estimated to the nearest mile per hour. In drawing the aph assume that the velocities corresponding to scale number 2 nge from 3.5 to 7.5 and similarly for the other scale numbers.)

Beaufort number represents wind velocity in miles per hou

0	represents velocity	less than 1
1		1 to 3
2		4 to 7
3		8 to 12
4		13 to 18
5		19 to 24
6		25 to 31
7		32 to 38
8		39 to 46
9		47 to 54
10		55 to 63
11		64 to 75
12		above 75

Line Graphs

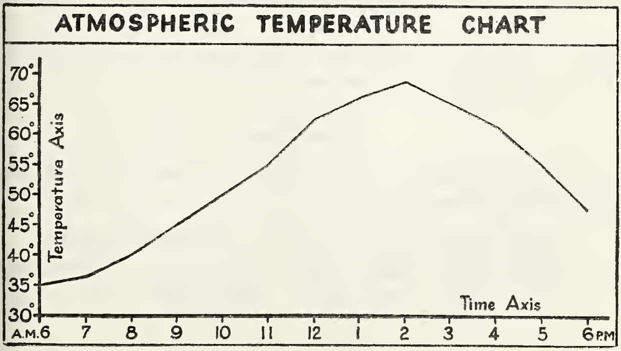
You see below a bar graph showing atmospheric temperature for 12 hours of a certain day. This graph has another featur The tops of successive bars are joined by line segments, so as t give a jagged path, with corners. Now if the bars were remove we would still be able to read the information from the jagge line. It is called a ***broken-line graph***.



If we are looking for the simplest kind of graph to construct, ve seem to have attained our objective. It is worth while then o ask what are the essential elements in a line graph.

You observe that this graph is enclosed in a rectangle. However, we can discard two sides of the rectangle, retaining only he bottom and left-hand sides. These lines we shall call ***axes***; hey are guide lines along which scales are marked and measurenents can be made. The point where they meet is a beginning oint for the whole graph; its mathematical name is the ***origin***.

In the temperature graph which we are studying, the horiontal axis may be called the ***time axis***. It is marked off in paces each of which represents one hour. The origin is marked A.M. and the point marked 12 represents noon. The vertical xis is the ***temperature axis***. The scale on it shows degrees ahrenheit which are marked at intervals of 5°, beginning ith 30° at the origin. Now suppose we wish to put on he graph the fact that the temperature at 9 A.M. is 44°. From he point marked 9 on the time axis we measure upward a disance corresponding to 14°. Since one vertical interval is 5° we equire $2\frac{4}{5}$ of these intervals. In this way we locate a point on ur graph. Having found successive points corresponding to he hours, we need only a ruler and pencil to complete the graph. ere, then, is the temperature graph reduced to its essential eatures.

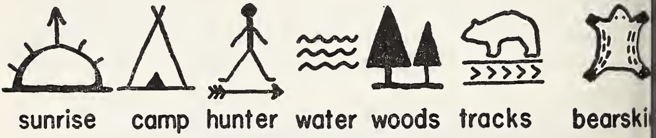


It will help you to gain familiarity with line graphs if yo answer the following questions. First, follow the graph fro left to right with your eye. Between what hours is the temperature rising? Between what hours is it falling? When is th temperature greatest? At what approximate time was the temperature rising most rapidly? Is the temperature likely to ris or fall between 6 and 7 P.M.?

Line Graphs versus Other Types

Are line graphs as useful as pictographs or other ingeniou graphs? At first glance they are not so striking, but as yo become used to them you will realize that they have decide advantages. (*a*) They are easier to make. (*b*) Their meanin is just as easy to read. (*c*) By the use of geometry and algebra we can draw important lessons from them.

When primitive people first learned to write, they did so b means of picture symbols. How tedious it would be today if al our books had to be written in picture writing! You will fin that line graphs have the same advantage over pictographs tha modern writing has over picture writing.



EXERCISES

Construct broken-line graphs, properly labelled, to represen the data in 1, 3 and 4.

1. The National Debt of Canada, in millions of dollars.
1871, 78; 1881, 155; 1891, 238; 1901, 268;
1911, 340; 1921, 2341; 1931, 2262; 1941, 3649.

2. Your graph of Ex. 1 should recall to you some facts of nadian history and provide the answers to the following estions.

(*a*) Which was the decade of railway building?

(*b*) In which decade did a Great War occur?

(*c*) In which decade did the government borrow the least money? What conclusion do you draw regarding the condition of the country in this decade?

(*d*) What is likely to be the character of the graph in the decade after 1941?

3. The average monthly output of central hydro-electric tions in Canada, in millions of kilowatt-hours.

1928, 1340; 1929, 1441; 1930, 1463; 1931, 1340;
1932, 1296; 1933, 1436; 1934, 1734; 1935, 1918;
1936, 2079; 1937, 2257; 1938, 2130; 1939, 2322;
1940, 2460; 1941, 2732; 1942, 3038; 1943, 3300;
1944, 3298.

4. The production of Douglas Fir, in Canada, in millions of t, board measure.

1927, 1437; 1928, 1517; 1929, 1605; 1930, 1255;
1931, 936; 1932, 642; 1933, 818; 1934, 1032;
1935, 1115; 1936, 1422; 1937, 1407; 1938, 1402;
1939, 1595; 1940, 1692; 1941, 1525; 1942, 1540;
1943, 1234.

5. From your graph of Ex. 4, answer the following questions:

(*a*) Describe how the production of Douglas Fir has varied from 1927 to 1943.

(*b*) How do you account for the depression in the curve?

(*c*) For what year is the graph rising most steeply? What does this mean?

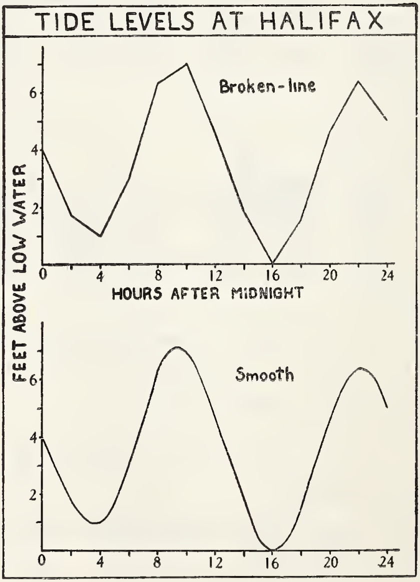
(*d*) For what year is the graph falling most steeply? What is the explanation?

Improving the Line Graphs

Do not read the next sentence after this until you have pau to think whether you could suggest any improvement in t broken-line graph.

* * *

Take the foregoing temperature graph, for example. As y trace the line with your pencil, from left to right, the rise and f of the line corresponds to the rise and fall of the temperatu Does it strike you that changes in temperature take place gradually and imperceptibly that the graph representing th should have no corners? In other words the graph would gi a better picture of the temperature if, instead of a broken li it were a ***smooth curve.*** As another example, compare t following broken-line graph and smooth graph which are dra from the same data.



Graphs of Formulas

You have learned in a previous chapter something of the agic of formulas. By examining closely the way in which the ters enter into a formula, you can often tell how a change in e of them affects another. When only two letters are changing is information can be displayed in a vivid manner by means a graph.

In drawing graphs we have made use of tables giving pairs numbers that match. How can we get such a table from a mula? Your previous work gives you the answer,—by sub-tution. By substituting numbers in the formula you build up able of values and from this table you construct a graph.

Formula* ⟶ *Table of Values* ⟶ *Graph

A simple example will best show the method. If a workman paid at the rate of 75c an hour then for h hours his wage, in llars is

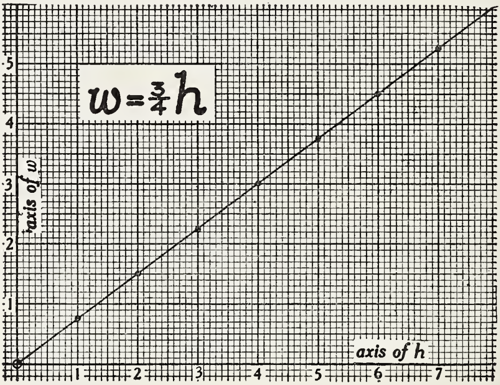
$$w = \frac{3}{4}h.$$

giving to h the values 0, 1, 2, 3, 4, etc., we calculate the responding values of w and so build up the following table.

h	0	1	2	3	4	5	6	7	8	. . .
w	0	0.75	1.5	2.25	3	3.75	4.5	5.25	6	. . .

To represent the relation graphically we shall take the hori-ntal axis for the time axis or ***axis of h***, and the vertical axis the wage axis or ***axis of w***. Now we are ready to plot points. For example, when $h = 3$, $w = 2.25$. This means t from the point marked 3 on the axis of h we measure upward listance of 2.25 to find the required point. If you use graph per, which is marked off in squares, it will be very easy to ate the points. When you have plotted a number of points s apparent that they all lie on a straight line. This straight is the graph of the formula $w = \frac{3}{4}h$.

Let us take another glance at this graph before we leave Even if we had no formula and no table, we could obtain mu information from the graph. For example, an increase of 2 in no matter where it is taken, will produce the same increa 3/2 in w. An increase of 3 in w always results from the sa increase 4 in h. Doubling h will always cause w to doub



Now it is your turn to answer. What is the effect on w adding the constant k to h? What is the effect on w of mul plying h by k?

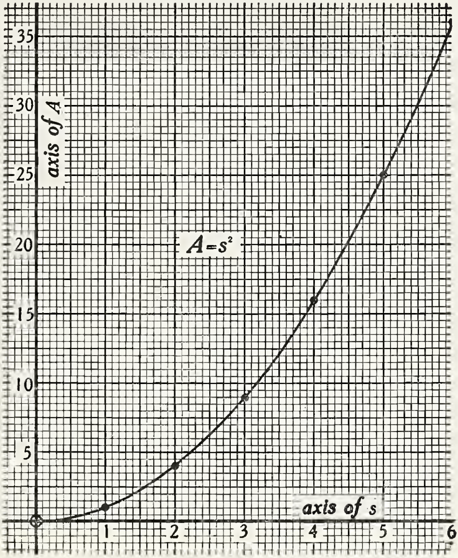
As a second example, consider the formula which gives t area of a square. If the side of a square is s units and the ar A square units, we have

$$A = s^2.$$

First, we construct a table in which values of A correspondi to given values of s are recorded.

s	0	1	2	3	4	5	6	7	8	. . .
A	0	1	4	9	16	25	36	49	64	. . .

ext, we take a sheet of squared paper, select an origin and axes d decide on the scales to be used on these axes. (You had better ke a look at how big the numbers are getting before deciding your scale. Since A eventually increases much faster than s, will be well to choose a smaller unit on the axis of A than on e axis of s.) Now transfer the table to the graph by plotting e points, one for each pair of values of s and A. The final step



to draw through these points a smooth curve, making as close it as we can to the plotted points. It is in this last step that u can train your hand and eye to produce a neat and workmane graph.

As this is not a straight graph it carries a somewhat different essage from that of the former one. As your eye follows the curve from left to right, you observe that it rises more and m steeply. From $s = 0$ to $s = 1$, the rise is only 1 whereas fr $s = 10$ to $s = 11$ the rise is 21! This means that A increa more and more rapidly with respect to s. The formula emph sizes this fact since, if s is doubled, A is multiplied by 4.

You learned in Chapter V that a formula such as $A =$ containing a second power, grows in a different manner fr that of one like $w = \frac{3}{4}h$ which contains only a first power. T two graphs show up this difference clearly, the first power bei represented by a straight line while the graph of the seco power curves upward more and more steeply as s grows.

As we take our leave of formulas and their graphs, let remind ourselves again that we are here dealing with thr agencies for presenting the same information. These are t formula, the table, and the graph. It may interest you notice that, in the study of graphs, three aspects of your stu of mathematics are fused together. We may say that the fo mula gives the information algebraically, the table gives it arit metically, and the graph gives it geometrically.

EXERCISES

1. Draw a graph of the formula for the perimeter of an eq lateral triangle, $p = 3s$. (*a*) What is the character of the grap (*b*) What is the effect on p of (i) increasing s by 4, (ii) increasi s by k, (iii) multiplying s by 3, (iv) multiplying s by k?

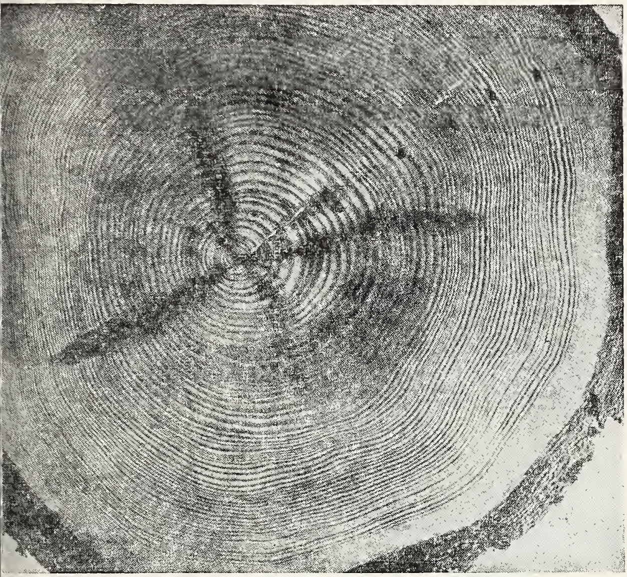
2. Draw a graph of the formula for the area of a circ $A = \pi r^2$. (Take $\pi = 3.1$). From the graph, describe how continuous increase in r affects A.

3. A certain job requires 36 days' work of one man. If men do the job in t days then the number of days' work is Hence $nt = 36$ and $n = 36/t$. Plot a graph of this formul From your graph read off (*a*) how many men would do the j in 8 days, (*b*) how many days 5 men would require. (In the fo mula $n = 36/t$, the growth of t affects n inversely. Compa the discussion of Chapter V, page 191.)

A Postscript on Graphs

This is a postscript on the subject of graphs.

A natural graph. In the course of our education, one of our ncipal teachers is nature. Did it ever occur to you that she netimes makes use of graphs? Here is an example of a graph



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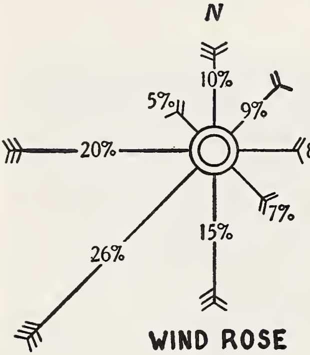
y Press.

m nature. It is a cross-section of a tree which was exposed en the tree was cut down. As with other graphs we can read ne information from this. How do you read from it the age the tree? Pick out certain years of the tree's life in which there was an abnormal amount of rainfall. How would y detect a year of drought?

An artificial graph. It would be impossible to describe h all the ingenious and fanciful types of graph that have be invented.* One more exa ple will have to suffice. Tl is a *wind rose* such as is us at an airport to indicate t frequency and velocity of t winds from different dir tions. From it we read th the wind blows from t south-west 26% of the tin from the west 20% of t time, etc. The number barbs on each arrow stan for a number in the Be fort scale for which you ha already drawn a graph. Th give the approximate wind velocity for each direction. Th we read that south-west winds have velocities ranging from to 38 miles per hour.



WIND ROSE

REVIEW EXERCISES

A

1. **Canadian Immigration.** Represent by a bar gra Canada's immigration in the year 1937-38: from the Unit States, 5,643; from Central Europe, 4,773; from the Unit Kingdom, 3,351; from other countries, 1,878.

2. **Canal Traffic.** Represent on a bar graph the followi comparison of three canals, by net tons of traffic (1937).

Sault Ste. Marie,	87,637,000;
Suez,	36,129,000;
Panama,	28,108,000.

---

*A book called *Graphic Presentation*, by W. C. Brinton, contains a la collection of graphs of all kinds.

3. **Where is merchandise sold?** Represent on a rectangular aph the following distribution of sales by retail stores of dif- ent types: Food stores, 25.2%; General merchandise stores, .1%; Automotive sales, 17.5%; All other stores, 38.2%.

4. (*a*) Draw a graph of the formula for the *perimeter of a uare*, $p = 4s$. (*b*) What is the character of the graph? (*c*) Two uares have sides of 10 inches and 100 inches, respectively. hat is the effect on the perimeters of (i) doubling all the sides,) increasing all the sides by 2 inches?

5. **Canada's Population.** (*a*) Make a bar graph to show the owth in population of Canada according to the last 6 censuses.

891, 4,833,239; 1901, 5,371,315; 1911, 7,206,643;
921, 8,787,949; 1931, 10,376,786; 1941, 11,506,655.

) From your graph, tell in which decade the growth of popu- ion was most rapid.

6. **Passenger Cars in Canada.** (*a*) Represent by a line graph e number of passenger cars in Canada in the period 1905 to 40.

1905, 553; 1910, 5,890; 1915, 60,688;
1920, 251,945; 1925, 641,186; 1930, 1,055,514;
1935, 989,744; 1940, 1,234,637.

) Give an explanation for the depression in your curve.

7. **High Buildings.** Represent by a bar graph the heights the following buildings:

Great Pyramid,	480 feet;
Eiffel Tower,	984 feet;
Empire State Building,	1250 feet;
Chrysler Building,	1046 feet;
R.C.A. Building, Rockefeller Centre,	850 feet;
Woolworth Building,	792 feet.

8. Refer to the *wind rose* on page 296. (*a*) According to this aph, tell for what per cent of the time the wind blows from the east, (ii) the north-west. (*b*) Tell the range of velocities (i) east winds, (ii) north-west winds.

B

9. **Three Industries in War time.** (*a*) Represent the foll ing information by means of bar graphs, one for each indust

Number of persons employed in three industries in Canada.

	Aircraft	Farm Implements	Shipbuilding
1938	1,617	6,458	3,596
1939	3,596	5,306	3,491
1940	10,348	6,857	9,707
1941	26,661	8,804	21,240
1942	44,886	11,853	50,132

(*b*) In which of the three graphs do the bars lengthen m slowly? What is the explanation?

10. **How far away are the planets?** Representing the sun a point, draw lines radiating from it to show the distances of t planets. The distance from the sun to the earth is taken as

Mercury,	0.39;	Venus,	0.72;	Earth,	1.00;
Mars,	1.52;	Jupiter,	5.20;	Saturn,	9.54;
Uranus,	19.18;	Neptune,	30.05;	Pluto,	39.51.

11. **The Circumference of a Circle.** Draw a graph of the fo mula for the circumference of a circle, $C=2\pi r$, taking $\pi=22$ What is the character of the graph? Compare it with the gra of the perimeter of a square (Ex. 4).

12. **Safety First!** The number of accidental deaths Canada each year reaches tragic proportions. In 1943 the to number was 7,385, distributed according to the following figur Represent this information on a circle graph. Falls, 23%; Mo Vehicle, 19%; Drowning, 14%; Burns, 11%; Railroad, 4 Poison Gases, 2%; Other, 27%.

13. **Income Tax.** An increasingly important source of D minion revenue has been the tax on incomes. In the followi

le the number after each year gives the income tax in millions dollars.

1936, 83; 1937, 102; 1938, 120; 1939, 142; 1940, 134;
1941, 248; 1942, 510; 1943, 910; 1944, 1152.

Represent this information by a broken-line graph. (*b*) Be-
een what years is the graph steepest? What is the explanation?

14. **We Go to School.** Out of each 100,000 pupils in Canada Grade I the numbers of those who continue to the higher grades as follows.

ade I, 100,000; Grade II, 99,284; Grade III, 98,274;
ade IV, 96,112; Grade V, 92,237; Grade VI, 85,936;
ade VII, 77,163; Grade VIII, 66,829; Grade IX, 48,241;
ade X, 33,329; Grade XI, 20,191.

present this information by a line graph.

15. **Where do apples come from?** The average exports of apples different exporting countries for the 5-year period, 1928 to 32, are listed in percentages below. Show this information on ircle graph.

United States, 57.4%; Canada, 15.2%; Australia, 10.9%; Italy, 5.7%; Yugoslavia, 5.1%; New Zealand, 3.5%; Others, 2.2%.

16. **The Girl Guides.** The total number of members of the anization of Girl Guides in Canada, over a 10-year period, is en by the following figures. Round off these numbers to 2 ures and represent them on a line graph.

36, 48,392; 1937, 63,308; 1938, 57,266; 1939, 59,206;
40, 61,888; 1941, 67,531; 1942, 58,612; 1943, 60,160;
44, 64,166; 1945, 64,236.

17. **The Falling Body Formula.** (See Ex. 2, page 192.) Draw raph of the formula $d = 16t^2$, being careful to select scales on ur two axes so as to obtain a convenient curve.

C

18. A. Walker walks from his home, Spruce Nook, to Eaglet 8 miles away, at the rate of 4 miles an hour. He remains Eagleton for 2 hours and then returns home at 2 miles an hou Construct a line graph showing Mr. Walker's distance fro Spruce Nook from the time he left until he returned. Use horizontal *time* axis and a vertical *distance* axis. Should th graph be smoothed off?

19. Draw a graph of the formula

$$d = 0.045r^2 + 1.1r,$$

which gives the stopping distance of a car at different spee (See p. 195.)

20. The Jewel restaurant serves 60c meals. If the avera cost of a meal is 30c and the daily cost of labour and rent is $1 then the daily profit for n meals served is p cents where

$$p = 30n - 1200.$$

Choosing a convenient scale for each axis, draw a line graph show the relation between n and p. What is the interpretati of the fact that a part of the graph is below the axis of n?

21. The directors of Camp Almac planned trips for two grou of boys. Wilbur's group started from Almac, travelled $1\frac{1}{2}$ mil east and then turned northeast. John's group started fro Bilnuk, 5 miles east of Almac, travelled 1 mile south and th turned northwest. They were told that a party would me them in the bush with their supper. Find graphically the poi at which the provisioning party should place itself in order intercept both Wilbur's and John's groups.

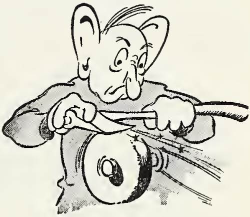
CHAPTER X

MORE ABOUT ALGEBRA

We Sharpen Our Tools

Our purpose in this chapter is to extend our knowledge of as already discussed, rather than to introduce new ones. For is reason, it will be helpful for us to recall what has been done algebra.

Our first contact with algebra was made when we began using ters to stand for numbers. We described these letters as thematical slaves, which give al-bra its power. In the beginning, r letters stood for the ordinary signed numbers of arithmetic. We d no other numbers when literal mbers first came upon the stage. ter, we widened our number system take in negative numbers. This tension of the number system of ithmetic gives algebra further scope dealing with the problems of everyday life.



From now on, the letters we use will stand for any numbers our extended number scale. We introduced literal numbers d signed numbers separately to give you a chance to master e idea at a time. Henceforth, you will understand that a ter, such as a, m, or x, may stand for any number, positive, gative, or zero. Here is a sample of the values which a letter may have: -1000, 7, $\frac{3}{4}$, -0.76, -40, 72, 0.

More About Substitution

Is $-5a$ positive or negative? * * * You see at it all depends on a. If $a = 7$, then $-5a = -5(7) = -35$, a negative number. On the other hand, if $a = -7$, then $-5a$ $-5(-7) = 35$, a positive number. Moreover, if $a = 0$, th $-5a = 0$. We cannot tell the sign of $-5a$ until we know t sign of a. Substitution of the value of a gives the answer.

You do not need to be told at this stage that substitution a common process in algebra. It is only necessary to recall o work with formulas and equations. Until now you have had practice in substituting negative numbers for letters. Now it time for such practice. Try the following examples witho assistance. Then compare your solutions with those given belo

Example 1. If a temperature of C° Centigrade corresponds one of F° Fahrenheit, then we have the formula

$$C = \tfrac{5}{9}(F - 32).$$

Use this formula to change the reading -4°F to degrees Cen grade.

Example 2. If $x = 1$, $y = -2$, find the value of $x^2 + y^2 - 2x$

Example 3. If $a = -1$, $b = -3$, and $c = -2$, find the val of $a^3 - b^3 + c^3 + 3abc$.

* * *

Solution 1. $C = \tfrac{5}{9}(-4-32) = \tfrac{5}{9}(-36) = -20$.

Therefore the temperature is -20°C.

Solution 2.

$$x^2+y^2-2xy = 1^2+(-2)^2-2(1)(-2) = 1+4+4 = 9.$$

Solution 3.

$$\begin{aligned} a^3-b^3+c^3+3abc &= (-1)^3-(-3)^3+(-2)^3+3(-1)(-3)(- \\ &= -1 + 27 - 8 - 18 = 0. \end{aligned}$$

Did you make a perfect score? Observe how important it always to keep in mind the *meanings* of the symbols. Here a some more exercises on which to improve your skill.

EXERCISES

d the value of:

1. $x^2 + 6x + 8$ when (*a*) $x = -2$, (*b*) $x = -3$, (*c*) $x = -5$.
2. $(x - 2y)^2$ when $x = -5$ and $y = -2$.
3. $x^3 - y^3 + z^3$ when $x = -2$, $y = -3$, and $z = -1$.
4. $2a^2b - 3ab^2$ when $a = 4$ and $b = -3$.
5. $x^4 - 3y^4 + 2z^4$ when $x = y = z = -1$.
6. $a^3 - a^2b + ab^2 - b^3$ when $a = -2$ and $b = -3$.
7. $\dfrac{x^2 - 3xy}{4xy + y^2}$ when $x = 3$ and $y = -2$.
8. $\dfrac{1}{x} + \dfrac{1}{2y} - \dfrac{1}{3z}$ when $x = -2$, $y = 3$, and $z = -4$.
9. $\dfrac{1}{x + 2y - 3z}$ when $x = -2$, $y = 3$, and $z = -4$.
10. $1 + x + x^2 + x^3 + x^4 + x^5$ when $x = -1$.

1. If $\dfrac{1}{R} = \dfrac{1}{r} + \dfrac{1}{s}$, find R when $r = 3$ and $s = -4$.

2. If $v^2 = u^2 + 2as$, find v when $u = 40$, $a = -32$, and $s = 9$.

3. Change the reading 14°F into degrees Centigrade.

4. Solve the formula of Example 1 for F. Use the resulting nula to change the reading −200°C to degrees Fahrenheit.

More About Simplification

If you were given the expression

$$xx + y + y$$

would exclaim (we hope) "What rious way to write $x^2 + 2y$!" latter expression means the e thing and it is simpler. Simi- y, the expression

$$5a + 2b - 3a - 6b$$

mplified by writing it as $2a - 4b$.

Which expression is simpler?

In the solution of algebraic problems, it often happens th expressions which occur can be simplified. This simplificati may be a part of the work of solving the problem. It is n always obvious which of two equivalent expressions is the si pler. For example,

$$\frac{1}{x}+\frac{1}{y} \text{ is equal to } \frac{x+y}{xy}.$$

Are we simplifying when we change the first expression into t second or when we change the second expression into the fir You see that it is really a matter of taste, or sometimes of t use which we expect to make of the expression.

There is no one method of simplifying algebraic expressio Different problems require different methods. Consider now t expression

$$2(x+y)-3x.$$

If you remove the parentheses by multiplying by 2, you ha

$$2(x+y)-3x = 2x+2y-3x = 2y-x.$$

You agree that the final expression is simpler than the origin

Now take the expression

$$-3(x+y-2z-3w).$$

Removing the parentheses, we get

$$-3(x+y-2z-3w) = -3x-3y+6z+9w.$$

The final expression is not simpler than the original. Since the are no like terms to collect, the form with the parentheses preferable. This is true of many expressions.

Now examine the following pairs of equal expressions:

$$\begin{aligned} 24(3x-y) &= 72x-24y; \\ a(m+n+r-s) &= am+an+ar-as; \\ 2ab^2(x-y) &= 2ab^2x-2ab^2y. \end{aligned}$$

This list suggests that sometimes we simplify an expression

erting parentheses. The process is not difficult. Try ting the following expressions with parentheses:

$$\begin{aligned} 2 + 2w &= \\ 6ab - 3ac &= \\ 4xy - 8xz + 12x^2w &= \end{aligned}$$

* * *

l you get these answers?

$$\begin{aligned} 2 + 2w &= 2(1 + w), \\ 6ab - 3ac &= 3a(2b - c), \\ 4xy - 8xz + 12x^2w &= 4x(y - 2z + 3xw). \end{aligned}$$

In order that inserting parentheses should produce a simplition, what must the terms have in common? What is placed ront of the parentheses? What goes inside? State a rule for rting parentheses.

* * *

The process involves the division of each term by the factors mon to all the terms. For this reason, the operation is ally called ***taking out a common factor***. You may prol as follows.

1. *Write the common factor of the terms in front of the parentheses.*
2. *Divide the common factor into the expression and place the quotient within the parentheses.*

EXERCISES

plify each of the following by inserting parentheses:

1. $3a + 6b - 9c$; **2.** $2a - 3ab$;

3. $x^2 - xy + 2x$; **4.** PWOR + RCMP;

5. UNRRA + RCNVR; **6.** $6mn^2 + 15m^2n$;

7. $a^2bc - ab^2c + abc^2$; 8. $2ut + at^2$;

9. $4\pi r^2 - 2\pi rh$; 10. $rah + r^2a^2h^2 + r^3a^3h^3$;

11. $6t^2o^2ma + 12car^2ot$; 12. $k^3l^4m - k^2l^4m^2 - k^3l^3m^2$;

13. $4xy^2z^3 - 6x^2yz^2 + 8xyz^3$.

Write an expression with parentheses for each of the followi

14. The sum of the areas of 5 circles each of radius r and circles each of radius s;

15. The total volume of 3 rectangular solids, of which the fi has edges a, b and c, the second has height b and a square b of side a, and the third has height a and a square base of side

16. The total volume of 3 spheres of radius r and 4 cylind of radius r and height h.

Fractions Graduate into Algebra

You do not get far in arithmetic without finding the need fractions. For this reason we have recalled in Chapter VII t basic principles of fractions and the rules for adding, subtracti multiplying, and dividing them.

As you may suspect, fractions arise in algebra as well. T fractions of algebra are no different from those of arithme except that letters may appear in them. Consider the followi example:

> A car goes from Bottleburg to Boxville, a distance of miles, at an average speed of x miles an hour and retur at an average speed of $2x$ miles an hour. You see that the first trip the car takes $23/x$ hours and on the return tr $23/2x$ hours. Thus the time for the whole journey is
>
> $$\frac{23}{x} + \frac{23}{2x} \text{ hours.}$$

Can this expression be simplified by adding the two fractio Try it.

* * *

The principles which apply to fractions involving lite numbers are the same as those we learned for the fractions ithmetic. All the methods of Chapter VII hold without ange. The novelty of finding letters in the terms of a fraction ll soon wear off. After all, these letters stand for numbers.

A Challenge to the Student

You will now be given an opportunity to master the methods of working with algebraic fractions. Here is what you are strongly urged to do.

Review the basic principles of Chapter VII. Then try by yourself the sets of examples which follow. Each set will give you practice in applying a familiar process to fractions containing literal numbers. After you have given the problems in each set a fair trial, study the solutions which follow the set. If there still remain any questions which you cannot answer for yourself, discuss these with your teacher.

This may seem to you a longer method than having someone tell you how to do the problems, but it is the method of mastery.

SET No. 1: FINDING FACTORS OF ALGEBRAIC TERMS

A note about factors. You have seen that factors are ded in arithmetic for the simplification of fractions like 2. It is no surprise to learn that we shall need factors in ebra to simplify fractions like $6x^2/2xy$.

In arithmetic we call 2 and 3 factors of 6 since $6 = 3 \times 2$. hough it is also true that $6 = 4 \times \frac{3}{2}$, we do not call 4 a factor 6. In algebra, the usage is more general. ***We call a and actors of ab, no matter what numbers a and b stand*** . Thus a might be 4 and b, $\frac{3}{2}$; then $ab = 6$. This difference ween the usage of algebra and that of arithmetic need not fuse you. If algebra were nothing but arithmetic, we should bother learning it. Here is one place where algebra strikes on its own!

The term $6a^2b^3$ can be expressed as a product in the form

$$2 \times 3 \times a \times a \times b \times b \times b.$$

The factors in this expression, since they cannot be factor further, will be called the ***simple factors*** of $6a^2b^3$.

Be sure that you know what is meant by a ***comm*** ***factor.*** If not, look on page 210.

Example 1. For the terms $2ab$, $4a^2b$, $6ab^2$, find (*a*) three comm simple factors, (*b*) an expression which is the product of all t common simple factors and no others (the highest comm factor).

Example 2. Find the highest common factor of $3xyz$, $5x^2$ $6xy^2z$.

* * *

Solution 1. We have

$$\begin{aligned} 2ab &= 2 \times a \times b, \\ 4a^2b &= 2 \times 2 \times a \times a \times b, \\ 6ab^2 &= 2 \times 3 \times a \times b \times b. \end{aligned}$$

Now pick out all the simple factors that are common to all thr terms. They are 2, and a, and b. The highest common fact is obtained by multiplying these common factors togeth Therefore, the

$$\text{highest common factor} = 2 \times a \times b = \mathbf{2ab}.$$

Solution 2. In the same way, we have

$$\begin{aligned} 3xyz &= 3 \times x \times y \times z, \\ 5x^2yz &= 5 \times x \times x \times y \times z, \\ 6xy^2z &= 2 \times 3 \times x \times y \times y \times z. \end{aligned}$$

Therefore, the highest common factor $= xyz$.

SET No. 2: REDUCING FRACTIONS TO THEIR LOWEST TER

Example 3. Reduce the following fractions to their lowe terms:

$$\frac{6}{9a}; \quad \frac{4x^2}{2x}; \quad \frac{3a^2b^3}{2ab}; \quad \frac{5mn}{m}.$$

* * *

lution 3. Recall that *the terms of a fraction can be divided by same number without changing its value.* The use of this basic nciple gives

$$\frac{6}{9a}=\frac{2}{3a};\quad \frac{4x^2}{2x}=2x;\quad \frac{3a^2b^3}{2ab}=\frac{3ab^2}{2};\quad \frac{5mn}{m}=5n.$$

r each fraction, state the number by which the terms were ided.

SET No. 3: FINDING MULTIPLES OF ALGEBRAIC TERMS

A note about multiples. In order to add and subtract ctions in arithmetic, we made use of multiples. We shall ed their aid for the same purpose in algebra.

The usage of algebra with regard to factors has already been ed. The algebraic notion of multiples follows the same pat- n. We say that ***ab is a multiple of a and of b, no mat- what numbers a and b stand for***. If you are not clear out the meaning of ***common multiple***, turn to page 211.

ample 4. For the three terms of Example 1, find an expression ich is the product of only so many of the simple factors as are essary to make it a multiple of each of the three given terms e lowest common multiple).

ample 5. Find the lowest common multiple of $3xyz$, $5x^2yz$, y^2z.

* * *

ution 4. Refer to the factoring in solution 1. We wish to ke up the multiple of the three terms that contains as few of simple factors as possible. For a multiple of the first term, need the factors 2, a, and b. The inclusion of the factors 2 d a gives us a multiple of the first two terms. The inclusion the factors 3 and b gives us a multiple of all three terms. erefore, the

owest common multiple $=(2\times a\times b)(2\times a)(3\times b)=12a^2b^2$.

Solution 5. Refer to the factoring in solution 2. Then proce as in the last solution and get:

lowest common multiple $= (3xyz)(5x)(2y) = 30x^2y^2z.$

SET No. 4: ADDING AND SUBTRACTING FRACTIONS

Example 6. Add: $\dfrac{3}{2x} + \dfrac{4}{3x}\cdot$

Example 7. Subtract: $\dfrac{5}{a^2} - \dfrac{2}{a}\cdot$

Example 8. Simplify: $\dfrac{n}{4mn} + \dfrac{3}{2n} - \dfrac{1}{m}\cdot$

* * *

Solution 6. We have

$$\frac{3}{2x} + \frac{4}{3x} = \frac{9}{6x} + \frac{8}{6x}$$
$$= \frac{9+8}{6x} = \frac{17}{6x}\cdot$$

The basic principle of page 224 enables us to change both deno inators to $6x$. The ***common denominator*** $6x$ is chosen b cause it is the lowest common multiple of the denominators and $3x$.

Solution 7. We have

$$\frac{5}{a^2} - \frac{2}{a} = \frac{5}{a^2} - \frac{2a}{a^2}$$
$$= \frac{5-2a}{a^2}\cdot$$

Solution 8. The lowest common multiple of the denominato is $4mn$. Hence we multiply numerators and denominators so to change all denominators to $4mn$. Then,

$$\frac{n}{4mn}+\frac{3}{2n}-\frac{1}{m}=\frac{n+6m-4n}{4mn}=\frac{6m-3n}{4mn}.$$

There is nothing very thrilling about adding and subtracting ctions. Neither is there anything thrilling about learning to e a saw. But a carpenter who has not learned to use a saw s a very poor foundation on which to build his practice of rpentry. Here is another opportunity for you to develop skill th your algebraic tools.

EXERCISES

nd the highest common factor and the lowest common multiple

1. $3a^3$, $6a^2$ and $9a$; 2. $6x^2$, $2x^4$ and $4x^6$;

3. $2a^2b$, $3abc$ and abc^2; 4. x^2y^3, xy^2 and x^3y^2;

5. $6mn$, $8m^2$ and $10mn^3$; 6. $RCAF$, CAO and WA^2C.

duce to their lowest terms:

7. $\frac{6a^2b}{3ab^2}$; 8. $\frac{mr^2h}{2mr}$; 9. $\frac{21x^3y^2}{3x^2y}$; 10. $\frac{r^4s^2t^3}{r^3s^3t}$.

nplify:

1. $\frac{x^2-ax}{2x}$; 12. $\frac{a^2}{a^2+2a}$; 13. $\frac{(x+1)(x+2)}{2x(x+1)}$; 14. $\frac{r^2-rs}{rs-s^2}$.

mbine into a single fraction:

5. $\frac{2x}{3}-\frac{3x}{5}$; 16. $\frac{1}{2y}+\frac{1}{3y}$; 17. $\frac{1}{2z}-\frac{1}{z^2}$; 18. $\frac{5}{ab}-\frac{4}{ab}$;

9. $\frac{h}{k}+\frac{k}{h}$; 20. $\frac{2}{mn}-\frac{3}{n^2}$; 21. $\frac{3}{x+1}-\frac{2}{x}$;

2. $\frac{b}{a}+\frac{a}{b}+\frac{2}{ab}$; 23. $\frac{t}{rs^2}-\frac{u}{r^2s}$; 24. $\frac{1}{p^2}+\frac{1}{q^2}-\frac{2}{pq}$.

SET No. 5: MULTIPLYING AND DIVIDING FRACTIONS

Example 9. Multiply: $\frac{2x}{y^2}\times\frac{y}{3x^2}$.

Example 10. Multiply: $\frac{2a^2}{b}\times\frac{2b^2}{a^3}\times\frac{a}{4b}$.

Example 11. Divide: $\frac{m^2}{n}\div\frac{m}{n^2}$.

* * *

Solution 9. Recall that the product of two fractions is obtain by multiplying the numerators and also the denominators (s page 228). Hence

$$\frac{2x}{y^2}\times\frac{y}{3x^2}=\frac{2\overset{1}{\cancel{xy}}}{3\underset{xy}{\cancel{x^2y^2}}}=\frac{2}{3xy}.$$

In this solution we multiplied first and then divided xy out of t numerator and denominator. We might have divided x and out before multiplying. This gives the shorter solution:

$$\frac{2\overset{1}{\cancel{x}}}{\underset{y}{\cancel{y^2}}}\times\frac{\overset{1}{\cancel{y}}}{3\underset{x}{\cancel{x^2}}}=\frac{2}{3xy}.$$

Solution 10. Using the shorter method, we have

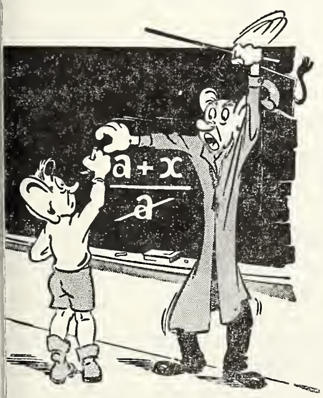
$$\frac{\overset{1}{\cancel{2a^2}}}{\underset{1}{\cancel{b}}}\times\frac{\overset{\overset{1}{\cancel{b}}}{\cancel{2b^2}}}{\underset{\underset{1}{\cancel{a}}}{\cancel{a^3}}}\times\frac{\overset{1}{\cancel{a}}}{\underset{\underset{1}{\cancel{2b}}}{\cancel{4b}}}=\frac{1\times1\times1}{1\times1\times1}=1.$$

Do this problem by the longer method for a check on the answe *lution* 11. Let us do this one in two ways. (1) Since, to vide by a fraction we multiply by the reciprocal of the frac-n (see page 231), we get

$$\frac{m^2}{n} \div \frac{m}{n^2} = \frac{\overset{m}{\cancel{m^2}}}{\underset{1}{\cancel{n}}} \times \frac{\overset{n}{\cancel{n^2}}}{\underset{1}{\cancel{m}}} = mn.$$

) From another point of view, we may regard the given quo-nt as a complex fraction. Then, multiplying the terms of this ction by n^2, we get

$$\frac{\dfrac{m^2}{n}}{\dfrac{m}{n^2}} = \frac{m^2n}{m} = mn.$$



Young Percy thinks he knows how to reduce the fraction on the blackboard. Why is the teacher horrified?

EXERCISES

Find the value of:

1. $\dfrac{ab}{c} \times \dfrac{ac}{b}$;

2. $\dfrac{x^2}{y} \times \dfrac{y^2}{x}$;

3. $\dfrac{3p^2}{qr} \times \dfrac{2q^2}{pr}$;

4. $\dfrac{2a^2b}{c^3} \times \dfrac{3bc^2}{4a}$;

5. $\dfrac{x^2}{yz} \times \dfrac{y^2}{zx} \times \dfrac{z^2}{xy}$;

6. $\dfrac{R^3}{PS} \times \dfrac{S^3}{VR} \times \dfrac{V^2}{P^2} \times \dfrac{P^4}{RS}$;

7. $\dfrac{a(a+2)}{(a+1)(a+3)} \times \dfrac{a(a+3)}{(a+1)(a+2)}$;

8. $\dfrac{a^2b}{4} \div \dfrac{ab}{8}$;

9. $\dfrac{4x^2y}{z^3} \div \dfrac{2xy^2}{z^2}$;

10. $\dfrac{r^5s^3}{t^2} \div \dfrac{r^3s}{t^4}$;

11. $\dfrac{hm^2}{k^2} \times \dfrac{k^3n}{h^3} \div \dfrac{km}{h^2}$;

12. $\dfrac{4ax}{5b^2} \times \dfrac{3by}{8c^2} \div \dfrac{3a^2}{10cz}$.

13. Give another way of writing

(*a*) $\left(\dfrac{a}{b}\right)^3$, (*b*) $\left(\dfrac{x}{2}\right)^4$, (*c*) $\left(\dfrac{2r}{3s}\right)^2$.

14. Name the fraction by which you must multiply $\dfrac{a}{b}$ to ge

(*a*) $\dfrac{2a}{3b}$, (*b*) $\dfrac{a^2}{b^2}$, (*c*) $\dfrac{a^2}{ab}$, (*d*) $\dfrac{x}{y}$.

15. Name the fraction by which you must divide $\dfrac{a^2}{9}$ to ge

(*a*) $\dfrac{a}{3}$, (*b*) 1, (*c*) a^2, (*d*) $\dfrac{b}{2}$.

The Equations Enlarge their Domain

In our study of equations in Chapter IV, we made no use f signed numbers. This was because we had not then admitted em into the family of numbers. Now, however, we need not strict ourselves. Using the rules for signed numbers, we may pply the four basic principles for solving equations (page 155) ith both positive and negative numbers.

Moreover, if the equations of Chapter IV had not been selected for you with some care, you might have run into some fficulties. For example, how would you solve the equation

$$x + 4 = 3\ ?$$

is simple enough now. You subtract 4 from each member d get

$$x = -1.$$

ithout the use of negative numbers this equation would have en impossible to solve. No positive number satisfies it.

With signed numbers at our disposal, we can solve equations much greater variety than before. Here is an easy one for u to try.

xample 1. Solve for x: $-3x - 7 = 8 + 2x.$

* * *

lution **1.** Apply the rules for solving equations. Then from

$$-3x - 7 = 8 + 2x$$

get $-7 = 8 + 5x.$ (Add $3x$ to each side.)

ence $-15 = 5x$ (Subtract 8 from each side.)

d $-3 = x.$ (Divide each side by 5.)

ow substitute -3 for x in the given equation and verify the swer.

xample 2. It began to snow at x hours after noon. The snow ll at 1/3 of an inch per hour for 3 hours and then 1/5 of an ch per hour. At 5 P.M. there were $1\frac{1}{2}$ inches of snow on the ound. When did it begin to snow?

* * *

Solution 2. In 3 hours 1 inch of snow fell. During the ne $(5-3-x)$ hours, the fall was $\frac{1}{5}(5-3-x)$ inches. Therefore, w have

$$1+\tfrac{1}{5}\ (2-x) = \tfrac{3}{2},$$

or

$$10+2\ (2-x) = 15.$$

This gives

$$14-2x = 15,$$

whence

$$-2x = 1, \text{ or } x = -1/2.$$

Hence, it began to snow at 11:30 A.M.

In the next example, see if you can justify each step in th solution.

Example 3. Solve for x:

$$2.5x-(x-3) = \tfrac{3}{2}(4-x)-4.5.$$

Solution 3. We have

$$5x-2\,(x-3) = 3\,(4-x)-9, \qquad \text{(Why?)}$$

whence

$$5x-2x+6 = 12-3x-9. \qquad \text{(Why?)}$$

This gives

$$3x+6 = 3-3x \qquad \text{(Why?)}$$

or

$$x+2 = 1-x. \qquad \text{(Why?)}$$

Therefore

$$2x = -1 \qquad \text{(Why?)}$$

and

$$x = -1/2. \qquad \text{(Why?)}$$

The final step is to verify this solution by substituting $x = -1/$ in the given equation.

This last example may appear to you rather involved. I however, you keep in mind that the object is to get x alone o one side of the equation, the four rules of page 155 will enabl you to achieve this object. The order in which the rules are use is not fixed. You can often get the final result by using them i many different orders. In the foregoing solution, the equatio was first cleared of fractions by multiplying each side by 2. The parentheses were removed and like terms collected. A metho similar to this is often a good plan of attack.

The unknown letter in an equation is often called for short ***he unknown.*** The object, in the solution, is to find the value the unknown which will make the equation true. Hence, the nal test of the correctness of a solution is to substitute the value und for the unknown in the original equation, in order to see hether it makes the two sides equal.

EXERCISES

lve the following equations:

1. $5x + 2 = 2x - 7.$ **2.** $2(5x - 4) = 3(2x + 4).$

3. $4 + (2y - 3) = 2y - (y + 5).$ **4.** $\frac{x}{2} - 3 = \frac{x}{3} + 1.$

5. $\frac{2x + 1}{3} = \frac{2x + 3}{4}.$ **6.** $\frac{3r}{5} + 2 = \frac{2r}{3} - 2.$

7. $\frac{1}{2}(7y - 5) = \frac{2}{3}(8y - 23).$ **8.** $\frac{3}{4}x + \frac{7}{12} = \frac{2}{3}x + \frac{1}{6}.$

9. $(7z - 1) + 2(3z - 4) = (5z - 6) - (4z - 5).$

10. $\frac{x + 5}{4} - (3x + 2) = 8 - \frac{1}{2}(4x + 7).$

11. A certain number is doubled and the product increased by . If the result is 26, find the number.

Some Parting Advice

This chapter has given you an opportunity to improve your astery of some important algebraic skills. These are:

1. Substitution of signed numbers for letters.

2. Taking out common factors.

3. Addition, subtraction, multiplication, and division of simple pressions, including fractions.

4. Solution of equations with one unknown.

These skills may all be brought into play when you make use algebra. If you have mastered them, you have not only an important instrument for the solution of everyday problems bu you have also built a good foundation for the further study c algebra. The most interesting aspects of algebra are not i evidence when you are drilling at the basic skills as in thi chapter. Yet the spade-work must be done. If you hope t gain ease in using formulas and solving problems, and, finally, you hope to *think* in algebra, the routine operations must b mastered. They are like the scales and arpeggios by which th musician prepares himself to interpret great works of music.

Skill Needs Drill

REVIEW EXERCISES

A

Time Test No. 1. (Allow 12 minutes.)

1. If $x = -2$, find the value of $2x^3 - 3x^2 - 7x + 5$.

2. If $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$, find f when $p = 5$ and $q = -10$.

Insert parentheses in the following:

3. $6x^2y + 2xy^2 - 4x^2y^2$.
4. $3a^2b^3c^4 - 6a^4b^3c^2$.

Combine into a single fraction:

5. $\frac{8}{3x} + \frac{5}{4x}$.
6. $\frac{1}{ab} + \frac{1}{bc} - \frac{1}{ca}$.

Find the value of:

7. $\frac{5ab}{c} \times \frac{c^2}{a^2}$.
8. $\frac{6m^2n}{5k} \times \frac{10k}{12mn}$.
9. $\frac{abc}{x} \div \frac{b^2c}{2x^2}$.
10. $\frac{x}{y} \times \frac{2y^2}{z^2} \div \frac{xyz}{z^2}$.

Solve the following equations:

11. $3x - 5 = 5x + 5$.
12. $8 - 3(x - 1) = 14$.

ime Test No. 2. (Allow 18 minutes.)

13. If $a = 3$, $b = -2$, and $c = -1$, find the value of

$$a^3 + b^3 + c^3 - 3abc.$$

14. If $s = ut + \frac{1}{2}at^2$, find the value of s when $u = -40$, $a = 32$, d $t = 10$.

15. Insert parentheses in $12k^3m^2n^2 + 30k^2m^4n - 18k^2m^3n^2$.

16. Make use of the formula $C = \frac{5}{9}(F - 32)$ to express the mperature 5°F in degrees Centigrade.

mbine into a single fraction:

17. $\dfrac{3a}{a^2} + \dfrac{4bc}{b^2}$.   18. $\dfrac{xy}{x^2} - \dfrac{xz}{yz}$.

19. $\dfrac{4}{m^2} - \dfrac{3kn}{km} - \dfrac{2m^2n}{m^3}$.

aluate:

20. $\dfrac{(x + 1)(x + 5)(2x - 1)}{(2x - 1)(x + 5)(x + 1)}$.   21. $\dfrac{a^2b^2}{4m^2n} \times \dfrac{m^2n^2}{2a^2b} \div \dfrac{bn}{12}$.

22. Solve for x: $10 + 4(x - 7) = -50 - 5(x - 1)$.

23. Solve for a: $3a - 10 = \frac{1}{2}a - 15$.

* * *

B

24. If a certain number is tripled and the result then increased 10, the sum is equal to twice the number. Find the number.

25 Find a number such that the sum of its half, its double, d its triple exceeds six times the number by 1.

26. Solve the formula of Ex. 16 for F. Use the new formula express the temperature −20°C in degrees Fahrenheit.

27. A rectangular lot has an area of 20 units and depth 2 An adjacent lot has an area of 25 units and depth $3x$. Expre the total frontage of the two lots as a single fraction.

28. A large truck can empty a coal-car in m hours. How muc of the car does the truck empty in one hour? If a smaller truc can empty the coal-car in $2m$ hours, how much of the car woul both trucks empty in one hour if they work together?

29. A 1000-gallon gasoline tank can be emptied through certain tap in $3x$ hours. How many gallons pour from the ta in 1 hour? If another tap can drain a 500-gallon tank in 2 hours, express as a single fraction the number of gallons that ca pour from both taps in 1 hour.

30. John walks along a highway for 6 miles at an avera speed of r miles per hour. A motorist gives him a lift for 2 miles at an average speed of $12r$ miles per hour. Express as single fraction the total time for John's journey. (Recall th $d = rt$.)

31. Find the area of a sheet of paper which is $3k^2/mn$ uni long and $m^2n^2/6k$ units wide.

32. The area of a barn door is $8x^3/3y^2$ square feet. If th length is $4x^2/6y^3$ feet, find the width.

33. Mr.MacTavish's new clerk reported to him that the sel ing price on an article was \$20 more than 5 times the profit, an the cost price was \$18 more than 3 times the profit. MacTavis became somewhat depressed. What was his profit?

C

34. Evaluate: $\dfrac{3\,(x+2)}{x-1} \times \dfrac{2x+1}{3\,(x-3)} \div \dfrac{(x+2)(2x+1)}{(x-1)(x-3)}$.

35. Solve for x: $\frac{3}{4}x + 3 = \frac{1}{2}(x + 10) - 3$.

36. Beginning at ground level, an elevator moved x floors. he elevator then went up 15 floors and stopped at the 12th oor. Find x.

37. The temperature on Tuesday was 15°F. Henry comented, "That's 25 degrees more than twice the number of egrees Fahrenheit on Monday." What was Monday's temerature?

38. Two ships sail on the same course. One, sailing at 30 .p.h. passes a buoy at 1 P.M. The other sailing at 20 m.p.h. asses the buoy at 2 P.M. At what time did the first ship pass he second?

39. An enemy craft was located x feet above sea-level. After he craft descended 150 feet, it was hit by a bomb which had ropped 800 feet from a height of $600-x$ feet. Was the enemy raft an aeroplane or a submarine?

RECREATIONS

1. **How fast is the train going?** As you ride in a train, you often find yourself listening to the clicks as the wheels of the car pass over the spaces between the rails. If the rails are 30 feet long, for how many seconds should you count clicks in order that the number of clicks should equal the measure of the speed of the train in miles per hour?
2. **How fast is the wind blowing?** A balloon is released from a tower in the town of Rainville just as an aircraft flies overhead. The aircraft flies directly into the wind for 15 minutes, then turns and flies directly down wind, overtaking the balloon over Snowburg, a town 10 miles down wind from Rainville. If the aircraft loses no time in turning, find the speed of the wind.
3. **Time and tide.** A rope is attached to the side of a ship so that twenty feet of the rope are under water at low tide. If the tide rises two feet an hour, how much of the rope is submerged after three hours?
4. **Find the fallacy.** Stanley Bright informed his classmates that he could prove that any number is equal to any other number. Here is his "proof".

   Let x and y be any two numbers, and a and b be two equal numbers.

   Then we have $a = b$,

   $$ax = bx,$$

   and $ay = by$.

   Subtract and get

   $$ax - ay = bx - by,$$

   or $ax - bx = ay - by$.

   Therefore $x(a - b) = y(a - b)$,

   or $x = y$.

   Where did Stanley make his error?

CHAPTER XI

SOME EVERYDAY USES OF ARITHMETIC

> *"It was an awful sight of money when it was piled up. Well, Judge Thatcher he took it and put it out at interest, and it fetched us a dollar a day apiece all the year round—more than a body could tell what to do with."*
>
> Mark Twain, in *Huckleberry Finn.*

Here are some items selected from newspaper reports. Can u tell their meaning?

A discount of 5% for cash;

A sample of milk testing 3.5% butterfat;

An increase of 4.5% in population for a 10-year period;

A rise of 19% in the cost of living;

An issue of government bonds bearing 3% interest;

A local tax rate of 40 mills on the dollar.

These items suggest a number of ways in which arithmetic ters into our everyday affairs. We would all admit the useful- ss of arithmetic to the banker who invests our money, the tuary who insures our houses, and the engineer who designs r bridges. Its uses, however, are not confined to such occupa- ns as these. It is used also by the carpenter who builds shelves your kitchen, the merchant who buys and sells stoves, the rmer who takes his milk to the factory, and the boy on his ening route delivering the *Northern Watchword-Chronicle.*

Minding One's Own Business

It is now many centuries since people first thought of using kens as money to save themselves from the necessity of always rtering one commodity for another. Nowadays it is a part of the everyday life of each of us to make simple business calcu tions involving the use of money. We should, for example, able to check our grocery bills, our bank balances, and our t sheets. We should also be able to make calculations to determi whether it is wise to invest in a house, a car, or insurance. In su calculations we commonly make use of percentage.

How to Use Percentage

In dealing with fractions we have seen the advantage having the denominator a power of 10. A fraction with deno inator 10, 100, 1000, or some higher power of 10 can be at on expressed in decimal form. In business affairs this advanta shows up in the use of ***per cents***. The term per cent, an abb viation of the Latin ***per centum***, means ***per hundred***; it m be read as ***hundredths***.

You have already made simple calculations of percentag To calculate 3 per cent of 250, you write

$$3\% \text{ of } 250 = 250 \times 0.03 = 7.5.$$

Three per cent of a number is 3 *hundredths of the number*, that

$$\boxed{3\% = 0.03.}$$

In the number 3, as well as in other integers such as 35, or 11 a decimal point is understood, although not written, after t units digit. To change a per cent to a decimal, you merely mo the decimal point 2 places to the left and, to change a decim to a per cent, move the decimal point 2 places to the right. It easier to do this by thinking of the meaning of per cent than b memorizing a rule. Here are some examples:

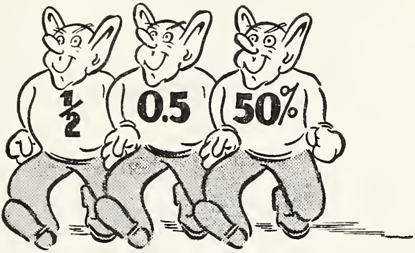
$$0.04 = 4\%, \qquad 0.025 = 2\tfrac{1}{2}\%, \qquad 0.0125 = 1\tfrac{1}{4}\%,$$
$$5\tfrac{3}{4}\% = 0.0575, \qquad \tfrac{1}{2} \text{ of } 1\% = 0.005, \qquad 3\tfrac{1}{3}\% = 0.03\tfrac{1}{3}.$$

In an earlier chapter you learned certain important fractio in both common and decimal form. It will be useful for you t know these fractions also as per cents. Here they are.

$\frac{1}{2} = 0.5 = 50\%$
$\frac{1}{4} = 0.25 = 25\%$
$\frac{3}{4} = 0.75 = 75\%$
$\frac{1}{3} = 0.33\frac{1}{3} = 33\frac{1}{3}\%$
$\frac{2}{3} = 0.66\frac{2}{3} = 66\frac{2}{3}\%$

$\frac{1}{5} = 0.2 = 20\%$
$\frac{2}{5} = 0.4 = 40\%$
$\frac{3}{5} = 0.6 = 60\%$
$\frac{4}{5} = 0.8 = 80\%$

$\frac{1}{8} = 0.125 = 12\frac{1}{2}\%$
$\frac{3}{8} = 0.375 = 37\frac{1}{2}\%$
$\frac{5}{8} = 0.625 = 62\frac{1}{2}\%$
$\frac{7}{8} = 0.875 = 87\frac{1}{2}\%$



EXERCISES

(for oral answers)

ange the following decimals to per cents.

1. 0.07
2. 0.12
3. 0.025
4. 0.0125
5. 0.086
6. 0.245
7. 1.5
8. 0.628
9. 0.105

ange the following per cents to decimals.

10. 9%
11. $4\frac{1}{2}\%$
12. 5%
13. $6\frac{1}{4}\%$
14. 32%
15. 3.2%
16. 24%
17. 2.4%
18. 10.8%
19. 20%
20. 33%
21. 18%
22. 150%
23. 99%
24. $\frac{1}{4}\%$
25. $\frac{2}{5}\%$

ange the following fractions to per cents.

26. 3/10
27. 5/4
28. 7/8
29. 2/3
30. 5/6
31. 9/16
32. 3/32
33. 7/3
34. 17/20

Three Uses of Percentage

Let us see what is involved in a simple example in percenta such as

$$4\% \text{ of } 600 = 24.$$

This example contains three numbers, which may be disti guished by names:

4% or 0.04 is the ***rate***;

600 is the ***base***;

24 is the ***percentage***.

The same equation can be written

$$24 = 0.4 \times 600.$$

Since this is only one of hundreds of such examples, it is wor while to bring in our mathematical slaves, the letters, to give the relation once and for all. If the numbers which measure th rate, base and percentage are denoted by r, b, and p, respectivel we have the equation

$$\boxed{p = rb.}$$

One advantage of condensing this relation into a formula that it suggests three kinds of problems. For, since the formul involves three numbers, it follows that if two of these numbe are known and the third unknown then the equation can solved for the unknown. Now, letting each letter in turn repre sent the unknown, we see that the three problems are these:

(*a*) *Given the base and rate, to find the percentage.*
Here r and b are known and $p = rb$.

(*b*) *Given the base and percentage, to find the rate.*
If b and p are known, then $r = p/b$.

(*c*) *Given the rate and percentage, to find the base.*
If r and p are known, then $b = p/r$.

Each of these problems will now be illustrated by an exampl *xample* 1. To encourage trade, a sporting-goods merchant lvertised a 15% discount sale. What should be the sale price a tennis racquet regularly priced at $14?

Here the base number is 14, the rate 15% and the percentage required. The formula $p = rb$ gives

$$p = 0.15 \times 14 = 2.10.$$

ence the sale price is $\$14 - \$2.10 = \$11.90.$

xample 2. The value of the total Canadian domestic exports r May 1943 is listed as $250,607,000 and for May 1944, as 68,357,000. What is the rate per cent of increase from May'43 May '44?

Here we shall first round off the numbers to 3 figures, obtaing 251 million and 368 million. The increase, 117 million, is the rcentage. The formula now gives

$$117 = r \times 251,$$

ence $r = 117/251 = 0.47$, correct to 2 figures. This indicates 47% increase.

xample 3. A customs office found that it had taken in a total $1260 in duties on goods on which the rate of duty was 24%. hat was the value of the goods cleared through the customs?

In this problem we are given $r = 0.24$, $p = 1260$ and the lue of b is required. The formula gives

$$1260 = 0.24 \times b,$$

ence $b = 1260/0.24 = 5250$ and the required value is $5250.

EXERCISES

The first fifteen may be answered orally. It is often an advan-ge to use the fractional equivalent of the rate per cent.

nd the values of:

1. 2% of 175
2. 20% of 725
3. 75% of 256
4. $1\frac{1}{2}$% of 248
5. $12\frac{1}{2}$% of 96
6. $33\frac{1}{3}$% of 12.9
7. What per cent of 72 is 24?
8. What per cent of 80 is 16?

9. What per cent of 20 is 19?

10. What per cent of 200 is 25?

11. 85 is 50% of what number?

12. 6 is 1% of what number?

13. 30 is 5% of what number?

14. 36 is $33\frac{1}{3}\%$ of what number?

15. 500 is 40% of what number?

For the next ten exercises keep track of your time. From a tot of 50 *obtain your score by deducting* 5 *for each incorrect answer a* 2 *for each minute over* 10.

16. Find 15% of 725. **17.** Find 21% of 240.

18. Find $4\frac{1}{2}\%$ of 96. **19.** Find 3.6% of 42.8.

20. What per cent of 142.5 is 57?

21. What per cent of 728 is 273?

22. What per cent of 424 is 31.8?

23. 8.4 is 8% of what number?

24. 122.1 is $16\frac{1}{2}\%$ of what number?

25. 41 is $3\frac{1}{3}\%$ of what number?

After reading each of the following, note down your estimate the answer. Then make the calculation. Are your estimates clos

26. Harry had $4.50 in his allowance budget for shoes. H went to a store which advertised a 20% discount sale. Two pai of shoes were marked, without the discount, at $6 and $5.5 Could he buy either of these?

27. An *ad valorem* customs duty of $2.25 was charged on a article worth $7.50. What was the rate of duty?

(An *ad valorem* duty is a percentage of the value of the good collected by customs officials.)

28. The total national income of Canada for 1943 was 14 billion dollars. Of this total, agriculture accounted for 14 billio mining for $2\frac{1}{2}$ billion, and manufacturing for 48 billion. Expres each of these as a percentage of the total.

29. If money can be invested at a yearly interest of 3%, what ivestment would be necessary in order that the interest would rovide a scholarship of $240 a year?

30. If the butterfat content of milk is 3.5%, what daily weight f milk would furnish butter for a family which consumes 6 ounds of butter per week?

31. After the employees of a certain factory had been given a 2% rise in wages, Mr. Dean's weekly wage was $48.33. What as his wage before the increase?

Let's Have a Chat with a Merchant

ON SALE
SALE

Here are some of the facts which we find out as a result of ur chat.

When a retail merchant buys goods from a manufacturer or holesaler, he adds to his ***cost price*** a certain percentage to btain his ***selling price***. The excess of the selling price over he cost price is his ***margin***. That is,

Selling Price − Cost Price = Margin

ut of the margin from his year's operations he must first pay he expenses of running his business. These expenses, such as wages, rents, interest on invested or borrowed money, and cos of heat and light, make up his ***operating expenses*** or ***ove head***. The remainder is his ***profit***. That is,

$$\boxed{\textit{Margin} - \textit{Overhead} = \textit{Profit}}$$

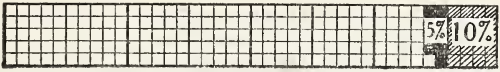
A merchant may express his profit as a percentage either of h cost price or, more commonly, of his selling price.

At various times a merchant may advertise a sale of all h merchandise or of certain articles at a reduction from the usu selling price. This reduction is called a ***discount***. He may, fo example, give a discount of 20% on summer goods at the openin of the fall season. Sometimes a further discount is allowed fo cash payment or for some other reason. Consider the followin example.

Example. Baillie and Co., Outfitters, bought a consignment o overcoats at $22.50 each. They marked them for sale at a advance of 30% of the cost price. Their margin was therefor 0.3 of $22.50 or $6.75 and their selling price was $29.25 pe garment. They estimate their overhead at 10% of their tota sales. Hence, the overhead charge for each overcoat is 0.1 o $29.25 or $2.93. This leaves a profit of $3.82 on each overcoat

At the end of the season, the firm found that it had a numbe of overcoats left over and, not wishing to keep them in storag for a year, it advertised them at a discount of 10%. The discoun was therefore $2.93 and the reduced sale price was $29.25 − $2.9 or $26.32.

Double Discounts. This question of successive discounts re quires a bit of careful thinking. Suppose that a price of $20 is reduced by two successive discounts of 10% and 5%. Th first discount is 10% of $200 or $20, and the reduced price is $180 The second discount is 5% of $180 or $9, and the second reduce price is $171. The figure illustrates the two reductions.



Now, by making calculations for yourself and illustrating hem by figures, find out whether two successive reductions of 0% and 5% give the same result as (*a*) a single reduction of 5%, (*b*) two successive reductions of 5% and 10%.

Do You Pay Rent for Money?

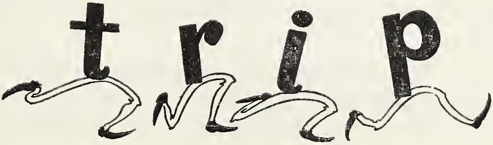
Just as rental is paid for the use of a house, a store, or a farm, o rental is paid for the use of money, with which these and other hings could be bought. The following example gives the terms hich are used in such a transaction.

xample. In order to buy seed grain, Mr. Sterling borrowed 200 on which he agreed to pay ***interest*** at the ***rate*** of 5% ***er annum.***

The $200 is the ***principal*** of his loan. For the use of it he ays a rental or interest charge which he computes at the rate f 5% for one year. A year's interest is therefore $\$(200 \times 0.05) =$ 10.00; 6 months' interest is $5.00. The ***amount*** of his loan t the end of 6 months is $205.

At the end of 6 months Mr. Sterling paid the interest which as due and also $100 of the principal. For the next 6 months e paid interest on his reduced principal of $100. At the end of he year he discharged the remainder of the debt by a payment f $102.50.

Now we are ready to condense a simple interest transaction nto a formula. For this purpose we call again on the letters for heir magic act.



You have guessed their meaning already. The letter p in tends to stand (when he stops running) for the number of dollar in the principal of the loan. Similarly r measures the rate, t th time, that is the number of years for which the loan bears interest and i the interest. For one year the interest would be pr an for t years it would be t times as much, so that

$$\boxed{i = prt.}$$

The amount a of the loan is the sum of principal and interest, o

$$\boxed{a = p + prt = p(1 + rt)}$$

Interest calculated in this way is called ***simple interest*** It is used chiefly for short term loans, that is for loans which ar repaid in the course of a few months. The following example show how the formulas may be used. Although the formula are a convenient means of condensing rules, it is possible for yo to do these problems without explicit use of formulas, by thinkin of the *meaning* of interest and percentage.

Example 1. In order to pay the college fees for his son Jock Mr. Grant borrowed \$250 on a note, agreeing to pay interest a 4% per annum. After 8 months he paid back the loan in full What amount did he pay?

For the interest due in 8 months, or 2/3 of a year, we find from the formula $i = prt$,

$$\begin{aligned} i &= 250 \times 0.04 \times \tfrac{2}{3} \\ &= 6.67, \end{aligned}$$

so that the interest is \$6.67 and the amount which he has to pa is \$256.67.

Example 2. Mrs. Andrews bought a washing machine for whic she paid all but \$80 of the purchase price. She wished to complete payment at the end of 9 months and was told that she coul o this by paying \$85 at that time. What rate of interest would ıe be paying?

At the time of the purchase the company, in effect, lent Mrs. ndrews \$80 for 9 months. On this she paid interest of \$5. ence we have $i = 5$, $p = 80$, $t = \frac{3}{4}$, and r is to be found. The ormula $i = prt$ gives

$$5 = 80 \times r \times \tfrac{3}{4}$$

om which $r = 0.08\frac{1}{3}$. This means that the rate is $8\frac{1}{3}\%$ per nnum.

xample 3. At the end of the harvest season Farmer Shannon ecided that he would need a new binder for the next season's ops and, being a thrifty man, he saved the necessary money om the sale of his grain. He found the price of the binder he anted to be \$160. He therefore calculated what sum he should eposit in the local bank which paid 2% interest in order that ıe amount to his credit in 9 months' time should be \$160. Show is calculation.

Let us denote the required sum by \$$p$. The interest on \$$p$ r 1 year is \$$(p \times 0.02)$ and for 9 months \$$(p \times 0.02 \times \frac{3}{4})$. ince the amount is \$160, we have

$$p(1 + 0.02 \times \tfrac{3}{4}) = 160,$$

hence

$$1.015p = 160$$

nd

$$p = \frac{160}{1.015} = 157.64.$$

e should therefore deposit \$157.64.

EXERCISES

ind the simple interest on:

1. \$348 for 2 years at $3\frac{1}{2}\%$ per annum.
2. \$67.50 for 10 months at 4% per annum.
3. \$160 for $1\frac{1}{4}$ years at $4\frac{1}{4}\%$ per annum.
4. \$82.40 for 9 months at $5\frac{1}{2}\%$ per annum.
5. \$200 for n years at 3% per annum.

6. \$$k$ for 3 years at 4% per annum.

7. \$$x$ for y years at 2% per annum.

Find the rate per annum at simple interest when:

8. \$96 amounts to \$99.84 in 1 year.

9. \$125 amounts to \$133.75 in 2 years.

10. \$75 amounts to \$77.50 in 8 months.

11. \$344 amounts to \$354.75 in 15 months.

12. \$$x$ amounts to \$$1.1x$ in 1 year.

13. \$$x$ amounts to \$$\frac{9}{8}x$ in 2 years.

14. \$100 amounts to \$$(100 + x)$ in 6 months.

Find the sum which must be invested to yield simple interest o

15. \$300 at 6% for 6 months.

16. \$96 at 5% for 9 months.

17. \$510 at $4\frac{1}{4}$% for $1\frac{1}{2}$ years.

18. \$21.35 at $3\frac{1}{2}$% for 20 months.

19. \$$k$ at 3% for 1 year.

20. \$$x$ at 2% for n years.

After reading each of the following problems make an estima of your answer. Then make your calculation. Do your estimat improve?

21. On goods which cost him \$4800, what is Mr. Doolittle margin if he marks them up 30% and then gives a discount 12%?

22. A year's volume of business (that is, the total sales) of th Come-and-Get-It Groceteria was \$148,200. If its average margi was 20% of cost and its operating costs 12% of sales, find th year's profit.

23. The Steele Hardware operates on a margin of 25% of co and its overhead amounts to 15% of its sales. What per cen of the total sales is profit?

24. Epstein Bros., Clothiers, marked goods at 20% abov cost and then gave a discount of 5%. If 10% of the selling pric is to be allowed for overhead, what was the profit on an article hich sold for $63?

25. Find the selling price of goods which were originally arked at $80 if the merchant gave (*a*) a discount of 15%, (*b*) vo successive discounts of 10% and 5%, (*c*) two successive disunts of 5% and 10%.

26. Before selling a truckload of apples which were marked $100, the Red and Gold store gave two discounts of 10% each. hat was the total discount? What single rate of discount is uivalent to two successive discounts of 10%?

27. What single rate of discount is equivalent to two successive discounts of 10% and 5%?

We Calculate Our Taxes

You may have heard the saying that "nothing is sure except ath and taxes". Taxes are one of the oldest institutions in the rld and many have been the means, fair and foul, for collecting em. The name *taxgatherer* was at e time a term of disrespect as shown the frequent linking together, in the ew Testament, of publicans (or taxtherers) and sinners. Most grown-s still do a great deal of good-natured umbling about paying their taxes, but ey would have much more occasion to umble if they were to be deprived of e things which their taxes were colcted to provide.

In your study of social science, you have probably discussed e purposes for which governments collect taxes. Also you will on learn, if you have not learned already, about the many nds of taxes by which the federal government, the provinces, d municipalities obtain their revenues. We confine attention re to one important kind, ***property taxes***. As you might expect, the arithmetic involved in the calculation of taxes is close allied to percentage.

Example. When Mr. Claymore sold his farm with the intenti of retiring, he made extensive inquiries regarding houses in th neighbouring town of Indigo. The house which he finally select was assessed for $3200 and he learned that the town tax rate f the year had been struck at 36 mills on the dollar. Being a pr dent man, Mr. C. made a calculation of the tax he would hav to pay on the house, as follows:

$$36 \text{ thousandths of } 3200 = 3200 \times 0.036 = 115.20.$$

He then gave a long low whistle and informed Mrs. C. that th would have to pay $115.20 taxes on the house for the year.

Property taxes, which are the chief source of revenue of town and cities, are levied on the assessed valuation of the real estat which means the land and buildings. The tax may be express as a percentage, but it is more often expressed in mills. *A mi is one-tenth of a cent or one-thousandth of a dollar.* A tax rate $3\frac{1}{2}\%$ is the same as 35 mills on the dollar or, as it is often abbr viated, 35 mills. You have learned to compute a percentage converting the rate into a decimal: 6% = 0.06 or 6 hundredth You can also convert mill rates into decimals: 6 mills on th dollar = 0.006 or 6 thousandths.

Example. Mr. Rogers owned a town house which was assess for $2800 on which he paid a tax of 38 mills on the dollar. H sold this house and bought in its place a city house assessed f $3500, with a tax rate of 32 mills. What difference did the chan make in Mr. Rogers' property tax?

His town tax was 0.038 of $2800 = $106.40.
His city tax is 0.032 of $3500 = $112.00.
His tax is therefore increased by $5.60.

An advantage of the use of mills is in their convenience f expressing separately the taxes for the various services which th city provides. The following schedule shows how the progressiv city of Fredonia computes its tax rate.

Unemployment relief	0.76	
General debt	2.93	
Local improvement rate	0.61	
General purpose rate	14.48	
Garbage and ash collection rate	1.50	
Total city council rate		20.28
High school rate	2.52	
Technical school rate	1.98	
Public school rate	8.75	
Total for educational purposes		13.25
Public library board		0.97
Total mill rate		34.50

ample. Mr. Simpson's house and lot are assessed for $4800. cording to the foregoing tax rate of 34.5 mills, his tax is

0.00345 of $4800 = $165.60.

the tax rate, 13.25 mills are for the upkeep of the city schools. r. Simpson's share of the schools' tax is

0.01325 of $4800 = $63.60.

On his tax slip, Mr. Simpson is advised that he will receive

(*a*) a discount of 2% on all taxes paid on or before May 21,

(*b*) a discount of 1% on all taxes paid after May 21 but on or before July 21.

o he is informed that, if any amount remains unpaid after tober 21, he will be charged an additional 4% of the unpaid ance. If he pays one-third of his taxes on May 15, one-third July 15 and the remaining third on December 15, he computes three payments as follows.

May 15	1/3 of taxes	=	$55.20
	discount = 0.02 of $55.20	=	$ 1.10
	payment	=	$54.10
July 15	1/3 of taxes	=	$55.20
	discount = 0.01 of $55.20	=	$ 0.55
	payment	=	$54.65

Dec. 15 1/3 of taxes = $55.20
charge for late payment = 0.04 of $55.20 = $ 2.21
payment = $57.41

It pays to pay your taxes early!

EXERCISES

1. The property of the Eagle Tennis Club was assessed $1200 until the members built a club-house which raised t assessment to $2100. The tax rate was 30 mills on the doll Find the amount of the taxes before and after the club-house w built.

2. Mr. Penley's tax sheet showed the assessed valuation his house to be $4250 and the tax rate, 33 mills. Find the t which he has to pay.

3. Mr. Penley paid his tax in three equal instalments. the first he was allowed a discount of 2%, on the second a di count of 1%, and on the third no discount. Find the sum of three payments.

4. My friend Swinnerton bought a city lot for $600. It w assessed at three-fifths of the purchase price, with a tax rate 32 mills on the dollar. He kept the lot four years before buildi a house on it. If the tax rate remained constant during this tim how much had he expended on the lot in purchase price and tax before building his house?

5. In an argument over tax rates the question was aske "Would you rather have your house assessed for $3000 and tax at 40 mills on the dollar or assessed for $4000 and taxed at mills?" Find the taxes in these two cases. How do they co pare?

6. Find the total amount of revenue which the township Allcorn raises by a tax of 34 mills on an assessed valuation $7,350,000.

7. Mr. Jacobs owns three pieces of real estate which are ssessed for $3400, $4100, and $5200. The tax rate is 32 mills, which 15 mills are for the support of the schools. Find (*a*) the nount of Mr. Jacobs' school tax, (*b*) the amount of his taxes for her purposes.

8. The budget of the town of Silverton calls for a year's penditure of $136,000. If the real estate of Silverton is assessed $4,256,000, what tax rate should be struck by the town council?

9. By means of a tax rate of 36 mills, the town of Itaxia raised 6,420 to finance its year's operations. What was the assessed luation of its property?

10. Using the following items from the treasury departments the three towns of Ayton, Beaton and Seaton, calculate the issing item for each.

	Assessed valuation	Tax rate	Revenue from taxes
Ayton	$8,345,000	45 mills	. . . .
Beaton	$2,863,000	. . .	$108,800
Seaton	. . . .	33 mills	$528,000

What is Insurance?

Let us suppose that each 500 house owners contrib- ed $10 to a common fund, eir agreement being that any one of their houses re burned the owner would ceive the whole $5000. This uld be a way of ***insuring*** eir houses against loss by e.

The actual operation of fire insurance is not so simple. In first place a company must be incorporated, with officials d offices. Then a study must be made of fire losses over a riod of years. Based on this a calculation is made of the amounts which owners will need to pay for insurance on th buildings. These must be sufficient (*a*) to cover the operati expenses of the company, (*b*) to pay probable losses from fi (*c*) to furnish a reserve fund in case future losses are greater th expected, and (*d*) to pay dividends on money invested in t company.

The written contract between the company and the own of a building is an ***insurance policy***. On the policy is stat the amount for which the building is insured; this is called t ***face*** of the policy or sometimes the ***risk***. The sum whi the policy-holder must pay the company for the insurance is t ***premium***. It is calculated at a certain ***rate*** per $100 insurance carried. This rate depends on the type of constructi of the building, the facilities available for fighting fires and oth factors which you can think of. Most premiums are paid for 3-year term. *In practice the rate for 3 years is 2½ times the r for one year.*

You will learn the main facts about the working of fire insu ance by studying the following examples.

Example 1. A dwelling worth $6000, with solid brick co struction and a first-class roof, is insured for $4500. Since th type of construction makes the risk of fire relatively small, t insurance is quoted at the low rate of $0.36 per $100 for a 3-ye period. The 3-year premium is then $(45 × 0.36) or $16.2 If the house is destroyed by fire, the company will pay the own $4500. If it is damaged to the extent of $500, the company w pay $500. In other words, the company will pay the amount the damage, provided it is not greater than the face of the polic

Example 2. The Clover-Leaf Creameries took out three polici on their warehouse, in three different fire insurance compani the Sun, Moon, and Star. The three policies were for $400 $6000, and $8000, respectively, the total being below the val of the property. The rate charged by each company was 85c p $100 for one year. If the owners paid their insurance for o year, the sum of their yearly premiums would be $(180 × 0.8 or $153.00. If they paid by the year their premiums for thr ears would amount to 3 times $153 or $459. If they paid in lvance for a 3-year term the premium would be $2\frac{1}{2}$ times $153 hich amounts to $382.50. It is not hard to see why most insur- ice is paid for 3-year periods.

The Clover-Leaf warehouse suffered damage which was ap- raised by the proper official at $9600. Then each company paid s proper share of this loss. Since all the policies totalled $18,000 hile the Sun company's was $4000, then the Sun company paid /18 or 2/9 of $9600, which is $2133.33. Similarly the Moon iid 1/3 of $9600, or $3200, while the Star paid 4/9 of $9600, hich is $4266.67.

xample 3. The furniture in a storehouse is valued at $12,500. he Atomic Insurance Co. insures the stock for $6000 and the licy contains a co-insurance clause which specifies that 80% the value of the stock shall be covered by insurance. This ipposed amount of insurance is $(12,500 × 0.8) or $10,000, so iat the owners would be expected to carry another $4000 of surance.

Now suppose the stock suffered damage of $4000. The amount r which the Atomic Company is liable is then $\frac{6000}{10000}$ of $4000 $2400. This is the amount hich the company will pay, hether or not the owners carried ie full $10,000 of insurance. his example shows how a co- surance clause works. Such auses are sometimes contained policies on buildings such as arehouses and factories but sel- om in those on dwellings.

UNIVERSAL
INSURANCE
COMPANY
★ FIRE
★ HAIL
★ MARINE
★ ACCIDENT
★ BURGLARY
★ AUTOMOBILE
★ PLATE GLASS

What kinds of property are in- ıred and why? It is possible to ıy insurance on other kinds of roperty than buildings and for her kinds of loss or damage than by fire. Several kinds of insurance are listed on the frame poster. For each of the items, can you tell what is insured an against what hazard it is insured? If not, a question to yo parents or grown-up friends will bring the information.

EXERCISES

1. The face of the insurance policy on my house is $4500 an the rate is 48c per $100 for three years. What is my 3-ye premium?

2. A company which issues insurance in a rural area quote a rate of 30c per $100 per year on barns and outbuildings equipp with lightning-rods. What is farmer Mann's yearly premium o his barn which has lightning-rods and is insured for $3800?

3. Mr. Wood has a house worth $4000 and a barn wort $2500. He has the house insured for 3/5 of its value and th barn for 4/5, the rates being 20c and 35c per $100 on house an barn respectively. Find the annual cost of Mr. Wood's in surance.

4. Copy the following items into your work-book and fill i the premiums.

Face of policy	3-year rate	Premium
$5500	50c per $100	....
$3600	45c per $100	....
$8200	60c per $100	....

5. A dwelling worth $12,000 is insured for 80% of its valu The 3-year rate is 54c per $100. Find the cost of the insuranc over a period of 15 years.

6. A business block is insured in three companies, the face of the policies being $5000, $7500, and $9000. A fire did damag appraised at $8000. What amount does each company pay?

7. Mr. Case's suburban house valued at $4800 is insured fo 2/3 of its value, the 3-year premium being $17.28. Find the rat at which the insurance is quoted.

8. Mr. Jenness takes out $4500 of insurance on his house and 1800 on its contents. The rates are 45c per $100 and 54c per 100 on house and contents, respectively, for 3-year policies. ind his total 3-year premium. If he paid his insurance for one ear at a time, what would the yearly cost be?

9. The Plutonic Fire Insurance Co. issued a policy for $7500 a business property valued at $11,000. The policy contained co-insurance clause requiring that the property be insured for % of its value. If a fire does damage of $4500, for what nount is the Plutonic liable?

REVIEW EXERCISES

A

1. A pamphlet on sea power contains the statement: If you mpared the earth with a chess board, 46 out of 64 squares ould represent water. What percentage of the earth's surface water?

2. Copy the following table into your work-book and fill in e blanks.

raction	$\frac{9}{8}$	$\frac{1}{6}$	$\frac{3}{20}$	..	..	..	..	..	..
ecimal	..	..	..	0.045	0.06	0.008	..	..	..
er Cent	..	..	..	..	..	..	$\frac{3}{4}$	$12\frac{1}{2}$	2

3. Dave was offered a job of selling magazines, either for a eekly wage of $6 or on a commission of 25%. To make the cond offer the more attractive, how much would his average eekly sales have to be?

4. On a commission of $1\frac{1}{2}$%, how much does an agent receive r selling 4 dozen cases of dried fruit at $18 a case?

5. Copy the following equations into your work book and fill the blanks.

$4\frac{1}{2}\%$ = . . . mills on the dollar.
$3\frac{3}{4}\%$ = . . . mills on the dollar.
2.56% = . . . mills on the dollar.
. . . % = 28 mills on the dollar.
. . . % = $31\frac{1}{4}$ mills on the dollar.
. . . % = $40\frac{1}{2}$ mills on the dollar.

6. I have bought a cultivator for $66.50, agreeing to pay f it in 8 months with interest at 5% per annum. What will m debt be at the end of the 8 months?

7. The price of a tractor is $175 cash or $25 down and $1 in 10 months. If a purchaser makes his payments on the secon plan what rate of interest does he pay?

8. What sum must I invest at $5\frac{1}{4}\%$ per annum in order th it may amount to $300 in 16 months?

9. Mrs. De Tour, on a visit to the United States, bought vanity case for $6.75 in U.S. currency. If Canadian currency quoted at a discount of 12% in the United States, what did s pay in Canadian funds?

10. War savings certificates are purchased at the rate of $8 f a $10 certificate, redeemable in 7 years. At simple interest, wh rate does this represent?

11. If I mark goods at 10% above cost and then give a di count of 10% do I gain or lose? Express the gain or loss as per cent of (*a*) the cost, (*b*) the selling price. Neglect overhea

12. An automobile dealer employs Geo. Ross as a salesma on a commission of 3%. If his fee for the sale of a car is $22.3 how much does the dealer receive for the car?

13. Mr. Fields, on giving up his farm, held an auction sal After paying the auctioneer's fee of $2\frac{1}{2}\%$, he received procee of $1485.90. What was the auctioneer's fee?

14. In Prosperity county the tax rate is rather low—8.9 mill for county purposes and 5.5 mills for township purposes. Th township rate includes a 2.5 mill school rate. Find the total taxe n a farm assessed at $4300. How much of this goes to the ounty? How much is for the upkeep of the township schools?

15. A 10 million dollar issue of government bonds, bearing ½% coupons, became due and was paid by the issue of new onds bearing 3% coupons. By how much per annum were the overnment's interest payments thereby reduced?

16. A rented house is assessed at $3400, the tax rate being 33 ills on the dollar. The cost of insurance is $33.60 every three ears. The cost of repairs is estimated at $80 a year. If the ouse rents at $40 a month, how much of the year's rent is left ter expenses are paid?

17. If the interest rate is doubled and the principal is doubled, ow is the interest affected? If the interest rate is doubled and he principal is halved, how is the interest affected?

18. According to the 1941 Canadian census the Canadian-born opulation amounted to 9,487,808, which was 82.46% of the total opulation. Other British-born persons were 8.72% of the total nd the United-States-born were 2.72%. (*a*) How many people ere British (but not Canadian)-born? (*b*) How many were born n the United States?

19. A common cement mixture has the following specification: parts of gravel (or crushed stone) to 2 parts of sand and 1 part f cement. In making this mixture how much sand should be mixed with 2 cubic yards of gravel? What per cent of the mixure is cement?

20. Some chemical fertilizers are sold commercially with umerical labels, such as 4-8-10. This means that the fertilizer ontains 4% of nitrogen, 8% of phosphoric acid, and 10% of otash. What is the amount of each of these ingredients in a on of 4-8-10 fertilizer?

21. Find the weight of each of the three ingredients in 9 100-ound bags of 7-5-9 fertilizer.

B

22. A sales tax of 8% on the selling price of goods was reduc to 4%. Before the reduction a customer paid a total of $129. for a cabinet. What would he have paid after the reduction?

23. Mr. Allen: "Does our insurance premium come due th year?"

Mrs. A. (who keeps the family accounts): "We pay t premium on our $3200 policy this year and on our $30 policy next year."

Mr. A.: "Are the rates the same?"

Mrs. A.: "No, we pay 43c per $100 on the first policy ar 48c per $100 on the second. We have now bought enoug Victory Bonds to pay our insurance with the coupons ea year and have a little left over."

How many $100 bonds bearing 3% coupons have the Alle bought?

24. Mr. Matthews insured his house for $5500 at the rate 42c per $100 for 3 years. His insurance dated from Apr. 1, 194 He sold the house on July 1, 1943. What rebate of his insuran premium should he receive from the new owner?

25. After saving money for a residence for some years, M Service finally purchased one on May 1, 1945. The house wa covered by a 3-year insurance policy taken out on Sept. 1, 194 for which the premium was $45. No payment had been ma before the purchase on the 1945 taxes which amounted to $12 What adjustment should the former owner make with M Service with respect to taxes and insurance?

26. In the automobile repair shop where he is employe James Keys works on a commission basis receiving 50% of th first $50 of work done and 40% of the amount over $50. In on week James' work bills amounted to $135.24. What was hi cheque for that week?

27. When his house needed a new roof, Mr. Hill went to th bank to borrow $100 for a year. He was told that the ban arged interest on loans at 6% per annum. He signed a note r $100, payable in one year, and was given $94. Find the ctual rate of interest he paid on his loan from the bank.

28. Mr. Furness was persuaded to have his home insulated, e argument being that he could thereby save 15% of his coal ills. The cost of insulating was $175. He had been accus-med to burn 9 tons of coal yearly at a cost of $15.50 a ton. f the insulation proves equal to the claims made for it, in how any years will the cost of insulation be paid for by the saving coal? (Do not calculate interest on the cost of the insulation.)

29. A traveller, about to buy a home, has the choice of two roperties at the same price in different towns. He finds that ne of these houses is assessed for $4400 and that the tax rate is 2 mills, while the second is assessed for $3900 with a tax rate of 1 mills. Which property has the advantage in tax bills? If the revailing tax rates are maintained, what will be the difference the taxes on the two houses for a 5-year period?

30. The municipality of Suburbia has decided to build a school nd to pay for it by equal yearly instalments over a 20-year eriod. It finds that this yearly charge would be $4260. If the ssessment of the municipality is $2,240,000, what increase in the x rate on the assessed property will be sufficient to defray the ost of the school?

31. If nitrate of soda contains 16% by weight of nitrogen, how uch of this substance must be used to give sufficient nitrogen r a ton of 4-8-10 fertilizer?

32. The Falcon Brand Condensed Milk plant pays for milk ccording to its butterfat content, the rate being $2.05 per cwt. r milk which tests 3.5% butterfat. For each 0.1% variation the test the rate varies by $5\frac{6}{7}$c. A dairy farmer delivered uring one month 12,375 lb. of milk of which 8250 lb. tested 3.8 nd the remainder 3.6. Calculate his month's milk cheque.

33. If, with the rates of Ex. 32, a farmer received $91.02 for 200 lb. of milk what average per cent of butterfat did his milk est?

C

34. In a certain state there is a sales tax of 2%. A groce store sells for $10 a book of coupons to be exchanged for groceri The grocer tears out 20c worth of coupons for the tax when sells the book. A customer insisted on paying the 20c in cas whereupon the grocer replied that there was no difference. W he right? How much profit, if any, does the grocer make on t sale of 100 books by his system?

35. (*a*) Find the interest on $$x$ for y months at z% per annu

(*b*) At what rate per annum, simple interest, will amount to $$b$ in n years?

(*c*) What sum must be invested at r% per annum, simp interest, to amount to $$s$ in t years?

36. To make one ton of fertilizer the following materials we used:

100 lb. sulphate of ammonia,	containing	20%	nitrogen;
63 lb. nitrate of soda,	"	16%	"
48 lb. cyanimid,	"	21%	"
1200 lb. superphosphate,	"	20%	phosphoric aci
333 lb. muriate of potash,	"	60%	potash;
256 lb. dolomite limestone filler.			

What numerical label should be attached to this fertilizer?

RECREATION

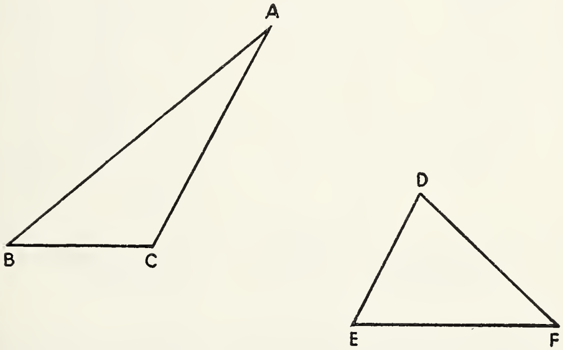
Which of the following plans would you choose for you spending allowance? Plan 1: Monthly payments beginning wit $1 on January first with a rise of 8c in each month thereafte Plan 2: Semi-monthly payments beginning with 50c on Januar first with a rise of 2c on the fifteenth and the first of each mont thereafter.

CHAPTER XII

N WHICH YOU EXTEND YOUR KNOWLEDGE OF GEOMETRY

Some Properties of Triangles

Do triangles differ from one another in regard to the sum of ıeir angles, just as they do in regard to the sum of their sides? Vhich is the greater, the sum of the angles of the triangle ABC r the sum of the angles of DEF?



You may naturally reply, "It is a waste of time to pour out vords on a question which can be so easily answered by experiıent. Let us ask the protractor."

It is suggested that each of you draw five triangles, measure he angles of each and find the five sums. Let the triangles be f many shapes and sizes. Find the highest and lowest angle ums obtained by the class and arrange the sums in order from reatest to least as indicated below.

Angle-sums	*Tallies*	***Frequency*** (i.e. number of triangles)
186°		
$185\frac{1}{2}$°		
185°		
$184\frac{1}{2}$°	𝍷	
184°		
$183\frac{1}{2}$°		
183°		
$182\frac{1}{2}$°	𝍷𝍷	
182°		
$181\frac{1}{2}$°		
181°	𝍸 𝍷	
$180\frac{1}{2}$°	𝍷	
180°	𝍷𝍷	
$179\frac{1}{2}$°	𝍷	
179°		
$178\frac{1}{2}$°		
178°		
$177\frac{1}{2}$°		
⋮		
174°		
Total no. of triangles		

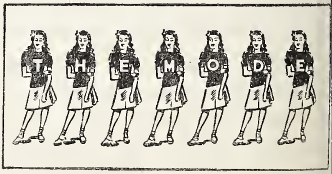
Then make a tal mark in the prop place for each su Each group (𝍸) i dicates *five* angle sum (N.B. We have insert in our table only a f of the tallies obtain from a large class.)

When the tally complete, the tot number of tallies f each angle-sum is r corded under the hea ing "Frequency".

In making this tabl you are not only throw ing a little light on th sum of the angles of triangle. You are als making what is know as a ***frequency dis tribution***.

Opposite which angle-sum is the greatest number of tri angles? This angle-sum is the most "fashionable" one and i appropriately called the ***mode***. There are different kinds o averages; the mode is one of them.

A more familiar average, known as the ***arithmetic mean***, is obtained by finding the total value of all the angle-sums and dividing by the total number of cases.



As an illustration the mode and arithmetic mean of the ıeights of 48 girls are found below.

Height in inches	*Frequency* (i.e. number of girls)	*Totals*
66	1	66
$65\frac{1}{2}$	1	$65\frac{1}{2}$
65	2	130
$64\frac{1}{2}$	5	$322\frac{1}{2}$
64	3	192
$63\frac{1}{2}$	5	$317\frac{1}{2}$
63	6	378
$62\frac{1}{2}$	7	$437\frac{1}{2}$
62	6	372
$61\frac{1}{2}$	4	246
61	3	183
$60\frac{1}{2}$	2	121
60	1	60
$59\frac{1}{2}$	1	$59\frac{1}{2}$
59	1	59
	48	$3009\frac{1}{2}$

$$\text{Mode} = 62.5 \text{ in.}$$

$$\text{Arithmetic mean} = \frac{3009\frac{1}{2}}{48} \text{ in.} = 62.7 \text{ in.}$$

Later, you will be taught a shorter method of finding arithnetic means.

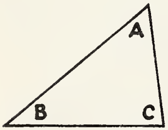
What is the arithmetic mean of the values for the angle-sum btained by the class?

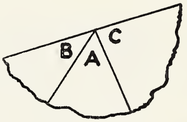
Someone may say, "What about the values far away from he arithmetic mean? Perhaps they are correct." We suggest hat pupils who found sums differing from the mean by a degree r more should re-measure the angles very carefully. What is he general effect of this re-measurement? What is the maximum rror one would expect in the angle-sum? Is it possible for an error made in the measurement of one angle to be cancelled b an error made in the measurement of a second angle?

Even if every member of the class found that the sum is 180 would you be justified in asserting that the sum of the angles o all triangles is two right angles? Would you feel certain of th for a triangle whose vertices were at the sun, the moon and th earth? Are you entitled to make the leap from 100 or 200 tr angles to every triangle that has ever been, or will be, made Since exceptions might conceivably occur, you cannot be abso lutely certain. All that can be concluded ***from your measurements*** is that it is probable that the sum of the angles i 180° or close to 180°.

We may add that the sum ***is*** 180°. But this assertion i based on grounds other than measurement. Your slight deviations from 180° are the result of unavoidable errors of measurement. Your larger deviations are caused by lack of practice i measuring angles, and—we are reluctant to say it—perhaps a little carelessness.

An interesting method of demonstrating roughly that th angle-sum of a triangle is two right angles will now be described





Draw a triangle and the cut it out. Tear off tw corners and place th vertices as shown in th illustration. Do the angles A, B, and C appea to form a straight angle

Let us combine ou knowledge of algebra and geometry in doing the following exercise:

The angles at the base of a triangle are equal, each being twice as large as that at the vertex. What is the size of each angle?

Let the number of degrees in the vertical angle be x.

Then the number of degrees in each base angle is $2x$.

Since the sum of the three angles is 180°,

$$\begin{aligned} x + 2x + 2x &= 180. \\ \text{Therefore,}\quad 5x &= 180. \\ \text{and}\quad x &= 36. \end{aligned}$$

Hence, the vertical angle is 36° and each of the base angles s 72°. Verify this solution.

EXERCISES

1. In the triangle ABC, $\angle A = 30°$ and $\angle B = 50°$. What s the size of $\angle C$?

2. In the triangle ABC, $\angle A = \angle C = 45°$. What is the size f $\angle B$?

3. In the triangle PQR, $\angle P = x°$, $\angle Q = 2x°$ and $\angle R = 3x°$. ind the size of each angle.

4. In the triangle XYZ, $\angle X = \angle Y = \angle Z$. What is the ize of each angle?

5. In a triangle, one angle is 54° and the other two angles re equal to each other. What is the size of the equal angles?

6. In a triangle each base angle is four times the size of the ertical angle. What is the size of each?

7. In a triangle DEF, $\angle D$ is three times as large as $\angle F$, and E is the arithmetic mean of $\angle D$ and $\angle F$. Find the size of), E and F.

8. Fifteen pupils measured an angle in their school grounds. he frequency distribution was as follows:

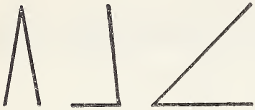
Angle	*Frequency*
72°	1
71½°	3
71°	0
70½°	5
70°	3
69½°	2
69°	1
	15

(*a*) What is the mode?

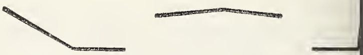
(*b*) Calculate the arithmetic mean.

Another Classification of Triangles

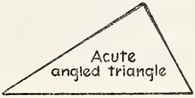
You will recall that we classified triangles according to t number of equal sides. (Page 47 Now we shall classify them accordi to their angles.

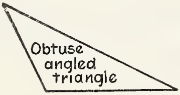


First, you should know that an *acute angle* is one that is less th a right angle.



Whatever lacks sharpness is obtuse. If an angle is greater than a right angle, but less than two right angles, we call it an ***obtuse angle***.





If a triangle contai three acute angles, it an ***acute-angled tr angle.*** If it contains a obtuse angle, it is an ***ob tuse-angled triangle.*** Hence we have the following classif cation of triangles according to their angles:

- ***Triangles***
  - ***Acute-angled triangles***
  - ***Triangles which are not acute angl***
    - ***Obtuse-angled triangles***
    - ***Right-angled triangl***

The phrase "right-angled triangle" is frequently abbreviate to "right triangle".

EXERCISES

1. (*a*) In which of the following cases is it impossible for the ngles A, B and C to belong to a triangle?

	$\angle A$	$\angle B$	$\angle C$
(i)	30°	70°	80°
(ii)	90°	12°	98°
(iii)	86°	90°	4°
(iv)	110°	30°	40°
(v)	110°	8°	112°

(*b*) Indicate which of the possible triangles of (*a*) above e (i) acute-angled, (ii) obtuse-angled, (iii) right-angled.

2. Copy the following table in your work book. In each se, give the value of the third angle and identify the triangle acute-angled, obtuse-angled, or right-angled.

Triangle	*The angles of the triangle*			*Classification according to angles*
	A	***B***	***C***	
i	75°	25°	. .	
ii	45°	48°	. .	
iii	. .	27°	53°	
iv	. .	45°	45°	
v	36°	. .	44°	
vi	72°	. .	18°	

3. What is the greatest number of obtuse angles a triangle n have? Explain.

4. If you subtract an acute angle from two right angles, what nd of angle do you get?

If the sum of two angles is 180°, the angles are said to ***supplementary***. For instance, 80° and 100° are supplemen ary angles. 60° is the supplement of 120°; 120° is the suppl ment of 60°.

5. What is the supplement of (*a*) 50°, (*b*) 130°, (*c*) 78°, (90°?

6. Can two obtuse angles be supplementary? Explain.

7. If two angles are supplementary, is the one necessari obtuse and the other acute?

8. Can a triangle contain an angle and its supplemen Explain.

9. Is the sum of two acute angles necessarily obtuse?

10. If A, B and C are the angles of a triangle what is th supplement of (*a*) $B + C$, (*b*) $A + B$, (*c*) B, (*d*) C?

If the sum of two angles is 90°, the angles are said to ***complementary***. Thus, 30° is the complement of 60°; 60° the complement of 30°.

11. Give the complement of (*a*) 20°, (*b*) 70°, (*c*) 30°, (*d*) 45

12. Can a triangle ABC have two pairs of complementar angles, e.g., the angles A and B and also the angles B and Explain.

13. If two angles of a triangle are complementary, what is th size of the third angle?

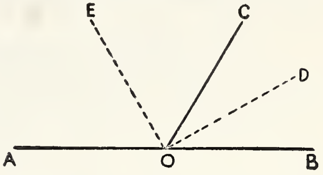
14. One angle is double another and the two are compl mentary. What is the size of each angle?

15. Two angles are supplementary and one is three times th other. What is the size of each?

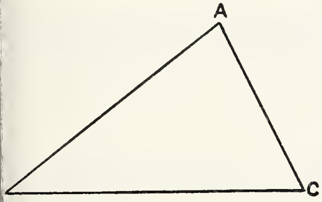
16. The difference of two complementary angles is 20°. Wh are the angles?

17. The difference of two supplementary angles is 20°. What e the angles?

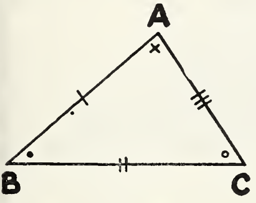
18. AOB is a straight line. D is the bisector of $\angle COB$ d OE is the bisector of COA. Without measuring e angles, show that $\angle EOC$ the complement of $\angle COD$.

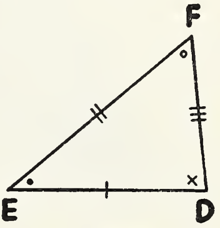


What information must we have before we can make a copy a triangle? Let us suppose that you need to transfer the triangle ABC from this book to another book, or from one part of the page to another. You wish to make a new triangle DEF with its sides and angles respectively equal to those of the triangle ABC, and with the same area as ABC.



You are not concerned whether the triangles lie on the paper the same way. For instance, if BC lies in the E-W direction, u do not care if the line corresponding to BC lies in the E-W





ection. The two triangles ABC and DEF are exact copies cording to our present interpretation of the word "copy".

The use of devices such as cutting out or tracing is not alved in these exercises.

Now what information about the triangle ABC would y like to have in order to make the copy? Don't ask for supe fluous information. You should work with the bare necessiti

It would be ridiculous to ask for the lengths of AB, BC, C and DA if you were required to construct a rectangle ABC The length and width are enough.

* * *

If you know the lengths of the three sides AB, BC and C have you enough information to enable you to copy the triang ABC?

Draw a triangle ABC in your work book and find out b experiment if it is sufficient to know the lengths of AB, BC an CA to construct a copy DEF of the triangle ABC. Compa your copy with the original.

The three sides of DEF are equal to the three sides of AB because you made them equal. Let us say that $DE = A$ $EF = BC$, and $FD = CA$.

Is $\angle D$ equal to $\angle A$? (Measure the angles.)

Is $\angle E$ equal to $\angle B$?

Is $\angle F$ equal to $\angle C$?

Is the area of DEF equal to that of ABC? (Draw th altitudes and measure them.)

If these seven magnitudes of the triangle DEF are equal the seven of the triangle ABC, then DEF is a copy of AB Instead of saying "the triangle DEF is a copy of the triang ABC", we sometimes say "the triangle DEF is **congruen** with the triangle ABC".

Your work would not be accepted as proof that two triangl are congruent if the three sides of the one are respectively equ to the three sides of the other. Later you will come to the proo At present, please accept the statement as one capable of proo

1. *Congruence of triangles*

Two triangles are congruent if the three sides of the one are respectively equal to the three sides of the other.

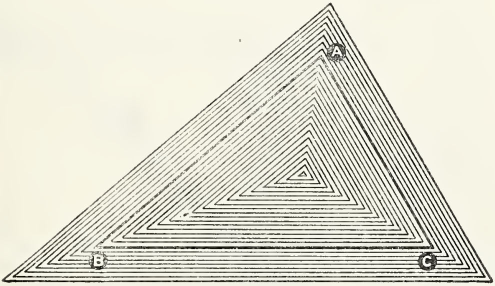
* * *

Now see if you can copy a triangle ABC if you know the zes of the angles. (You may use your protractor.)

Take a triangle ABC in which you know only that $\angle A = 80°$, $B = 40°$, and $\angle C = 60°$. Try to make a copy of the triangle.

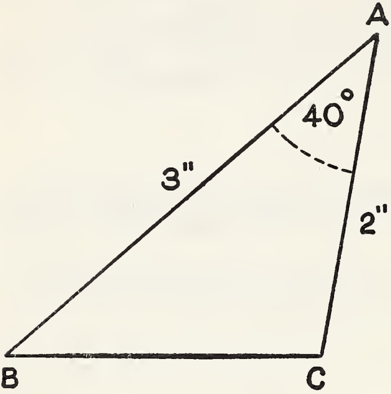
* * *

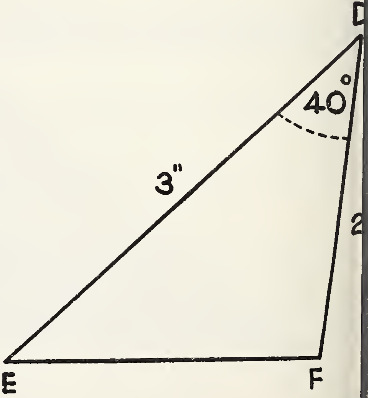
As you have found out, it is possible to construct thousands triangles with angles of 80°, 40° and 60°. To know the angles not enough.



* * *

Try to construct a copy of a triangle if you know two sides an the contained angle. Let the known sides be AB and AC and th known included angle be A. We shall leave this exercise wi you in the belief that you can construct a triangle DEF whi is congruent with the triangle ABC.



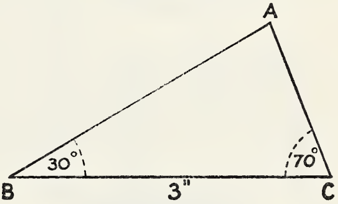


2. *Congruence of triangles*

Two triangles are congruent if two sides and the included angle of one are respectively equal to two sides and the included angle of the other.

* * *

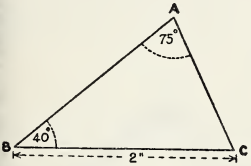
If you know the length of one side and the size of each of two ngles of a triangle, can you onstruct a copy? Try this xercise, using only the in- ɔrmation conveyed by the umbers in the figure.



* * *

That was child's play for you, but here is a slight variation hat may be more worthy of your intelligence. Let us take as a triangle to be copied one in which you know a side and two angles—wait a moment, *please*—but this time not both the given angles are adjacent to the given side. Look at the illustration.

If you find the going hard, we shall drop a hint. Be independent nd don't use the hint unless you are completely baffled and kely to lose sleep through anxiety.



* * *

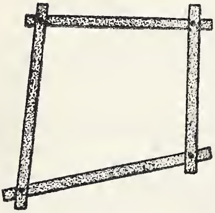
Hint for those who have become feverish: What is the sum f the angles A, B and C? Can you calculate C? . . .

When two angles and one side of a triangle are known, it i possible to make a copy of the triangle.

3. *Congruence of triangles*

Two triangles are congruent if two angles and a side in one are respectively equal to two angles and the corresponding side in the other.

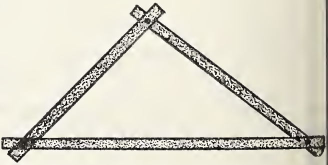
A note on rigidity. Cut four narrow strips of cardboar and fasten them together by one pin a each corner. Is the shape of the fram work fixed?



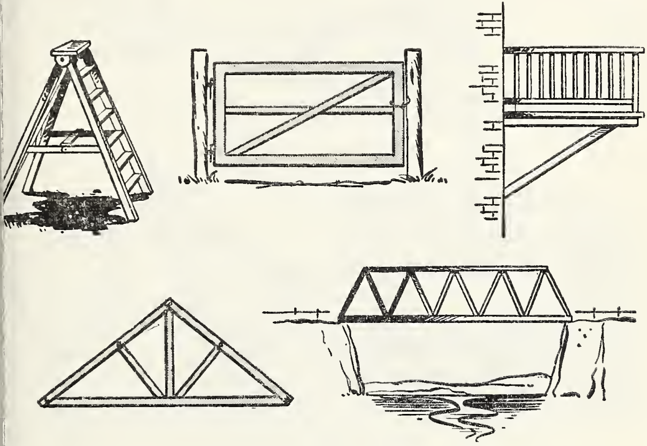
Construct a quadrilateral ABC with the following dimensions: $AB = 2$ in.; $BC = 2$ in.; $CD = 1\frac{3}{4}$ in.; $DA =$ $1\frac{1}{2}$ in.

Compare your quadrilateral wit those drawn by the other pupils. Yo notice that, in spite of the fact tha the corresponding sides are equal, there is a variety of shape Fixing the lengths of a quadrilateral's sides does not fix the siz of the angles or the area.

Take three strips of cardboard and fasten them, as shown in the illustration, with one pin at each corner. Is the triangular framework rigid, or is its shape changeable as was that of the four-sided linkage?



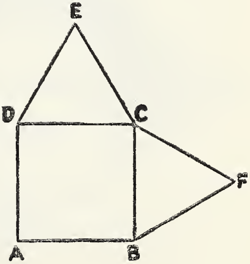
A copy can be made of a triangle if the lengths of the sides ar given; a copy cannot be made of a quadrilateral if only the length the four sides are given. The former statement can be ex-essed in another way—when the sides of a triangle are fixed in ngth, everything about the triangle is finally determined. Of hat practical use is this?

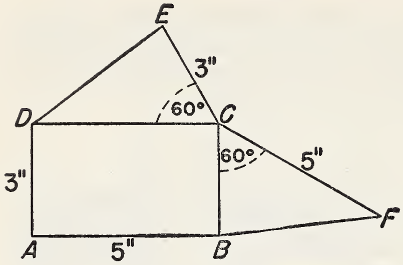


Whenever a structure must be rigid, look for a triangle.

EXERCISES

1. *ABCD* is a square. Show at the equilateral triangles *DC* and *FCB* are congruent. Jse your knowledge of squares d the conditions you have arned governing the congru- ce of triangles.)

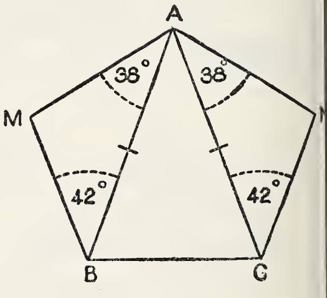


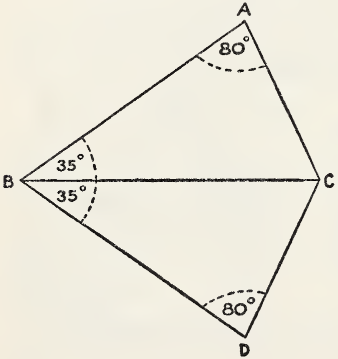

E
3"
60°
D
C
60°
5"
3"
F
A
5"
B

2. *ABCD* is rectangle. Sho that the triangl *EDC* and *BFC* ar congruent.

3. (*a*) *ABC* is an isosceles triangle. Show that the Δs *AMB* and *ANC* are congruent.

(*b*) What two points would you join in order to obtain another isosceles triangle?

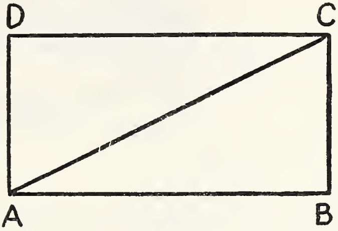

A
38°
38°
M
42°
42°
B
C


A
80°
35°
B
35°
C
80°
D

4. (*a*) Show that the tri angles *ABC* and *DBC* ar congruent.

(*b*) What two point would you join in order t produce two isosceles tri angles?

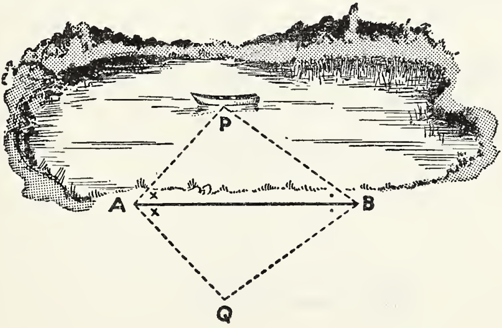
5. *ABCD* is a rectangle. (*a*) Show that the triangles *ABC* nd *CDA* are congruent. (*b*) Name the angle equal to the angle *CD*. (*c*) Name the angle equal to the angle *DAC*. (*Hint:* In ongruent triangles, equal angles are opposite equal sides.)



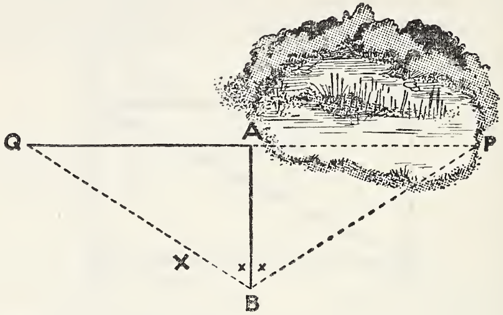
6. A friendly dispute arose concerning the distance of a boat from a point *A* on the shore of a lake.

To settle the matter, a distance *AB* was measured along he shore. The angles *BAP* and *ABP* were measured. Then he angle *BAQ* was made equal to the angle *BAP* and the angle *BQ* was made equal to the angle *ABP*. *AQ* was measured and he length was taken as the length of *AP*.

What principle of congruence was used?



7. (*a*) To find the distance between two points A and P, th
following method was used:



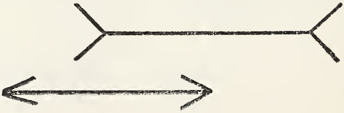
At A, a line perpendicular to PA was marked out.
point B, from which P was visible, was taken on this line. Th
angle ABP was measured. Then an angle ABX equal to AB
was constructed on the other side of BA. The arm BX wa
extended until it intersected at Q the line through PA.

The length AQ was measured and taken as equal to A
Justify this procedure.

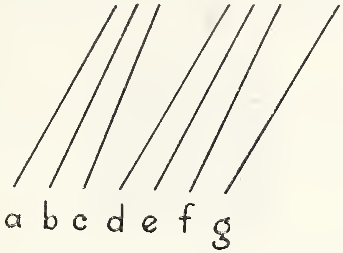
(*b*) If AB = 200 yards, and $\angle ABP$ (or $\angle ABQ$) = 40
find the length of AQ.

A digression on the "obvious". Do this little quiz withou
using instruments or other aids.

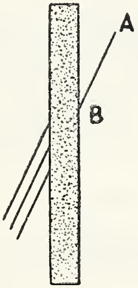
1. Are the two horizontal lines equal in length? If no
which is the longer?



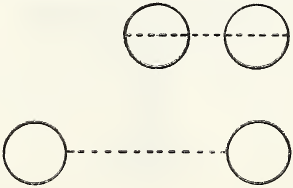
2. Which line, if any, is parallel to the line marked *a*?



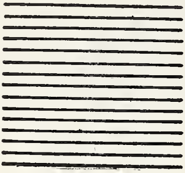
3. If *A B* were produced, along which of the three lines would lie?



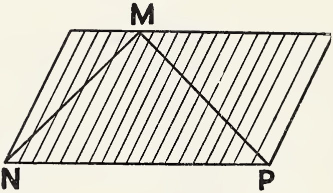
4. Which of the two dotted lines is the longer?



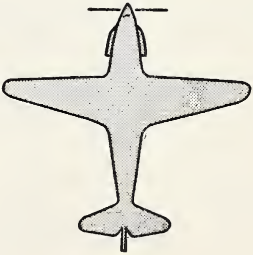
5. Does the outline form a square? If not, which is longer the length of the lines or the depth of the figure?



6. Is MN equal to MP?



7. Is the wing-span greater than the over-all length of the aeroplane?



It is told of the mathematician Laplace that he spent almost hour trying to understand one of his own proofs. In this proof had lightly passed over some of the steps with an "It is easy see. . . . ."

Some modern mathematicians have devoted years to the rutiny of statements which earlier workers accepted unquesningly. " 'Obvious' is the most dangerous word in mathetics."[1]

Perhaps we have now made the point that in mathematics one should not jump to conclusions. Let us add quickly that this caution should not be confined to mathematics. It should be displayed equally in everyday affairs.

We Resume Our Study of Triangles

Another glance at isosceles triangles. Draw a number of sceles triangles differing in size and shape.

By this time you know that, when requested to draw a certain ure, you should always draw the most general form of that ure. For instance, it would be considered amateurish to draw special kind of isosceles triangle, namely the equilateral one, en you are told merely to draw an isosceles triangle.

Measure the angles and enter the results in your work book a table similar to the following:

---

[1] E. T. Bell in *The Queen of the Sciences.*

Isosceles triangle	*The equal sides*	*The three angles*		
		1	*2*	*3*
ABC	AB, AC			
DEF	DE, DF			
GHK	HK, KG			
Etc.				

What do you notice about the sizes of the angles? *Do n jump to a general conclusion.* Compare your results with tho of other pupils. Are the results in agreement?

In wording your conclusion, it may be convenient for y to know that, in a triangle ABC, the angle B is described *opposite to* AC. The angle A is opposite to BC.

The proper conclusion to be drawn from your work is follows: *In the* ———————— (20, 26, 30, etc.) *differe isosceles triangles drawn, the angles opposite to the equal sid were equal or nearly equal.* We shall *tell* you that the followi generalization has been proved:

All isosceles triangles have equal angles opposite to t equal sides.

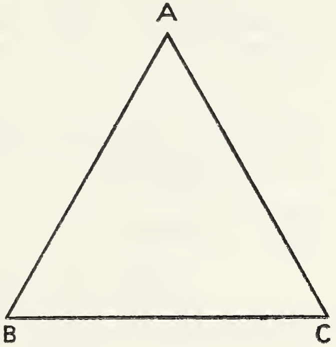
or,

If a triangle is isosceles, then the angles opposite the equal sides are equal.

* * *

Now let us turn to the special kind of isosceles triang namely the equilateral triangle.

Since $AB = AC$, what conclusion do you draw regarding an- es? Since $CB = CA$ what conclusion do you draw regarding ıgles? Now what is your conclusion regarding all the angles the triangle?



* * *

You have seen that the angles of an equilateral triangle are ual. The size of each angle is 60°. (Page 353; Exercise 4.)

EXERCISES

1. With straight edge and compasses, construct an angle of) 60°, (*b*) 30°, (*c*) 15°, (*d*) $7\frac{1}{2}$°, (*e*) 120°, (*f*) 150°, (*g*) 165°, (*h*) 5°, (*i*) 75°.

2. The angles of a triangle are $x°$, $x°$, and $4x°$. The equal les are each 2 inches in length. Construct the triangle. Mea- re the length of the side opposite to the largest angle.

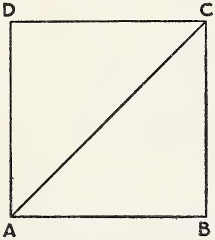
3. Each of the equal angles of an isosceles triangle is double e third angle. The side opposite to the smallest angle is 5 cm. ıg. What is the length of one of the equal sides?

4. Draw an isosceles triangle. Bisect the angle contained by e equal sides. Produce the bisector to meet the side opposite the angle bisected.

(*a*) Measure the two parts into which the side is divid by the bisector. What do you notice? Compare your findi with those of other pupils.

(*b*) Measure the angle that the bisector makes with th side opposite to the angle bisected. What do you observ Compare your result with those obtained by other members the class.

N.B. Later you will be able to give proofs where no you are using measurement. The results of 4(*a*) and 4(*b*) a important and should be remembered.



5. *ABCD* is a square.

(*a*) Name two isosceles triangles.

(*b*) Name two pairs of equal angle

(*c*) In an isosceles right triangl what is the size of each of the equ angles?

A tricky little point. Consider the statement, "All circl with equal radii have equal areas". Let us turn it around an say, "All circles of equal area have equal radii."

Each of these two statements is called the ***converse*** of th other.

Instead of saying, "All circles with equal radii have equ areas", we may use the 'If-Then' form, "If circles have equ radii, then they have equal areas". Express the converse in th 'If-Then' form.

In our example, both the statement and its converse can b proved. Now take these, "All right angles are equal", and "A turkeys are red-heads". Are the two converses of these stat ments true?

Now do the following exercises:

EXERCISES

To the best of your knowledge, describe each statement and so its converse as *true* or *false*.

1. If a figure is a square, then its four sides are equal.

2. All numbers that end in 0 are exactly divisible by 5.

3. If you are standing on your head, your face becomes red.

4. If two triangles are congruent, the three angles of the one e respectively equal to the three angles of the other.

5. All congruent triangles are equal in area.

6. All daughters of sisters are cousins.

7. All great-grandfathers are men.

8. If the perimeters of two circles are equal, their areas are ual.

9. All swimmers are sharks.

10. All bipeds are human beings.

11. If the perimeters of two rectangles are equal, the areas e equal.

12. If a baby is fed 'Herculene', it is healthy.

13. All negroes possess wavy hair.

14. If a man is of good character, he does not steal.

15. If A, B and C are the angles of a triangle, the sum of the gles A, B and C is two right angles.

If a statement is true, its converse may be true or false.

You should be as careful about converses in everyday life as you are about converses in mathematics.

To find out if a certain converse is probably true. Yo have accepted the statement, "If a triangle is isosceles, the angl opposite to the equal sides are equal". You are now wary enoug to realize that this does not justify your acceptance of the converse: "If there are equal angles in a triangle, the sides opposit to these angles are equal".

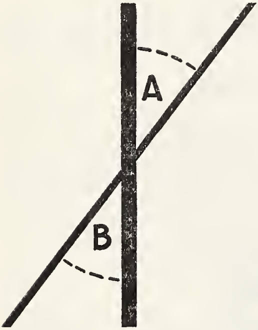
By working co-operatively, find out if the converse is probably true.

* * *

Since you cannot be sure of the truth of this converse, n matter how many triangles are examined, we shall inform yo that the converse can be established by other methods.

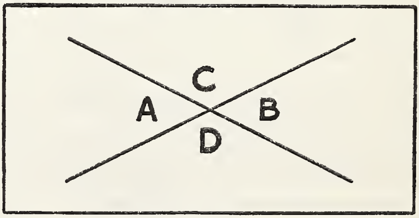
In a later book, we shall have something to say about 'trut in mathematics.

Parallel Lines



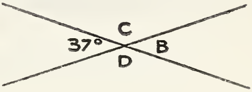
Where we are writing there is chair with a sloping back—one i which we seldom have time to slop The edge of the back meets the edg of the arm as we have shown in th sketch. The angles *A* and *B* *loo* equal, but we are cautious. (Are we

Draw intersecting lines and measure the ***vertically opposite angles*** as *A* and *B* are called. D your results agree with those o your fellow pupils?

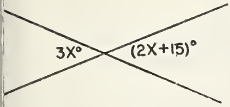


A and *B* are vertically opposite angles. *C* and *D* form another pair o vertically opposite angles.

EXERCISES



1. Find the number of degrees each of the angles *B*, *C* and *D*.



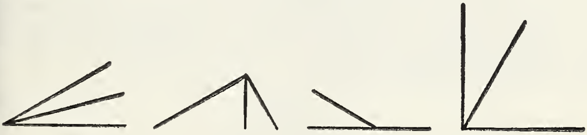
2. The number of degrees in two vertically opposite angles is indicated in the figure. Find the four angles.

3. Two vertically opposite angles contain $3x^\circ$ and $(x + 80)^\circ$. hat is the size of each of these two angles?

4. If two vertically opposite angles contain $8x^\circ$ and $(2x+90)^\circ$, ıat is the size of each?

5. Under what circumstances would two straight lines in the me plane not form vertically opposite angles?

A few new terms. When two angles have a common arm tween the other two arms, they are called ***adjacent angles.*** ɛre are four pairs of adjacent angles:



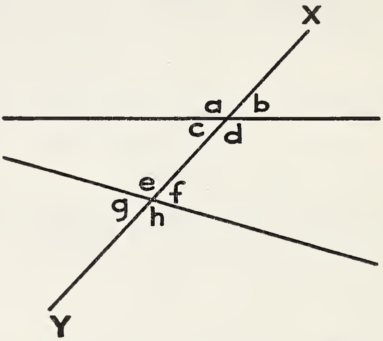
Our next illustration shows a ***transversal*** (*XY*) cutting o other straight lines. Find eight pairs of adjacent angles. nd four pairs of vertically opposite angles.

The angles *c* and *f* are called ***alternate angles.*** The an- ɛs *d* and *e* form another pair of alternate angles.

The angles *a*, *b*, *g* and *h* are known as ***exterior angles.***

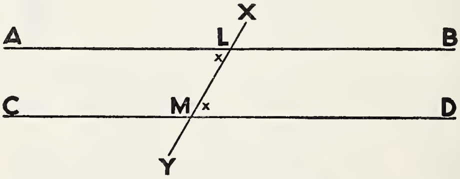
Two of the ***interior angles*** are *c* and *d*. Name two other ːerior angles.

In this figure, angles with similar positions at the two vertic are called ***corresponding angles.*** The angles b and f are o



pair of corresponding angles; d and h are another pair. Nam two other pairs of corresponding angles.

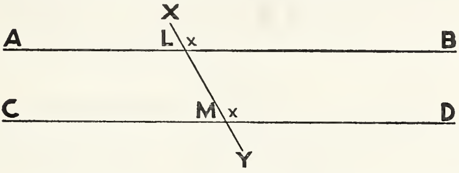
Two straight lines in the same plane are parallel, if . . It will be recalled that we have assumed that when a transve sal XY meets two straight lines AB and CD making the alte nate angles ALM and LMD equal, then AB and CD are pa allel. (See p. 36 and p. 407.)



If the angle LMD is equal to the exterior opposite ang XLB, are the lines AB and CD parallel?

The $\angle XLB$ equals the vertically opposite $\angle ALM$. Henc if $\angle LMD$ is equal to $\angle XLB$, $\angle LMD$ must be equal to $\angle AL$

But we have already taken for granted that if $\angle ALM$ is qual to $\angle LMD$, then AB and CD are parallel.



Hence, if the exterior $\angle XLB$ equals the interior opposite $\angle LMD$, we must take it for granted that AB and CD are arallel.

> *To sum up: Two straight lines are parallel if a transversal makes*
>
> (i) *alternate angles equal, or*
> (ii) *corresponding angles equal.*

What Does 'Ratio' Mean?

If pupil A has \$6 and pupil B has \$2, we may compare these ums in two different ways. We may say that A has \$6 − \$2, r \$4, more than B. Or, we may say that A has $\frac{6}{2}$ or 3 times as uch money as B.

If a line AB is 20 centimetres long, and a line BC is 5 centi-etres long, AB is $\frac{20}{5}$ or 4 times as long as BC. We may express his as follows:

$$\frac{AB}{BC} = \frac{20}{5} = \frac{4}{1}.$$

$\frac{AB}{BC}$ is called the ***ratio*** of AB to BC. Similarly, $\frac{BC}{AB}$ is the atio of BC to AB.

The ratio of one number to another, is the fraction hat the first is of the second.

The ratio of one quantity to another of the same ind, is the fraction that the measure of the first quan-ity is to the measure of the second.

EXERCISES

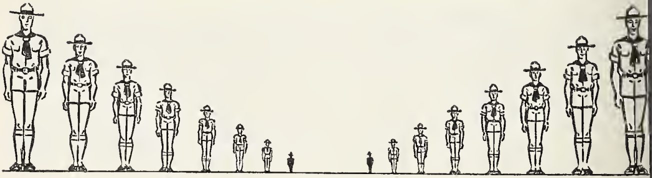
1. Give the ratio of the first to the second quantity: (*a*) 1 inches, 3 inches; (*b*) \$4, \$24; (*c*) 135°, 45°; (*d*) 3 inches, 6 inche (*e*) 3 yds., 6 ft.; (*f*) 2 tons, 400 pounds; (*g*) 6 yds., 6 yds.; (*h*) 4 minutes, 4 hours.

2. Compare the ratio of the first quantity to the second wit the ratio of the third to the fourth: (*a*) \$4, \$2; 400 cents, 20 cents. (*b*) 3 yds., 2 yds.; 9 ft., 6 ft. (*c*) 2 pounds, 6 pound 20 cents, 60 cents. (*d*) 8 cm., 10 cm.; 40°, 50°.

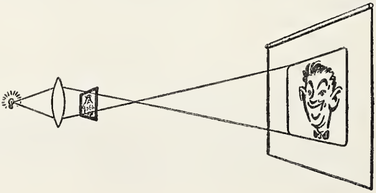
3. Express to the nearest second place of decimals the fo lowing ratios: (*a*) $\frac{5}{7}$; (*b*) $\frac{78.6}{9}$; (*c*) $\frac{3}{22.6}$; (*d*) $\frac{0.67}{0.08}$.

4. Find which is the greater of the following two ratio $\frac{28}{59}, \frac{27}{58}$.

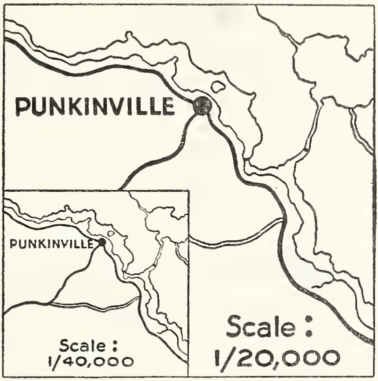
Enlargements and Miniatures



You are familiar with the fact that, when a projection lanter is used, the picture on the screen is a reproduction of the pictur



n the slide but on a larger scale. The shapes are preserved, ut the sizes are altered. In a photograph and its enlargement,



nd in a scale drawing and its original, we have also a preser ation of shape and a change in size.

Two figures of the same shape, but not necessarily f the same size, are said to be similar.

EXERCISE

Which of the following figures are similar? (Give your rea-ons.) 1. All circles. 2. All squares. 3. All rectangles. 4. All quilateral triangles. 5. All isosceles triangles.

If in a figure there are two perpendicular lines, are these lines erpendicular when the figure is enlarged? Of course they are. f the right angle were changed, the shape would be altered.

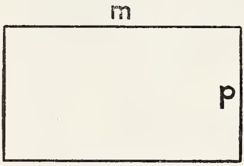
Draw a triangle of any size you please. Now construct a econd triangle the sides of which are respectively double the ides of the first one. Measure the angles of both triangles. Vere the angles changed when the sides were doubled?

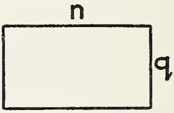
Whenever two figures are similar, the angles of the ne are respectively equal to the angles (taken in the ame order) of the other.

The following exercise illustrates another important propert of similar figures.

EXERCISE

The two figures are drawings to different scales of the sam formal pool. These figures are similar.





(*a*) Is there any difference between corresponding angle in the two figures?

(*b*) Measure in centimetres the sides m, p, n and q. Is th ratio $\frac{m}{n}$ equal to the ratio $\frac{p}{q}$? Is the ratio $\frac{m}{p}$ equal to the rati $\frac{n}{q}$?

Whenever two figures are similar, the ratios of corresponding sides are equal.

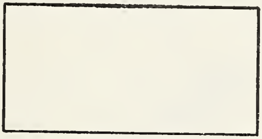
To sum up, there are two conditions which must be satisfie before figures can be described as similar.

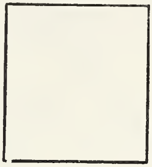
The two properties of similar rectilineal figures.

1. *The angles of the one are respectively equal to the angles of the other.*
2. *The ratios of corresponding sides are equal.*

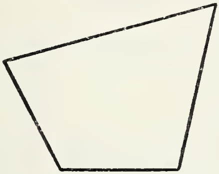
EXERCISES

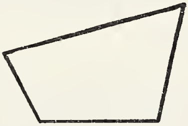
1. Are the following two rectangles similar? Apply the two ests of similarity.



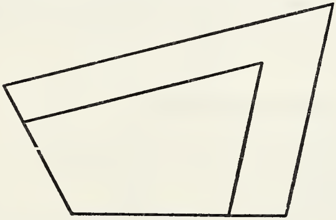


2. Are the following two quadrilaterals similar?



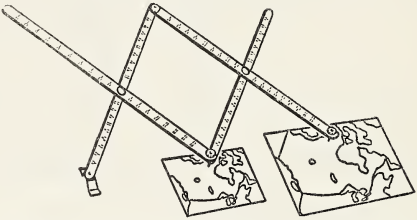


3. Are the following quadrilaterals similar?



4. Draw two non-congruent triangles in which the angles of ne are respectively equal to the angles of the other. Find the atios of corresponding sides. Repeat this with other pairs of uch triangles. Try to draw two such triangles in which the atios are unequal.

What you suspect as a result of experimentation has bee proved by other means. *Two triangles which have the angles of th one respectively equal to the angles of the other are invariably simila triangles.* The converse is also true because we have used th equality of the angles as one of the two tests of similarity.



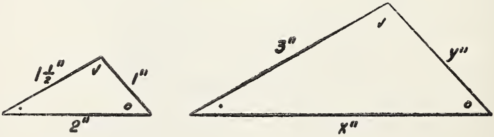
The pantograph is a device for making similar figures.

EXERCISES

1. If you know that two angles in one triangle are respectively equal to two angles in another triangle, can you say a once that the two triangles are similar? Why?

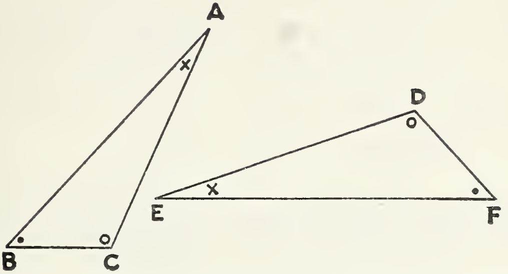
2. If one of the acute angles in a right triangle is equal t an acute angle in a second right triangle, are the triangles similar? Why?

3. Find x and y by calculation.



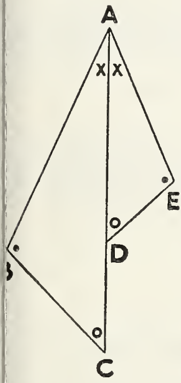
4. What ratios are equal to $\frac{AB}{BC}$ in each of the following pairs of similar triangles?

(*a*)

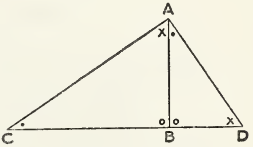


Hint: Corresponding sides are opposite equal angles.

(*b*)



(*c*)



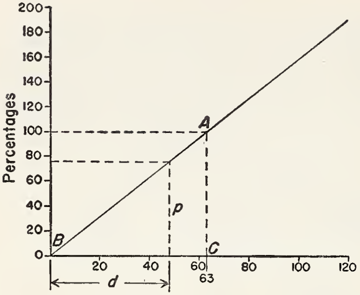
5. In 4 (*c*) if $BC = 9$ units and $BD = 4$ units, calculate the length of AB.

6. (*a*) What two tests for similarity have been used?

(*b*) If *triangles* have their angles respectively equal, is it necessary to apply the second test for similarity?

Prefabricated percentages. Sometimes it is necessary to find oughly many percentages of the same number. For example, vhat percentage of 63 is each of the following numbers? 17, 29, 4, 42, 61, 98.

This is easily done graphically by making use of similar riangles.



The point corresponding to 63 on the horizonta axis and 100 on the vertica axis is joined to the origin (It is not necessary to us the same scale on the tw axes.)

Take any point on th resulting line and draw perpendicular to the horizontal axis.

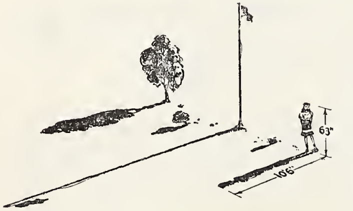
$$\text{From similar triangles, } \frac{p}{AC} = \frac{d}{BC}.$$

$$\text{That is, } \frac{p}{100} = \frac{d}{63}.$$

$$\text{Hence, } \frac{p}{100} \times 63 = d, \text{ or } d \text{ is } p\% \text{ of } 63.$$

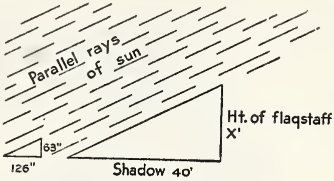
Thus, to find what percentage a number, say 28, is of 63, find 28 on the horizontal axis. Then, from the line *BA* find the corresponding reading on the vertical axis. Neat, isn't it?

EXERCISES

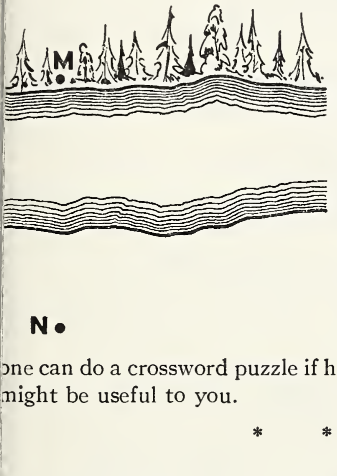


1. A pupil looked at her shadow and at the shadow cast by a flagstaff and said (in that superior manner we all admire), "I can easily calculate the height of that flagstaff." How would you find the height of the flagstaff from the information conveyed by the sketch?

We believe that you need no additional help to solve this problem.



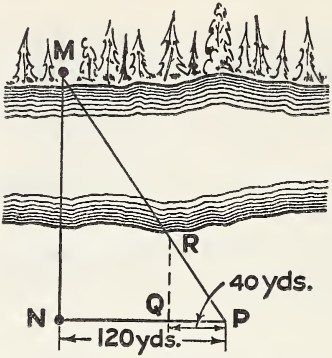
2. The shadow of a 3-foot stick was 1 foot 9 inches long. At the same time, the shadow of a tree was 42 feet long. How high was the tree?



3. You are standing at N and can see a rock M on the other bank of the river. If it is not feasible to measure MN directly, how would you find the distance? We shall deal with the matter below, but don't spoil the game by looking at the answer. Anyone can do a crossword puzzle if he has the key. Similar triangles might be useful to you.

* * *

From N mark out a line NP such that $\angle N = 90°$. Choose a point P so that it is possible to see M from P, and so that NP is a convenient length (say 120 yards). Take a point Q between N and P. Measure PQ. Let PQ be 40 yards. At Q mark a line perpendicular to NP, and on it find a point R in line with P and M. Measure QR. Let it be 60 yards long.



PQR and *PNM* are similar triangles. (Why?)

$$\text{Hence,}\quad \frac{MN}{RQ} = \frac{NP}{QP}$$

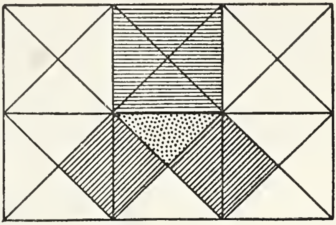
$$\text{Therefore,}\quad \frac{MN}{60} = \frac{120}{40}$$

$$\text{and}\quad MN = 180 \text{ yards.}$$

The Pythagorean Theorem

No doubt, many of you know that there has been discussion of the possibility that life exists on Mars. A proposal was made that a signal be sent to show the Martians, if any, that beings of some intelligence dwell on the planet Earth. There are a few minor difficulties. What kind of message would carry meaning to the inhabitants of another planet? Someone thought of signalling a certain geometrical figure, that for the theorem named after Pythagoras. The signal might be a little lacking in warmth but it would make a first-rate letter-head for intelligence corresponding with intelligence.

Let us tell you something about the famous theorem of Pythagoras.



earning from tiles. Twenty-ive or more centuries ago, a an sat staring at a tiled oor. He noticed that the argest of the three shaded quares had the same area as he two smaller ones com-ined. That wasn't very ex-iting; the dullest slave in the tilery knew all about it. However, e was not discouraged; he kept on noticing in a persistent man-er. Next, he observed that the large square was the square on he longest side of a right triangle and that the smaller squares ere the squares on the other two sides. A very remarkable hing!

But is this true of all right triangles or is it true only of the pecial case where the right triangle is isosceles? Read our next nstalment.

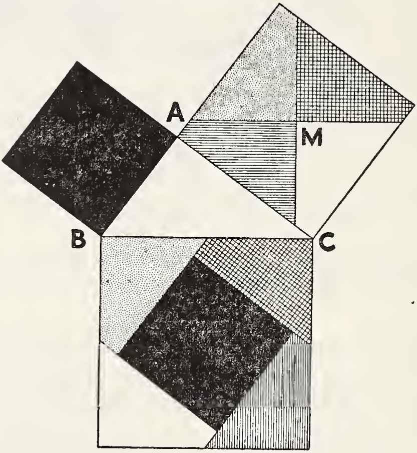
An experiment with right triangles. It is suggested that the embers of the class construct right triangles and co-operate in ompleting in the work books a table such as the following one. or the sake of convenience, in each triangle the angle C should e the right angle. The sides may be of any length.

Angle C	*Length of side*			*Area of square on AC*	*Area of square on BC*	*Sum of areas of squares on AC and BC*	*Area of square on AB*
	AC	*BC*	*AB*				
90°							
90°							
90°							

What inference may be made from the table? Do not gene alize rashly.

* * *

You may be interested in the following paper-cutting demo stration. Thin cardboard is preferable to paper.



Using any instruments you please, construct any right tri angle ABC. On each side make a square.

Find the point M where the diagonals of the square on A intersect. Through M, draw a line parallel to BC and a lin perpendicular to BC.

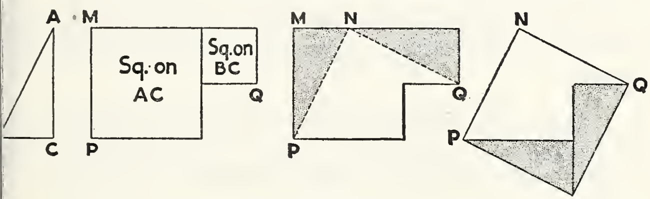
Cut out the square on AB and also the four pieces into whic the square on AC is divided. If these be assembled to form square, to what square is it equal?

There is evidence that this demonstration was used by th Chinese more than a thousand years B.C.

Neither your experiment nor your paper-cutting can serve a proof of the statement that ***the square on the longest sid***

f a right triangle is equal to the sum of the squares on he other two sides. However, we are able to assure you that has been proved. Indeed, more than three hundred proofs re now known. When the first proof was discovered, oxen were acrificed to the gods.

The longest side in a right triangle is called the ***hypotenuse.***



Another experimental demonstration.

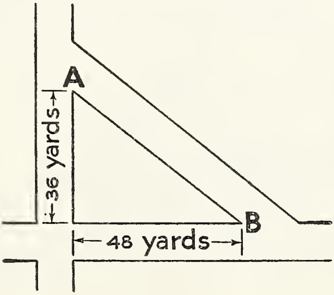
xample. Given that the two short sides of a right triangle are inches and 4 inches, find the length of the hypotenuse.

Let the number of inches in the length of the hypotenuse be x.

$$\begin{aligned} \text{Then } x^2 &= 3^2 + 4^2 \\ &= 9 + 16 \\ &= 25 \\ \text{Therefore, } x &= 5. \end{aligned}$$

Hence, the length of the hypotenuse is 5 inches.

EXERCISES



1. (*a*) How many yards long is the path AB?

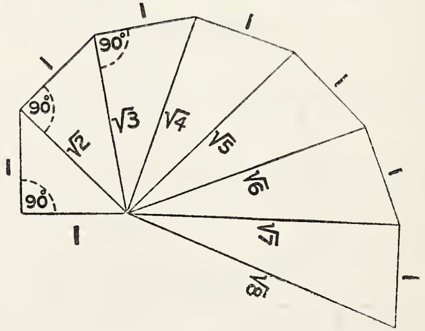
   (*b*) What distance is saved in using the short-cut?

2. A 25-foot ladder is leaning against a wall. If the foot the ladder is 7 feet from the wall, at what height above the groun is the top of the ladder?

3. Can a knitting needle fourteen inches long be laid on th bottom of a rectangular box one foot long and five inches wid Explain.

4. ABC is a triangle in which $\angle B = 90°$. Copy the fo lowing table and complete the entries. In doing this, you ma use the table of squares and square roots given on page 392.

Side	*Number of units of length*					
AB	5	15			60	12
BC	12		40	7		35
CA		17	41	25	61	



5. (*a*) Examine the accompanying figure and explain its construction.

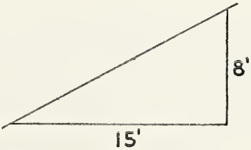
(*b*) How would you use this construction to find lines of ength $\sqrt{10}$ inches and $\sqrt{11}$ inches?

6. Winnifred Wheeler and Priscilla Pedlar rode 10 miles south-east, ate their lunch and then rode 6 miles due west. They were then due south of the point from which they originally set out. How far were they from that point?

7. (*a*) Find the ratio of the length of the diagonal of a square to the length of a side.

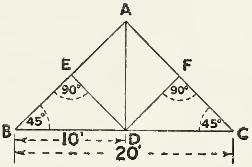
(*b*) Does this ratio hold good for all squares? Why?

(*c*) Show that the square on the diagonal of a given square has twice the area of the given square.



8. Calculate the length of the rafter, allowing 9 inches for projection at each end.

9. Find the lengths of DE, DF, AD, AE and AF.



10. Use the table to find (*a*) 29^2, (*b*) 26^2, (*c*) 23^2, (*d*) $\sqrt{1089}$, (*e*) $\sqrt{1681}$, (*f*) $\sqrt{961}$.

TABLE OF SQUARES AND SQUARE ROOTS

n	n^2	$\sqrt{n}$	n	n^2	$\sqrt{n}$	n	n^2	$\sqrt{n}$	n	n^2	$\sqrt{n}$
1	1	1.000	16	256	4.000	31	961	5.568	46	2116	6.782
2	4	1.414	17	289	4.123	32	1024	5.657	47	2209	6.856
3	9	1.732	18	324	4.243	33	1089	5.745	48	2304	6.928
4	16	2.000	19	361	4.359	34	1156	5.831	49	2401	7.000
5	25	2.236	20	400	4.472	35	1225	5.916	50	2500	7.071
6	36	2.449	21	441	4.583	36	1296	6.000	51	2601	7.141
7	49	2.646	22	484	4.690	37	1369	6.083	52	2704	7.211
8	64	2.828	23	529	4.796	38	1444	6.164	53	2809	7.280
9	81	3.000	24	576	4.899	39	1521	6.245	54	2916	7.348
10	100	3.162	25	625	5.000	40	1600	6.325	55	3025	7.416
11	121	3.317	26	676	5.099	41	1681	6.403	56	3136	7.483
12	144	3.464	27	729	5.196	42	1764	6.481	57	3249	7.550
13	169	3.606	28	784	5.292	43	1849	6.557	58	3364	7.616
14	196	3.742	29	841	5.385	44	1936	6.633	59	3481	7.681
15	225	3.873	30	900	5.477	45	2025	6.708	60	3600	7.746

Is the converse true? Give the converse of the following statement:

If a triangle contains a right angle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Is the converse true? Let us see. The three numbers 8, 15 and 17 would attract no special attention if they walked down the street arm in arm, but they are quite an interesting trio. Just look!

$$8^2 = 64$$

$$15^2 = 225$$

$$17^2 = 289$$

Therefore, $8^2+15^2 = 17^2$.

Who would have thought it of them? Do these little exercises.

EXERCISES

1. Some of the following groups of numbers have the property displayed by 8, 15 and 17. Pick them out.

(i) 3, 4, 5; (ii) 5, 12, 13; (iii) 5, 11, 12; (iv) 8, 6, 10; (v) 7, 24, 25; (vi) 12, 35, 37; (vii) 11, 59, 60; (viii) 11, 60, 61; (ix) 12, 16, 20; (x) 10, 24, 26; (xi) $2n$, n^2-1, n^2+1; (xii) m^2-n^2, $2mn$, m^2+n^2.

2. Try to discover other numbers which fulfil the requirement $a^2+b^2 = c^2$. (xi) and (xii) of Exercise 1 may be helpful.

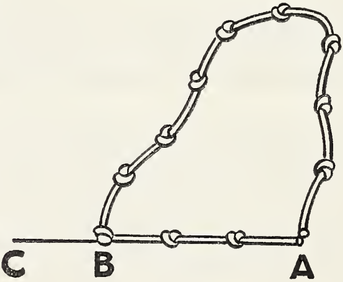
Take those groups of numbers which obey the law $a^2+b^2 = c^2$ and use them in drawing triangles. For example, you may draw a triangle ABC in which $AB = 8$ inches, $BC = 15$ inches, and $CA = 17$ inches or one in which $AB = 8$ quarter inches, $BC = 15$ quarter inches, and $CA = 17$ quarter inches, or one in which $AB = 5$ cm., $BC = 12$ cm. and $CA = 13$ cm.

In each triangle, measure the angle opposite to the longest side.

Your work should enable you to receive with composure the news that the converse of the Pythagorean theorem has been proved.

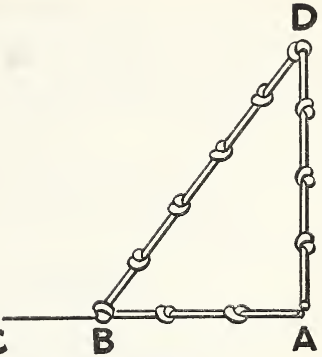
The Pythagorean theorem and its converse are two of the most important facts which you will learn in geometry. They are described in an old book as "of infinite uſe in moſt Geometricall Concluſions".

The rope stretchers. When the Egyptians wished to construct a right triangle out of doors, they used a loop of rope with eleven equally spaced knots. If they required a perpendicular to the line CA at the point A, they fastened both ends of the rope at A and stretched part of the rope along CA. At B, three spaces from A, the rope was again fastened. Then, someone held



the rope at D, five spaces from B and pulled the rope taut. DAB was a triangle with sides of 3, 4 and 5 units. The angle A opposite to the longest side was a right angle. (Converse of the Pythagorean theorem.)

This method of constructing a right angle out of doors is still sometimes used.



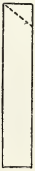
EXERCISES IN SPATIAL IMAGINATION

(To be performed mentally)

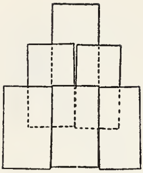
1. How many matches are required to make the phrase "My Valentine"? A whole match is to be used for each stroke.

2. The streets of Jitterburg are laid out in the N-S and E-W directions. A motorist drove north, turned to the right, turned to the right, turned to the left, stopped, and then drove in reverse. Towards what point of the compass was he then moving?

3. A strip of paper about six inches long and an inch wide was wrapped around a pencil. The paper was then removed from the pencil. When the tube was pressed flat, a piece was cut out as indicated in the sketch. Make a drawing to show the pattern seen when the paper was opened out.



4. If the paper tube were cut as shown in the second illustration, what pattern would be seen when the paper is unrolled?



5. Cards measuring 4 inches by 2 inches were arranged as shown in the sketch. The over-lap occurs along the line joining the middle points of the longer sides.

(*a*) If two more rows are to be added, how many additional cards will be needed?

(*b*) By how much will the area of the original figure be increased?

EXERCISES ON CHAPTER XI

A

1. Make a sketch of each of the following:

(*a*) an acute angle; (*b*) an obtuse angle; (*c*) two adjacent angles; (*d*) two adjacent complementary angles; (*e*) two adjacent supplementary angles; (*f*) vertically opposite angles; (*g*) alternate angles formed when a transversal meets two lines; (*h*) an exterior angle and an interior opposite angle formed when a transversal meets two lines

2. (*a*) If you double the size of an acute angle, do you necessarily obtain an obtuse angle?

(*b*) If you bisect an obtuse angle, do you necessarily obtain two acute angles?

(*c*) What is the least number of acute angles possible in a triangle?

(*d*) Can an isosceles triangle contain an obtuse angle? Give an example.

3. (*a*) What is the complement of (i) 30°, (ii) 89°, (iii) 0°, (iv) 1°, (v) $n°$, (vi) $(90-n)°$, (vii) $(m+n)°$?

(*b*) Is the complement of an angle necessarily acute?

(*c*) If the complements of the angles A and B are equal, what conclusion can be drawn?

(*d*) If the numbers of degrees in two complementary angles are $2x$ and $3x$, find x.

4. (*a*) Give the supplement of each of the following angles: (i) 30°; (ii) 89°; (iii) 0°; (iv) 1°; (v) 90°; (vi) 179°; (vii) n°; (viii) $(180-n)$°; (ix) $(A+B)$.

(*b*) Is the supplement of an obtuse angle necessarily acute?

(*c*) If the supplements of A and B are equal, what conclusion can be drawn?

(*d*) If an angle and half the angle are supplementary, what is the size of the angle?

(*e*) If the numbers of degrees in two supplementary angles are $2x-50$ and $4x-10$, find the two angles.

5. One angle in a right triangle is 22°. What is the size of the third angle?

6. In a certain right triangle, the two acute angles are equal. What is the size of each?

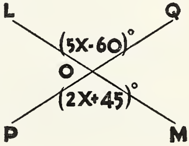
7. The angles of a triangle are $(2x+30)°$, $(5x-70)°$, and $(3x+10)°$. Find the size of each angle.

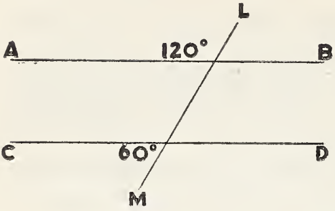
8. In a right triangle, one of the acute angles is five times the other. What is the size of each?

9. In an isosceles triangle, each of the equal angles is two and a half times the third angle. Find the size of each of the three angles.

10. An angle and its supplement are vertically opposite angles. What is the size of the angle?

11. LM and PQ are straight lines. Find (*a*) the value of x, (*b*) the size of $\angle LOP$, (*c*) the size of $\angle POM$.





12. AB and CD are straight lines and LM is a transversal. Are AB and CD parallel? Why?

B

13. (*a*) If two triangles are congruent, seven magnitudes of the one are respectively equal to seven of the other. What seven are they?

(*b*) In order to construct a triangle which is congruent with a given triangle is it essential to know the value of each of these seven features?

(*c*) What three things is it sufficient to know about a triangle in order to construct a triangle congruent with it? (You should be able to give three correct answers.)

14. (*a*) Give the converse of the following statement: If the sides of one triangle are respectively equal to the sides of another triangle, the angles of the one triangle are respectively equal to the angles of the other.

(*b*) Is the original statement true? Is its converse true?

15. The statue of a man 6 feet tall is 15 feet in height.

(*a*) What is the ratio of the height of the statue to the height of the man?

(*b*) If the nose of the man is $2\frac{1}{8}$ inches long, what is the length of the nose on the statue?

(*c*) If the middle finger on the left hand of the statue is $9\frac{3}{8}$ inches long, what is the length of the corresponding finger of the man?

(*d*) If the neck of the man is circular and is 16 inches in circumference, what is the diameter of the neck of the statue to the nearest inch? ($\pi = 3.1$).

(*e*) What is the ratio of the area of a cross-section of the ieck of the statue to the area of a cross-section of the man's neck?

16. (*a*) Construct a square equal to the sum of two given quares.

(*b*) Construct a square equal to the difference of two given quares.

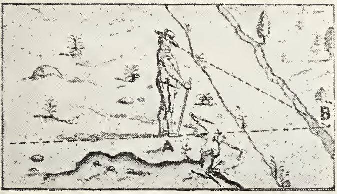
17. A pilot climbing at an angle of 45° finds that in a short ime he is vertically above a house which is three-quarters of a nile distant from his starting point. What is his altitude?

18. An aeroplane flew west from point A at 180 miles per hour or an hour and a half. Then it flew south at 150 miles per hour or two hours to a point C. If it had flown directly from A to C .t 150 miles per hour, how long would the journey have taken? Assume that there is no wind.)

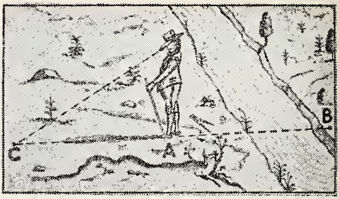
19. The following lengths are the sides of right triangles:

3, 4, 5 units
6, 8, 10 "
9, 12, 15 "
12, 16, 20 "
. . . "
. . . "

This series of lengths could be continued indefinitely. Give he two sets which should immediately follow the 12, 16, 20 set. Test your answers by using the converse of the Pythagorean heorem.



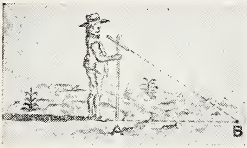
20. The illustration shows an old device for measuring a distance AB indirectly. A man pulled down the brim of his hat until he could look along its edge to the inaccessible point B. Then he turned around and, looking past the edge of the peak, observed a point C which was accessible. He measured AC and took the length as that of AB.



(*a*) What principle of congruence was used?

(*b*) What must be assumed about the levels of B, A and C? Why?

21. A variation of the preceding device will be described.



The short cross-stick is attached to the vertical staff so that it is movable. The cross-piece is adjusted until the observer can look along it and see the point B. Then, keeping the staff vertical, he turns it around and locates a point C as the man did with the brim of his hat.

How would the measurement be affected if the staff were not quite vertical when it was turned?

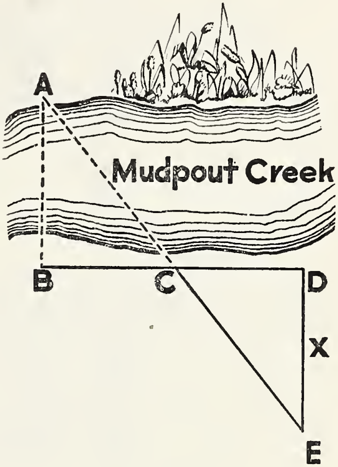
22. The angles of a triangle are $k°$, $2n°$ and $k°$. Show that $k°$ is the complement of $n°$.

23. Devise an instrument which could be used to give the difference between an angle and its supplement. Could the general instrument (page 42) be used for this purpose?

C

24. A square room of side 18 feet is 12 feet high, these measurements being correct to the nearest inch. A wire is stretched from one corner of the floor to the centre of the ceiling. What is the length of the wire to the nearest inch?

25. Ralph, who was standing at the point B wished to find he distance from A to B.

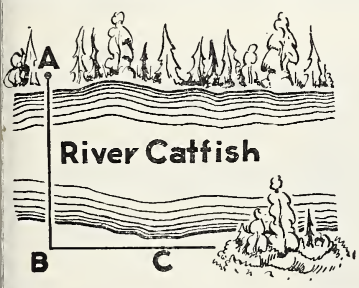


He marked out a line perpendicular to BA and on it ook a point C. Then he ound a point D on the line such that CD was equal to BC. Next, he walked along a line DX at right angles to CD until he came to a point E in line with C and A. He measured DE and took the ength as being equal to that of BA.

He gave a satisfactory explanation of his procedure.

What explanation would you give?

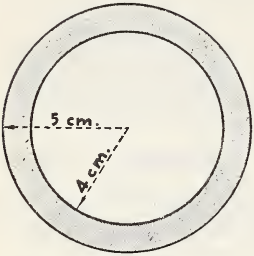
26. Margaret thought she would use Ralph's procedure in order to find the width of the lordly River Catfish.



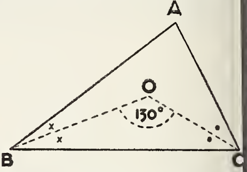
Everything went well until she came to C. Then she realized that a clump of trees prevented her from completing the plan. But she was a resourceful girl. She found the width of the Catfish by a modification of Ralph's plan. Instead of using congruent triangles, she used similar triangles. How did she obtain the distance from B to A?

27. Construct a circle equal to the sum of two circles.

28. Construct a circle equal in area to the ring.



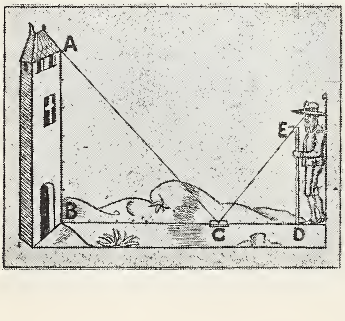
29. *BO* and *CO* are respectivel the bisectors of the angles *AB* and *ACB*. Calculate the size of th angle *A*.



30. The following are the lengths of the sides of right triangles

8, 15, 17 units
12, 35, 37 "
16, 63, 65 "
20, 99, 101 "
24, 143, 145 "
. . . "
. . . "

This series could be continued indefinitely. Give the set that immediately follows 24, 143, 145. Test the accuracy of your answer.

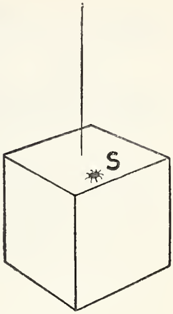


31. The illustration shows an interesting device once used in finding heights. The observer obtained from the mirror at *C* the reflection of the top of the tower. It is known that when reflection occurs the angle *ACB* is equal to the angle *ECD*. Explain how the height *AB* is calculated.

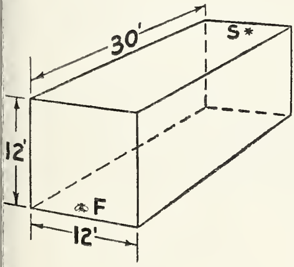
RECREATIONS

1. One day, Spiderella the mathematical ctipede was sitting on a face of a cube hich was suspended by a thread. She ished to make a tour of the six faces by he shortest route and return to her original osition?

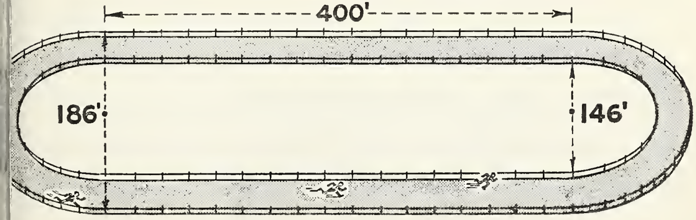
What route did she take?



2. Another time, she was on the vertical middle line of a all and one foot from the ceiling. (See illustration.) A fly was on the middle vertical line of the opposite wall and one foot from the floor. Spiderella crawled to the fly, taking the shortest route. What route was it? What was its length? This puzzle was proposed by H. E. Dudeney. You will find it and other interesting problems in "The Canterbury Puzzles" (Nelson).



3. (*a*) In a complete circuit, how much farther would be



run by the outside man than by the man in the middle of th track? ($\pi = 3.14$)

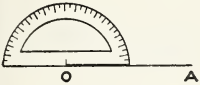
(*b*) How much farther would be run by the man in th middle of the track than by the inside man?

4. A hunter sees a huge bear about 100 yards due east of hi The hunter runs due north, picks up his gun, and kills the bea by shooting due south. Moreover, the bear did not move durin the entire proceedings. What was the colour of the bear?

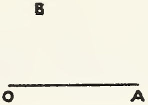
SOLUTIONS OF CERTAIN EXERCISES

Chapter I, Page 15

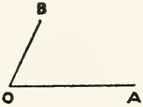
To make an angle of a given size. You are given a straight line OA, and are required to draw a line OB so that the $\angle AOB$ is 65°.



(i)



(ii)

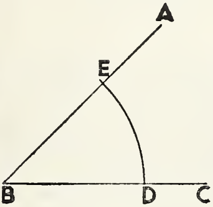


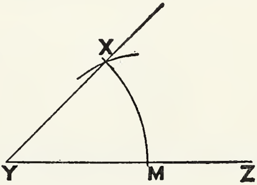
(iii)

Place your protractor accurately along OA so that the mark on the straight edge is exactly at O. (Fig. i). Using a sharp pencil, make a point at the 65° mark. (Fig. ii.) Join this point to O. (Fig. iii.)

Chapter I, Page 18

To construct an angle equal to a given angle. You are given the $\angle B$ and the straight line YZ, and are required to draw a straight line XY so that $\angle Y = \angle B$.





Open your compasses any distance BD. With B as centre, draw a circle. (You will find that you need only part of the circle. Part of a circle is called an arc.) Let this arc cut BA at E.

With centre Y and the same radius, draw an arc cutting YZ at M.

Place your compass point at D and adjust the instrumen until your pencil point is at E. With centre M and radius DE draw an arc. Let this arc intersect the other one at X. Joi XY.

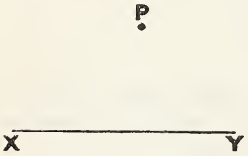
Measure the angles B and Y. Are they equal?

* * *

This operation can be performed out of doors by using a rop and pegs instead of compasses.

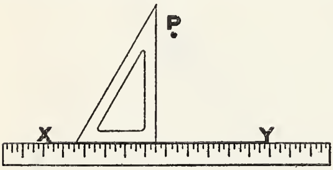
Chapter I, Page 26

With the aid of a set square, to draw a perpendicular to line from a point not on the line.



P is the given point and XY th given line. It is required to draw perpendicular from P to XY.

A ruler is laid carefully alon XY and a set square is placed s that one of the edges containin the right angle lies along the ruler.

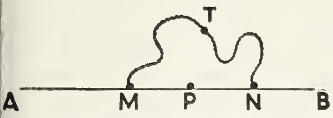


Then the set square is sli along the ruler until P i reached. A line is then draw along the edge of the se square to the ruler.

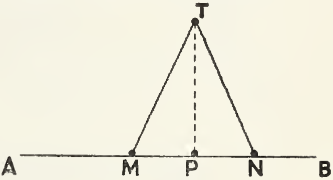
Chapter I, Page 28, Exercise 5

To obtain the perpendicular to a line on a tennis court at a given point on the line. AB is the given line and P is the point at which the perpendicular is required.

By means of the rope, equal distances PM and PN are measured off on each side of P. Pegs are placed at M and N. A loop is made at each end of the rope, and the middle point T is found. The loops are placed over the pegs and the rope, held at T, is drawn taut. A peg is inserted at the point where T meets the ground.



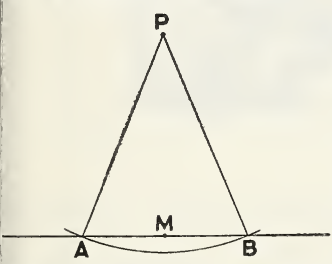
Compare the figure with the one obtained when compasses and straight edge are used to draw a perpendicular to a straight line at a point P on the line.



Chapter I, Page 32

Using rope and pegs, to obtain the perpendicular to a road from a post not on the road.

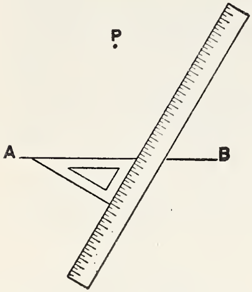
Attach the rope to a peg and drive the peg into the ground at the post. Then, using the peg and rope like a pair of compasses, obtain two points A and B on the side of the road and equidistant from the peg. Drive in pegs at A and B. Fasten the rope to the peg at A. Then, standing at B, draw the rope taut. By folding the rope, find M the middle point of AB. PM is the required perpendicular.



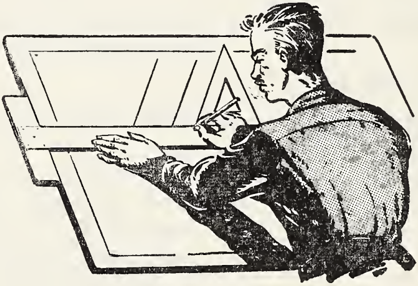
Chapter I, Page 36

To draw through a given point a parallel to a given line. P is the given point and AB is the given line. It is required to draw through P a line parallel to AB.

(*a*) **With set square**

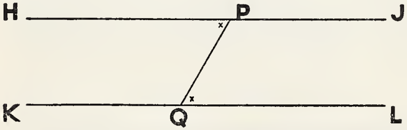


The set square is laid accuratel along AB. Then a ruler is placed i the position indicated. The nex operation is to slide the set squar along the ruler until the edge originally along AB reaches P. A lin drawn along this edge will be paralle to AB.

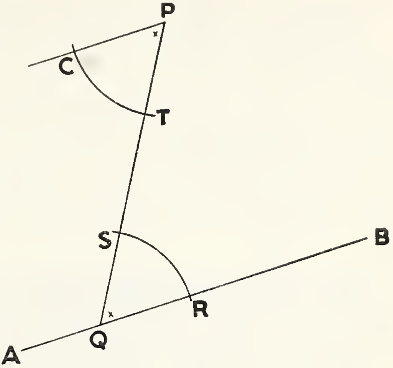


(*b*) **With compasses**

It is necessary for you to take for granted a geometrical property which will be proved later. If PQ meets the two straight lines HJ and KL making $\angle HPQ$ equal to $\angle PQL$, then HJ and KL are parallel.



Take any point Q in AB. Join PQ. At P we shall make an angle equal to $\angle PQB$. (See page 405.)

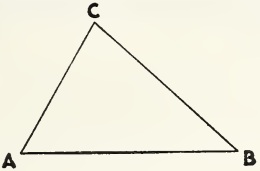


With centre Q and any radius, draw an arc cutting QB and QP at R and S respectively. With centre P and the same radius, draw an arc cutting PQ at T.

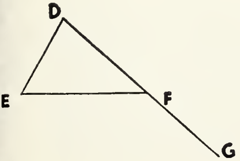
Place the compass point at R and open the instrument until the pencil is at S. With centre T and this radius draw an arc cutting the former one at C. Join CP. CP is the required parallel.

Chapter I, Page 48

A problem in measurement. The engineer went to a point C from which both A and B were visible. He measured the distances CA and CB and the angle C.



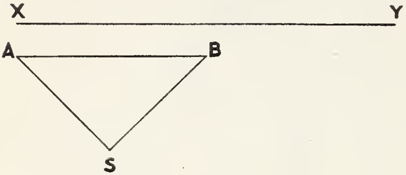
Then he chose a suitable scale and drew a line DE to represent CA. At D he drew DG making $\angle EDG$ equal to $\angle ACB$. From DG he cut off DF to represent CB on the scale chosen. The line EF represents the line AB.



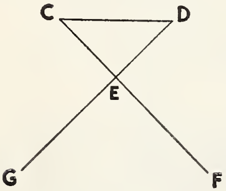
When constructing the triangle DEF, the engineer knew the lengths of two sides DE and DF, and also the size of the included angle D.

Chapter I, Page 51

Using your angle-measurer. It is required to find the distance from the shrub S to the front of the school XY. Select two points A and B from both of which S is visible and such that AB is parallel to XY. Measure the angles SAB and SBA and the length of AB.



Now draw the triangle SAB to scale. Let CD represent AB. At C make $\angle DCF$ equal to $\angle A$, and at D make $\angle CDG$ equal to $\angle B$. Let CF and DG intersect at E. Then ECD is a scale drawing of the triangle SAB.



Find the distance from E to CD, and hence obtain the distance from S to AB. If you know the distance from S to AB, how would you find that from S to XY?

ANSWERS TO EXERCISES

Page 7

11.

Actual length in feet	1	4	12	30	43	50	54	60	62.5
Scale length in inches	0.5	2	6	15	21.5	25	27	30	31.25

12.

Actual length in feet	1	2	5	27	98	100	113	169	205
Scale length in inches	0.1	0.2	0.5	2.7	9.8	10.0	11.3	16.9	20.5

Pages 11-12

1. (a) 90°, (b) 45°, (c) 180°.

2. (a) 15 minutes, (b) 3 hours.

3. (a) 720°, (b) 60°, (c) 45°, (d) 30°, (e) 1°.

4. 15°.

5. (a) 30°, (b) 180°, (c)120°, (d) 60°, (e) 150°, (f) 30°, (g) 60°, (h) 120°.

6. (a) $\frac{1}{3}$, (b) $\frac{1}{24}$, (c) $\frac{1}{16}$, (d) $\frac{1}{8}$, (e) $\frac{1}{6}$, (f) $\frac{1}{5}$, (g) $\frac{3}{10}$, (h) $\frac{3}{8}$, (i) $\frac{5}{8}$, (j) $\frac{2}{3}$, (k) $\frac{3}{4}$, (l) $\frac{5}{6}$.

7. (a) 30°, (b) 360°.

8. (a) 90°, (b) 180°, (c) 45°, (d) 45°, (e) 45°, (f) 45°, (g) 90°, (h) 180°, (i) 135°.

Pages 15-17

4. (a) 30°; (b) 95°; (c) 55°; (d) 180°; (e) 125°; (f) 150°.

5. $\angle DOC$ and $\angle COB$.

6. (b) No. If the measurement of one angle was 1° too large, and the measurement of the other 1° too small, the sum of these two measurements would be the same as that of the true measurements.

(c) Since $\angle DOB = \angle DOC + \angle COB$, the measurements, or the recording of them, must be inaccurate if the sum of $\angle DOC$ and $\angle COB$ is found to be unequal to $\angle DOB$.

9. (a) $\angle BOD - \angle BOC = \angle COD$,
(b) $\angle AOC - \angle BOC = \angle AOB$,
(c) $\angle AOD - \angle BOC = \angle AOB + \angle COD$.

10. (a) 3.9 miles, (b) $N\ 50° E$, (c) $S\ 50° W$, (d) E.

11. 3.9 bounds on bearing $S\ 38° W$ from stump.

Page 21

14. 13.1 miles; almost due east.

Pages 23-25

2. (a)The following figures exhibit symmetry about a line or lines: 1, 3, 4, 7, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 28, 30, 32, 33, 34, 35, 36, 37, 38.

3. Clockwise; counter-clockwise.

4. Yes; yes, if turned upside down.

5. *bpqd* or *dqpb*.

Pages 28-29

1. If the radius were equal to the half of MN, the circles would touch. This would make it difficult to determine the bisector. If the radius were less than the half of MN, the circles would not intersect.

3. It is perpendicular to the arms.

4. Hint: See answer to No. 3.

7. (i) Inaccuracy in the bisection.
(ii) Inaccuracy in the measurement.

8. (b) Hint: $135° = 90° + 45°$.

10. Yes.

Pages 31-32

1. Yes. The radius must be greater than half the given line.

5. Yes. The right bisector. The centre of the circle.

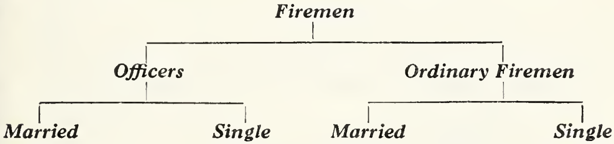
6. Use knowledge gained in doing Ex. 5.
7. Hint: Find centre of the circle.
10. The right bisector of AB.

Page 34

3. Square.

Page 38

1. The groups included under 'Firemen' should be mutually exclusive. An officer or an ordinary fireman may be married or single.
   A better classification would be



2. All birds are not included in the two classes 'Perching' and 'Insectivorous'. Moreover, the two classes are not mutually exclusive. The crested flycatcher, for example, is both perching and insectivorous.

Page 40

(a) Rash. The larch is a deciduous conifer.
(b) True.
(c) Rash. Some South American republics have been authoritarian.
(d) Rash. The pig has a cloven hoof, but does not chew the cud.
(e) If the conclusion is based on the one instance, it is rash.
(f) Rash.
(g) Rash. The four schools visited may not have been typical.

Page 44

4. 135°, 45°, 135°.
5.

	(i)	(ii)	(iii)	(iv)
(a)	Yes	Yes	Yes	Yes
(b)	No	Yes	Yes	Yes
(c)	Yes	Yes	No	No
(d)	No	Yes	No	No

Page 48

1. (a) $\angle E$, (b) $\angle D$, (c) $\angle F$.
2. (a) PQ and PR, (b) QP and QR, (c) RP and RQ.
3. $\angle X = \angle Y = 71°$.
4. $RP = PS = PT = 1.8$ in.

Pages 53-54

1. (a) $\angle P$ and $\angle Q$, (b) $\angle Q$ and $\angle R$, (c) $\angle R$ and $\angle P$.
2. Mark two points B and C from both of which a point A on the opposite bank is visible. Measure the angles ABC and ACB and the distance BC. Make a scale drawing of the triangle ABC. Find the length of the perpendicular from A to BC on this drawing. Then calculate the width of the river.
3. $\angle C = 90°$.
4. (a) $\angle A = 40°$, (b) $AB = 4.4$ in., (c) $AC = 4.4$ in.
5. (a) $\angle B = 60°$, (b) $AB = 2\frac{1}{2}$ in., (c) $BC = 2\frac{1}{2}$ in.
6. 2200 feet (approximately).

Pages 56-60

1. 4 miles to 1 inch.
2. (a) Hint: $112\frac{1}{2}° = 90° + \frac{1}{2}$ of 45°.

(b) Hint: 315° = 360° − 45°.

3. (b) It is symmetrical about both diagonals.
(c) No. (d) Yes; about *DG*.

4. It is a square.

5. About 34 ft.

6. $AD = 2.4$ in. $BD = 2.9$ in. $DC = 2.1$ in.

7. Dist. from N to $MP = 1.9$ in.
$MQ = 2\frac{1}{2}$ in., $PQ = 3$ in., $\angle MQP = 90°$.
MNPQ is a rectangle.

8. (a) $\angle YXZ = 90°$, $\angle YOZ = 135°$.

9. (b) East-West, (c) 4 in., (d) 40 ft., (e) 50 ft., (f) *S* 53° *E*, (g) Rectangle.

11. 6 inches. **12.** Equilateral.

13. (a) One if two sides are equal; three if the three sides are equal. (b) Three. (c) Two. (d) Four. (e) Two. (f) None. (g) An infinitely great number.

14. (a) Hint: $202\frac{1}{2}° = 180° + 22\frac{1}{2}°$; (b) $292\frac{1}{2}° = 270° + 22\frac{1}{2}°$.

15. 9.3 miles; 9.6 miles.

16. *ABCDEF* is a regular hexagon or six-sided figure.

17. (a) Cut out a triangular piece of cardboard with the three sides unequal. Find a line along which the cardboard can be balanced. Is this line of balance an axis of symmetry? (b) The figure is unsymmetrical.

18. (a) Hint: $11\frac{1}{4}° = \frac{1}{4}$ of 45°. (b) 405° = 360° + 45°.

19. (a) 1, (b) 3, (c) 6, (d) 10.

Pages 62-63

1. 14 ft. **2.** 5 ft. 6 in. **3.** $2m + 2n$.
4. 16 in. **5.** $4r$ ft. **6.** $4\frac{2}{3}$ ft.
7. 4k. **8.** 7. **9.** 4 in.

Page 64

1. 60. **2.** $3a + 3b + 3c$.
3. $5280m$. **4.** 4.

Pages 68-69

1. $16\frac{1}{2}$ in.
2. 6 ft. 5 in.
3. About 19 in.
4. 6.2 in.
5. About 18 ft.
6. *A* rotates half as rapidly as *B*.
7. **16.**

Pages 72-73

1. (a) 120, (b) 68 ft.
2. (a) 60 ft., (b) 225 sq. ft.
4. (a) Four times, (b) Nine times, (c) Twenty-five times.
5. (a) 18 sq. in., (b) 24 sq. in.
7. 112 sq. in.
8. (a) 27 sq. in., (b) 162 sq. in.
9. 150 sq. in.
10. 6 sq. in.

Pages 77-78

2. (a) 4.2 sq. in., (b) 4.2 sq. in.
3. (b) 4.4 sq. in.

Pages 80-81

3. $5\frac{1}{4}$ sq. in.
4. 5 sq. in.

Pages 83-84

1. 25-35 miles.
2. It should not be less than 84.5 pounds and should not be as great as 85.5 pounds.
3. It should not weigh less than 18.55 oz. and should not weigh as much as 18.65 oz.
4. (a) Not less than 36.5 inches and not as much as 37.5 inches.
(b) Not less than 5.15 feet and not as much as 5.25 feet.
(c) Not less than 8.45 inches and not as much as 8.55 inches.
(d) Not less than 18.535 yards and not as much as 18.545 yards.
(e) Not less than 13.65 miles and not as much as 13.75 miles.
(f) Not less than 5.95 inches and not as much as 6.05 inches.

Pages 85-86

1. 2. **2.** 93,000,000; the significant digits are 9 and 3.

3. 7.34; 5.84; 6.72; 5.00; 52.1; 327.

4.

3.14159	2.71828	57.3684
(a) 3.1416	(a) 2.7183	(a) 57.368
(b) 3.142	(b) 2.718	(b) 57.37
(c) 3.14	(c) 2.72	(c) 57.4
(d) 3.1	(d) 2.7	(d) 57
777.777	59.0909	8.99999
(a) 777.78	(a) 59.091	(a) 9.0000
(b) 777.8	(b) 59.09	(b) 9.000
(c) 778	(c) 59.1	(c) 9.00
(d) 780	(d) 59	(d) 9.0

Page 87

1. 3100 **2.** 2580. **3.** 470.
4. 176. **5.** 33.1. **6.** 2,800,000.

Pages 90-91

1. (a) 1200 (to two significant figures);
(b) The area would be multiplied by 4.

2. 980. **3.** (a) 3100 sq. in., (b) 3100 sq. in.

4. About 22 sq. in. **5.** About 150 sq. in.

6. (a) (i) 22 in., 22 in., 22 in., 22 in.
(ii) 21 sq. in., 30 sq. in., 28 sq. in., 39 sq. in.
(b) The circle.
(c) No. This would be a hasty generalization.

Pages 97-98 (2-figure accuracy).

1. 120 **2.** (a) 54 sq. in., (b) 27 cu. in.

3. (a) 96 sq. in., (b) 380 sq. in.; Four times; (c) 64 cu. in.; (d) 510 cu. in.; Eight times.

4. (a) 60 cu. in., (b) 120 cu. in.

5. (a) 100, (b) 6300 lb., (c) 630.

6. 96. **7.** (a) 560 cu. in., (b) 87 cu. in.
8. 370. **9.** $64.

Pages 99-100 (2-figure accuracy).

1. 580. **2.** (a) 240, (b) 320 pds.
3. 51. **4.** (a) 3000 cu. ins., (b) 900 pds.
5. 21 minutes.

Pages 101-102

1. (b) (i) 0.05 cm., (ii) 0.05 cm., (iii) 0.5 mm., (iv) 0.5 mm.
2. 25. **3.** 30. **4.** (a) 201 metres, (b) 109 yd.
5. 16.1 km. per hr. **6.** 224 miles per hr.
7. 155 cm. **8.** (a) 23, (b) 75.
9. (b) 4.9 cm. **10.** 9 miles.

Pages 103-104

1. 37 sq. cm. **2.** (a) 16 sq. cm., (b) 2.4 sq. in
3. (a) 0.75 sq. cm., (b) 0.75, (c) 190.

Pages 105-106

2. 2260 litres. **3.** (a) 10,500 cu. cm., (b) 10.5 litres.
4. (a) 0.1 cm., (b) 1 sq. cm., (c) 7 cu. cm.

Pages 107-113

1. $11\frac{1}{2}$ sq. in. **2.** (a) 11 in., (b) 6.9 sq. in.
3. (a) 226 in., (b) 24.8 sq. in. **4.** 14 sq. cm.
5. 6. **6.** (a) 12 in., (b) 11.7 in., (c) 1.0 sq. in., (d) 5.0 sq. in.
7. (a) 5 sq. cm., (b) 8.0 cm. **8.** (a) One-quarter.
9. (a) 880 yd., (b) 10,600 sq. yd. **10.** 450.
11. (a) (i) 7.55 cm., (ii) 7.65 cm., (b) (i) 9 sq. in., (ii) 9 sq. in.
12. (a) 350 sq. cm., (b) 120 cu. cm. **13.** 52 sq. cm.
14. $260. **15.** (a) 110 in., (b) 350 sq. in.
17. 391 sq. cm. **18.** 128 sq. ft. **19.** 28 cu. ft.

20. (a) 570., (b) 1700, to 2 figures.

21. (a) 66 ft. per sec., (b) 45 miles per hour.

22. Two triangles have the same area if the base of the first is half (one-quarter, one-eighth, one-sixteenth, etc.) that of the second, and if the altitude of the first is twice (four times. eight times, sixteen times, etc.) that of the second.

24. (a) (i) Circumference of first circle is twice that of second.
(ii) Circumference of first circle is three times that of second.
(iii) Circumference of first circle is four times that of second.
(b) (i) Area of first circle is four times that of second.
(ii) Area of first circle is nine times that of second.
(iii) Area of first circle is sixteen times that of second.

25. (a) (i) $12k$ cm., (ii) $8k^2$ sq. cm., (b) $9k^2$ sq. cm.

26. 21 per cent. **27.** 2 ft. **29.** 19 in.

Page 114. Recreations

1. If the surface is curved, the circumference is not equal to two π times the radius as measured along the surface.

2. Make a drawing of the Monad. Draw the diameter of the inner circle passing through the centres of the two semi-circles. These semi-circles have the same area. Hence, the two portions of the inner circle are of equal area.

3. Make a net of the cubical box. (See p. 75.)

Pages 123-124

1. $20x$. **2.** $25ab$. **3.** $30x^2y$.
4. $15k^3$. **5.** $38pqr$. **6.** $12z^3$.
7. s. **8.** $4.2mn$. **9.** $9y^2$.
10. $5r/4$. **11.** $\frac{35}{12}u^2v^2$. **12.** $9.75a$.
13. $6s$. **14.** $14x$ in. **15.** $21a$ in.
16. $162m$ rods. **17.** $14\pi r^2$ sq. in.; $22\pi r^2$ sq. in. smaller.

Pages 125-126

1. $4x$.
2. $7y^2$.
3. ab.
4. $3m^2n$.
5. $4klm$.
6. $13a^3$.
7. $4.1t$.
8. $0.08xy$.
9. p^2q^2.
10. $s/12$.
11. $16k$.
12. 0.
13. $8xyz$.
14. $7.9a$.
15. $\frac{5}{8}\,mn$.
16. Yes.
17. (i) 10; (ii) 9.
18. $7s$ in.
19. $5.5x$ ft.

Pages 127-128

1. 5.
2. 5.
3. 5.
4. 7.
5. 8.
6. 5.
7. 6.
8. 14.
9. 8.
10. 5.
11. 1.
12. 3.8.
13. 124.
14. 31.
15. 2.
16. 6.6.
17. 152.
18. 18.
19. 6.
20. 2.
21. $9.5p^2q$.
22. $3x^3$.
23. 154 sq. in.
24. 310.

Pages 130

1. $12x$.
2. $8mn$.
3. $24y$.
4. $3r^2$.
5. $4z^3$.
6. $25x^2$.
7. $3a$.
8. $39x^3y^3$.
9. $16abcd$.
10. $12.56r$.
11. $0.25m^3$.
12. $63k$.
13. pk^2r.
14. $\frac{3}{8}\,s^2t$.
15. $0.72a^2b$.
16. $hkmn/15$.
17. $70a$.
18. $6xy$.
19. $24xy$.
20. $24a^2b^2c$.
21. $4.6x^3y^3$.
22. $10pr^2$ sq. in.
23. $6x^3y$ sq. ft.
24. $24x$.
25. $10k^2m^2$.
26. $0.24a^2b$.
27. $14a^2b^2c^2$.
28. $6x^2y$ cu. in.

Pages 133-134

1. $rs + rt$.
2. $6m^2 + 2mn$.
3. $xy - xz$.
4. $20a^2 + 10ab$.
5. $6b^2 + 15b$.
6. $9a + 27b$.
7. $7a^3 + 7ab^2$.
8. $ab + a^2$.
9. $0.8a^2 - 0.2ab$.
10. 120.
11. 42.
12. $4r - 8s$.

13. $12m + 18n$. **14.** $6ab - 6ac$. **15.** $2x^3 - x^2y$.
16. $ax + ay + az$. **17.** $3km + 1.5kn$. **18.** $20a^2s + 20a^2t$.
19. $\frac{1}{12}ax^2 - \frac{1}{2}bx^2$. **20.** $7.2bm^2 - 2.4bm$. **21.** $\frac{1}{2}ha + \frac{1}{2}hb$.
22. $2ax + 4ay - 2az$. **23.** $14m^2 + 7mn + 28m$.
24. $2\pi R^2 - 2\pi r^2$. **25.** $20p^2 + 8pr$ sq. ft.
26. $3x^2 + 5x$ sq. in. **27.** $27m - rm$.
28. $32pn + 2qn$ oz. **29.** $100gx + gy$ cents.

Page 136

1. 135. **2.** $t^2 + 2t + 1$. **3.** $x^2 + 11x + 28$.
4. $xm + xn + ym + yn$. **5.** $a^2 + 10a + 16$.
6. $k^2 + 2km + m^2$. **7.** $6r^2 + 13r + 6$.
8. $9x^2 + 6x + 1$. **9.** $2a^2 + 7ab + 3b^2$.
10. $m^2 + 4mn + 4n^2$. **11.** $9a^2 + 42a + 49$.
12. $2x^2 + 11x + 15$. **13.** $3x^2 + 8xy + 4y^2$.
14. $c^2 + 5cd + 6d^2$. **15.** $b^4 + 3b^2 + 2$.
16. $s^2 + 10s + 25$. **17.** $6a^2 + 19ab + 10b^2$.
18. $4x^2 + 7xy + 3y^2$. **19.** $4k^2 + 4km + m^2$.
20. $x^2 + 6xy + 9y^2$. **21.** $9a^2 + 12ab + 4b^2$.

Pages 139

1. $26a$. **2.** $10ab$. **3.** $10x^2$.
4. $13m + 4n$. **5.** $7xy$. **6.** xyz.
7. $5xy + xz$. **8.** $9xy^2$. **9.** $3ab^2c$.
10. $6xyz$. **11.** $14mn$. **12.** $4x^2y$.
13. $10m^2 + 11mn + 3n^2$. **14.** $14r^2 + 23rs + 3s^2$.
15. $2b^2 + bc$. **16.** $34a$. **17.** $7pr + 5qr$.
18. $x^2 + 5xy$. **19.** $12am + bm$. **20.** $a^2 + b^2$.
21. $2x^2 + 5xy + 4y^2$. **22.** $2mn + n^2$.
25. $30xy$ sq. in.; $26xy$ sq. in. **26.** $13.5ab$ sq. in.

Page 142

10. 6. **11.** πr. **12.** $5ab$.
13. $4r$. **14.** $0.1s$. **15.** $10t$.
16. $2bc$. **17.** q. **18.** x^2.

19. $n/3$. **20.** 1. **21.** $0.8z$.
22. $b + c$. **23.** yz. **24.** $m - n$.
25. $3m + 2n$. **26.** $x + 2y$. **27.** $5x - 9$.
28. $3x$ ft. **29.** $(3m - 2)$ ft. **30.** $\$(2x - 1)$.
31. 12. **32.** $a - 4b$.

Review Exercises. Pages 144-146

A.

1. $3, 4a, \frac{3}{4}c^2, y, s, 0.6a^2b, 1$. **2.** (a) 1; (b) 2; (c) 3; (d) 1.
3. $11.3rs$. **4.** $9x$. **5.** $0.27t^3$.
6. $10a^2b$. **7.** $\frac{3}{4}a$. **8.** $4.5km$.
9. $2x^2$. **10.** pq. **11.** 0.
12. $7.5\,lw$. **13.** $0.5x^2y$. **14.** $4a + 5$.
15. $48k$. **16.** $8a^2b$. **17.** x^3y^3.
18. $0.15m^3$. **19.** $\frac{9}{16}x^2$. **20.** $6r + 15s$.
21. $a^2 + 9a + 14$. **22.** $x^2 + 2xy + y^2$. **23.** $2c^2 + 7cd + 3d^2$.
24. x. **25.** $5a$. **26.** $2m$.
27. $m^2 + n^2$. **28.** $x - 2y$. **29.** $3r + 1$.
30. 2. **31.** 3.

B.

32. 4. **33.** $x^2 + 5xy + 4y^2$. **34.** 0.
35. $8rs$. **36.** $13x^2y^2$. **37.** $28a^2 + 28b^2$.
38. $a^2 + 2b^2 + c^2$. **39.** 0. **40.** $15s^2 + 13rs + 8r^2$
41. $4ab$. **42.** $7x + 3y - 1$. **43.** $4x^2 + 7xy + 3y^2$.
44. $6ax + 4bx$ sq. yd. **45.** $4mn - 1$. **46.** $(x^2 - 2)$ ft.

C.

47. (a) $54s^4$ sq. in.; (b) $27s^6$ cu. in.
48. 130 sq. in. to 2 figs. **49.** (a) 8; (b) 18.
50. (a) 0; (b) -1. **51.** $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.
52. $15x^2 + 37xy + 20y^2$. **53.** $6ab$.
54. 3.6. **55.** 0.
56. $4s^2 - 2s + 6$ ft.

Recreations. Page 146

1. m^3i^2nu. (a) Nipissing; (b) Mississippi; (c) mammal; (d) remember. Mathematics assists science's progress.

Page 152

1. $x = 28$.
2. $a = 19$.
3. $m = 14$.
4. $x = 14$.
5. $x = 6$.
6. $c = 5$.
7. $k = 4$.
8. $r = 2$.
9. $y = 8$.
10. $y = 0$.
11. $x = 5.9$.
12. $s = \frac{1}{6}$.
13. $z = 3$.
14. $n = 13$.
15. $x = 27$.
16. $a = 9$.
17. $z = 20$.
18. $x = 20$.
19. $x = 4$.
20. $x = 5$.
21. $x = 30$.

Pages 156-157

1. $a = 3$.
2. $a = 30$.
3. $x = 10$.
4. $x = \frac{5}{7}$.
5. $n = 51$.
6. $x = 3$.
7. $x = 30$.
8. $r = 10$.
9. $m = 8$.
10. $a = 10$.
11. $x = 8$.
12. $a = 15$.
13. $x = 6$.
14. $P = 11$.
15. $a = 4$.
16. $x = 1.5$.
17. $k = \frac{1}{8}$.
18. $b = 1$.
19. $x = 2$.
20. $n = 0$.
21. $x = 10$.
22. $s = 0.56$.
23. $x = 6$.
24. $x = 1$.
25. $t = \frac{3}{5}$.
26. $x = 2$.
27. $x = 5$.
28. $n = \frac{5}{3}$.
29. $a = 8$.
30. $a = 12$.
31. $m = 9$.
32. $c = 6$.
33. $x = 15$.
34. $x = \frac{3}{4}$.
35. $x = 1$.
36. $a = 6$.
37. $r = 14$.
38. $a = 2$.
39. $x = \frac{3}{2}$.
40. $x = \frac{2}{3}$.
41. $n = \frac{1}{2}$.
42. $x = 10$.
43. $x = 6$.
44. $x = 49$
45. $a = 10$.
46. $x = 24$.
47. $m = 1$.
48. $y = 7$.

Page 163

1. 5.
2. 10 in.
3. 4 hours.

Review Exercises. Pages 165-168

A.

1. $x + 10$; $x - 10$; $10 - x$; $x/10$; $3x + 6$.
2. (a) Subtract 5; (b) add 5; (c) divide by 5; (d) multiplyby 5.
3. 4.1.
4. 5.2.
5. 22.5.

6. 45.
7. 56.
8. 17.
9. 96.
10. 32 ft.
11. \$25.
12. \$1.16.
13. \$875.00.

B.

14. $16\frac{2}{97}$.
15. $10\frac{2}{7}$ hours.
16. 25°, 50°, 105°.
17. 72.
18. 13 ft., 5 ft.
19. 90 rods.
20. 8 ft.
21. 10 in.
22. 36; 37.
23. 9; 52.
24. 6 in., 36 in.
25. \$137.50 to Bill; \$275.00 to Tom.
26. \$22.50.
27. 34.
28. 6.5 miles.
29. 9 in.
30. 15 min.
31. 14.

C.

32. 19 quarters; 8 dimes.
33. 11 in.
34. 1 hour and 50 min.
35. 256, 257, 258.
36. 1:30 p.m.
37. 4 hrs.
38. 4:10$\frac{1}{2}$ p.m.
39. 20 lb. at 90c.; 30 lb. at 75c.
40. $21\frac{9}{11}$.
41. 8, 24, 40.

Recreations. Page 169

1. 18 years.
2. \$1.02$\frac{1}{2}$.
3. 3:50 p.m.
4. Yes; 4, 4, 4, 16 (gets three \$4 nuggets back), 4, 4, 4, 32 (gets three \$4 nuggets and one \$16 nugget back), etc.
5. 1, 3, 9, 27.

Pages 174-175

1. $V = \frac{1}{3}\pi r^2 h$; radius and height measured in in. or cm., volume given in cu. in. or cu. cm., respectively.
2. $A = \pi(R^2 - r^2)$; radii in in. or cm. gives area in sq. in. or sq. cm., respectively.
3. $V = lwh$; length, width and height in in. or cm. gives volume in cu. in. or cu. cm., respectively.
4. $A = 4\pi r^2$; radius in in. or cm. gives area in sq. in. or sq. cm., respectively.
5. $d = 16t^2$.
6. $A = \frac{1}{2}bh$.
7. $c = \pi d$.

8. $V = 4\pi r^3/3$.
9. $V = \pi r^2 h$.
10. $A = 2\pi rl$.
11. $N = mdh$.
12. $A = s/n$.
13. $\frac{5}{9}(F - 32) = C$.
14. $d = rt$; units must be consistent, e.g. distance in *miles* equals rate in *miles per hour* times the time in *hours*.

Pages 178-180

1. $c = 15n$.
2. $m = d + 20$.
3. $d = 5r$.
5. $i = 12f$.
7. $n = m^2$.
8. $m = 6.3q$.

Page 182

1. $A = a^2(4 - \pi)$; $B = 2\pi a$.
2. $A = xy - 2ab$; $B = 2(x + y + 2a + 2b)$.
3. $A = lw - ab - s^2$; $B = 2(l + w + b + 2s)$.
4. $A = 2\pi r^2$; $B = 8\pi r$.
5. $A = \pi(R^2 - r^2)$; $B = 2\pi(R + r)$.
6. $A = \pi(R^2 - r^2)$; $B = 2\pi(R + r)$.
7. $A = a^2(1 + \frac{\pi}{4})$; $B = a(2 + \pi)$.
8. $A = r^2(4 - \pi)$; $B = r(8 + 2\pi)$.
9. $A = 2ab + ah - s^2$; $B = 2(a + b + s + k)$.

Pages 183-184

1. $V = 1540$.
2. $A = 22$.
3. $d = 400$.
4. $F = 212$.
5. $C = 100$.
6. $s = 2870$.
7. $K = 600$.
8. $V = 38{,}808$.
9. $A = 105$.
10. $y = 16$.
11. $l = 3/2$.

Page 186

1. 7.
2. 196.
3. 5.
4. 50.
5. 2.
6. 3.5 in.
7. 7 in.
8. 6.

Page 190

1. The length is (a) doubled, (b) halved, (c) increased 5-fold, (d) quartered.

2. The area is (a) increased 4-fold, (b) quartered, (c) increase 9-fold, (d) reduced to $\frac{1}{9}$.
3. The volume is (a) increased 8-fold, (b) reduced to $\frac{1}{8}$ (c) increased 27-fold.

Pages 192-193

1. C is (a) doubled, (b) tripled, (c) increased 6-fold, (d) halved
2. d is (a) increased 9-fold, (b) quartered, (c) increased 4-fold
3. W is (a) halved, (b) reduced to $\frac{1}{3}$, (c) increased 4-fold.
4. Pressure is (a) reduced to $\frac{1}{5}$, (b) doubled, (c) increased 4-fold (d) reduced to $\frac{1}{3}$.
5. T is (a) increased 4-fold, (b) doubled, (c) halved.

Review Exercises. Pages 196-200

A.

1. About 15,700 miles. 2. n is increased 4-fold; 22,000
3. $w = 6.5n$. 4. 143 lb.
5. 88 lb. 6. 21 ft.
7. About 5540 sq. ft. 8. $z = (P + a)/d$.
9. 20 lb. per sq. ft. 10. $y = x + 2.5$.
11. 6400 ft. 12. 1500 bu. of corn; 3000 bu. of wheat.
13. 400 m. 14. The number is tripled; 7200
15. 5 in. in circumference.

B.

17. $A = a^2 - 2s^2$; $B = 4(a + h - 2s)$.
18. 14 in. 19. 2 cents.
21. (a) $A = 6x^2$; (b) $V = x^3$. 22. 5 in.
23. $A = \pi(R^2 - 4r^2)$. 24. About 170 ft.
25. About 121. 26. $c = 5q + 10$.
28. 8 in.

C.

30. $V = 4\pi(R^3 - r^3)/3$; about 260 cu. in.
31. $V = \pi h(R^2 - r^2)$.
32. Hint: Enquire about the value of c.

33. d is (a) increased 16-fold, (b) increased 81-fold, (c) reduced to $\frac{1}{16}$. Energy is increased to $\frac{81}{16}$ of its original value.
34. About 114. 35. 0.0216 ft.; 0.72 ft.

Recreations. Pages 200-201

1. $\frac{2n+6}{2} - n = 3.$ 2. Yes, it would do nicely. (Note that $c = 6r$ approximately.)

Page 207

6. 123. 7. 398. 8. 754.
9. (a) 86,950; (b) 216,000; (c) 78,000; (d) 15,625.
10. 6450 ft. 11. (a) 1243 gals.; (b) 18 hours.

Page 208

1. 2, 3, 4; 2, 3, 6, 9. 2. 18. 3. 3.
4. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60.
5. Odd numbers. 6. Alternately odd and even.

Pages 209-210

1. The only prime even number is 2.
2. 9. 3. 2 and 5.
4. 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.
5. 51, 52, 54, 55, 56, 57, 58.
6. $18 = 2 \times 3 \times 3$; $24 = 2 \times 2 \times 2 \times 3$; $33 = 3 \times 11$; $56 = 2 \times 2 \times 2 \times 7$; $84 = 2 \times 2 \times 3 \times 7$; $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$.
7. Seven are prime, three composite.
8. $2 = 1 + 1$; $4 = 3 + 1$; $6 = 5 + 1$; $8 = 7 + 1$; $10 = 7 + 3$; $12 = 11 + 1$; $14 = 13 + 1$; $16 = 13 + 3$; $18 = 17 + 1$; $20 = 19 + 1$.

Page 212

1. $\underline{5}$. 2. 2, 4, $\underline{8}$. 3. 2, 5, $\underline{10}$.
4. $\underline{2}$. 5. 2, 3, 4, 6, $\underline{12}$. 6. 2, 7, $\underline{14}$.

7. $\underline{2}$.
8. $\underline{3}$.
9. 2, 5, $\underline{10}$.
10. 2, 3, $\underline{6}$.
11. 2, $\underline{4}$.
12. 3, 5, $\underline{15}$.
13. $\underline{63}$, 126.
14. $\underline{15}$, 30.
15. $\underline{40}$, 80.
16. $\underline{48}$, 96.
17. $\underline{75}$, 150.
18. $\underline{96}$, 192.
19. $\underline{66}$, 132.
20. $\underline{90}$, 180.
21. $\underline{120}$, 240.
22. $\underline{12}$, 24.
23. $\underline{36}$, 72.
24. $\underline{90}$, 180.

Pages 213

1. (a) 2500, (b) 64, (c) 81, (d) 1.
2. (a) 128, (b) 625, (c) 59,049, (d) 1,000,000.
3. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256 289, 324, 361, 400.
4. 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.
5. (a) 10,368, (b) 2, (c) 1266, (d) 100.
6. 16; 13 digits.

Page 215

1. 9, 16, 25, 36, 49, 64, 81, 100.
2. 13, 16, 17, 20, 60, 1000.
3. 6, 9, 20, 300.
4. 2, 3, 1, 10.
5. (a) 7 and 8, (b) 8 and 9, (c) 9 and 10, (d) 12 and 13, (e) 14 an 15, (f) 20 and 21, (g) 2 and 3, (h) 3 and 4, (i) 4 and 5 (j) 4 and 5, (k) 6 and 7, (l) 10 and 11.

Review Exercises. Pages 215-217

A.

3. (a) $2 \times 2 \times 3 \times 5$, (b) $2 \times 2 \times 2 \times 7$,
(c) $2 \times 2 \times 2 \times 2 \times 5$, (d) $2 \times 3 \times 5 \times 11$,
(e) $2 \times 2 \times 3 \times 3 \times 3$, (f) $2 \times 2 \times 5 \times 5 \times 7$.
4. (a) 2, 5, $\underline{10}$, (b) $\underline{3}$, (c) $\underline{11}$, (d) 2, $\underline{4}$, (e) $\underline{3}$, (f) 2, $\underline{4}$, (g) 5, $\underline{25}$, (h) 2, 3, $\underline{6}$, (i) 2, 3, 4, 6, $\underline{12}$.
5. (a) $\underline{21}$, 42, (b) $\underline{12}$, 24, (c) $\underline{66}$, 132, (d) $\underline{72}$, 144, (e) $\underline{60}$, 120, (f) $\underline{24}$, 48, (g) $\underline{60}$, 120, (h) $\underline{45}$, 90, (i) $\underline{105}$, 210.
6. 50c.
7. 72 inches.

8. (a) 7, 17, 29; (b) 6, 8, 9, 15, 16, 24, 25, 27;
(c) 7, 9, 15, 17, 25, 27, 29; (d) 6, 9, 15, 24, 27;
(e 9, 16, 25; (f) 8, 27.

11. (a) 3000, (b) 20,000, (c) 10,000, (d) 5000, (e) 60, (f) 400, (g) 100, (h) 7000.

B.

12. $1024 = 4^5$.   13. No.

16. Append two zeros after the right-hand digit and divide by 8. $87{,}200 \div 8 = 10{,}900$.

18. 704, 536, 3364.

19. (a) 8217, (b) 2574, (c) 41,580, (d) 1568.

C.

22. 1.   23. (a) 18, (b) 21, (c) 6.

24. (a) 4620, (b) 44,919, (c) 15,912.

27. The numbers are equal.

Recreations. Page 218

3. 1, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, 41, 43, 47, 49.

4. Since $37 \times 3 = 111$ therefore $37 \times 24 = 37 \times 3 \times 8 = 888$, etc.

Page 224

1. $\frac{4}{9} = \frac{12}{27} = \frac{32}{72} = \frac{20}{45} = \frac{240}{540}$.
2. $\frac{3}{16} = \frac{6}{32} = \frac{18}{96} = \frac{12}{64} = \frac{21}{112}$.
3. $\frac{4}{9}$.   4. $\frac{2}{5}$.   5. $\frac{1}{4}$.   6. $\frac{2}{5}$.
7. $\frac{9}{10}$.   8. $\frac{3}{4}$.   9. $\frac{3}{4}$.   10. $\frac{7}{9}$.
11. $\frac{7}{8}$.   12. $\frac{7}{11}$.

Pages 226-227

1. 1.   2. $\frac{3}{4}$.   3. $\frac{7}{10}$.   4. $\frac{1}{4}$.
5. $\frac{5}{6}$.   6. $\frac{5}{12}$.   7. $\frac{15}{56}$.   8. $\frac{1}{15}$.
9. $\frac{19}{12}$.   10. $5\frac{2}{3}$.   11. $3\frac{5}{7}$.   12. $3\frac{1}{10}$.
13. $4\frac{10}{11}$.   14. $5\frac{1}{15}$.   15. $2\frac{15}{16}$.   16. $\frac{20}{3}$.
17. $\frac{49}{9}$.   18. $\frac{47}{4}$.   19. $\frac{43}{12}$.   20. $\frac{146}{9}$.

21. $\frac{3}{20}$. 22. $\frac{13}{72}$. 23. $\frac{19}{60}$. 24. $\frac{5}{14}$.
25. $\frac{2}{245}$. 26. $\frac{2}{189}$. 27. $\frac{31}{252}$. 28. $\frac{7}{450}$.
29. $\frac{13}{12}$. 30. $\frac{1}{10}$. 31. $\frac{7}{12}$. 32. $\frac{89}{90}$.
33. $\frac{25}{168}$. 34. $\frac{31}{504}$. 35. $\frac{13}{60}$. 36. $\frac{23}{240}$.

Page 230

1. $\frac{3}{7}$. 2. $\frac{4}{9}$. 3. $\frac{2}{3}$. 4. $\frac{4}{3}$.
5. $\frac{15}{2}$. 6. $\frac{1}{20}$. 7. $\frac{15}{56}$. 8. $\frac{5}{9}$.
9. $\frac{2}{3}$. 10. \$1.10. 11. \$10. 12. 15c.
13. 60c. 14. 44c. 15. 63c. 16. $\frac{3}{10}$.
17. $15\frac{5}{7}$. 18. $7\frac{7}{12}$. 19. 24. 20. $\frac{1}{3}$.
21. $\frac{7}{55}$. 22. $\frac{1}{5}$. 23. $\frac{35}{72}$. 24. $\frac{1}{6}$.
25. \$1.24. 26. \$13.30. 27. $90\frac{2}{9}$ cu ft.. 28. $\frac{8}{35}$.
29. $\frac{17}{25}$.

Page 232

1. 8, $\frac{1}{21}$, $\frac{6}{5}$, $\frac{2}{7}$, $\frac{5}{42}$. 2. $\frac{2}{7}$. 3. $\frac{3}{16}$.
4. $\frac{5}{16}$. 5. $\frac{4}{27}$. 6. $\frac{1}{32}$. 7. $\frac{2}{39}$.
8. 10. 9. 9. 10. 24. 11. $\frac{3}{2}$.
12. $\frac{4}{5}$. 13. $\frac{5}{4}$. 14. 3. 15. $\frac{5}{6}$.
16. $\frac{2}{3}$. 17. 30. 18. 21. 19. 96.
20. $17\frac{37}{64}$. 21. $5\frac{1}{5}$.

Page 234

1. $\frac{49}{2}$. 2. $\frac{1}{2}$. 3. $\frac{2}{11}$. 4. $\frac{18}{11}$.
5. $\frac{3}{2}$. 6. $\frac{1}{2}$. 7. $\frac{3}{2}$. 8. 6.
9. $\frac{9}{14}$. 10. $\frac{8}{15}$. 11. $\frac{35}{24}$. 12. $\frac{5}{4}$.
13. $\frac{21}{16}$. 14. $\frac{42}{17}$. 15. $\frac{9}{7}$. 16. $\frac{1}{24}$.
17. $\frac{9}{10}$. 18. $\frac{4}{39}$. 19. $\frac{85}{78}$. 20. $\frac{1}{159}$.
21. $\frac{5}{6}$; $\frac{6}{5}$; $\frac{3}{10}$; $\frac{10}{3}$; $\frac{8}{15}$; $\frac{15}{8}$.

Page 236

1. 67.25. 2. 52.26. 3. 8.94. 4. 0.074016.
5. \$3.77. 6. 368.06. 7. 24.934. 8. 4.0894.
9. 1.8659. 10. 3.499.

Page 239

4. 191.66. **5.** 0.000,472,12. **6.** 0.000,000,000,1.
7. 0.6561. **8.** 1.092,727. **9.** 6.98.
10. 54.7. **11.** 0.26. **12.** 0.028.
13. 0.296,65. **14.** 11.4. **15.** 156.052,508.
16. 4790.

Page 241

1. 0.15. **2.** $0.\dot{4}$. **3.** $0.58\dot{3}$. **4.** 0.3125.
5. $0.\dot{4}2857\dot{1}$. **6.** $0.\dot{1}\dot{8}$. **7.** $0.4\dot{6}$. **8.** 0.768.
9. Each period has the same digits in the same circular order.

Pages 242-243

1. 0.30 in. **2.** 5.20.
3. (a) three, (b) two, (c) two, (d) one, (e) three.
4. (a) 41.5, (b) 2.3, (c) 0.024.
5. (a) 26.6, (b) 0.78, (c) 1.5.

Review Exercises. Pages 243-246

A.

1. (a) $\frac{3}{10}$, (b) $\frac{2}{9}$, (c) $\frac{5}{8}$, (d) $\frac{11}{16}$, (e) $\frac{4}{9}$, (f) $\frac{5}{11}$.
2. $\frac{7}{12} = \frac{91}{156} = \frac{84}{144} = \frac{175}{300} = \frac{0.007}{0.012} = \frac{4.9}{8.4} = \frac{6.3}{10.8}$.
3. (a) 0.46, (b) 1.375, (c) 0.452, (d) 0.136, (e) 0.4832.
4. (a) 1, (b) 2.
5. (a) 0.81, (b) 0.49, (c) 1.44, (d) 6.25, (e) 0.0121, (f) 1.0201, (g) 0.0009, (h) 0.000,025, (i) $2\frac{1}{4}$, (j) $6\frac{1}{4}$, (k) $11\frac{1}{9}$, (l) $1\frac{24}{25}$, (m) $4\frac{76}{81}$.
6. (a) 0.001, (b) 0.027, (c) 0.125, (d) 0.000,008, (e) 0.000,000,064, (f) 1.331, (g) $3\frac{3}{8}$, (h) $2\frac{10}{27}$.
7. (a) After the fifth digit; (b) After the second digit; (c) Two places to the left of the first significant digit; (d) After the

second digit; (e) One place to the left of the first significan digit; (f) After the first digit.

8. (a), (c), (f) give terminating decimals; (b), (d), (e) giv repeating decimals.
9. Numbers greater than one are increased; numbers less tha one are decreased.
10. One third; Two thirds. 11. 0.2 miles.

B.

12. (a) $7\frac{7}{30}$, (b) $1\frac{11}{12}$, (c) $7\frac{1}{8}$, (d) $25\frac{15}{36}$.
13. (a) $\frac{5}{7}$, (b) $\frac{3}{7}$, (c) $\frac{5}{8}$, (d) $\frac{11}{23}$, (e) $\frac{19}{23}$.
14. (a) 14, (b) $1\frac{99}{245}$, (c) $\frac{7}{22}$.
15. 0.701 gm. 16. 142 million miles.
17. 2490 cu. in. (assuming 3-figure accuracy in data).
18. (a) 181, (b) 0.0285, (c) 0.0101, (d) 240,000,000, correct to figs.
19. (a) 7.74, (b) 14.2, (c) 0.21, (d) 1.10.

C.

20. A fraction less than 1 is increased; a fraction greater tha 1 is decreased.
21. (a) No effect, (b) fraction increased, (c) fraction increased (d) fraction increased if $m < n$, decreased if $m > n$, (e fraction decreased if $m < n$, increased if $m > n$.
22. 1.1111. 23. $0.\dot{0}58,823,529,411,764,\dot{7}$.
24. (a) $\frac{127}{300}$, (b) $\frac{71}{330}$, (c) $\frac{497}{1100}$.

Pages 249-251

1. A deposit of $5; (a) -20, (b) $+8$, (c) $-b$, (d) $+15 - 18$.
2. -8; (a) -4.5, (b) 12, (c) -10, (d) 18, (e) 0.
3. B, $+0.75$; C, -8; D, $+2$; E, -1.50; F, -2.25; G, $+3.20$; H, -0.50.
5. (a) $+50$; (b) $+85$; (c) -10; etc.
6. Use positive numbers to denote income, and negative numbers to denote expenses. Etc.

Pages 257-258

1. +8. **2.** +4. **3.** −1. **4.** −6.
5. +7. **6.** −4. **7.** 0. **8.** −2.0.
9. $+1\frac{1}{2}$. **10.** −10.1. **11.** −11. **12.** +25.
13. +100. **14.** +2.4. **15.** $+\frac{1}{6}$. **16.** $-3\frac{1}{8}$.
17. −1.0. **18.** 0. **19.** $-a$. **20.** $-3a$.
21. a. **22.** +13. **23.** +6. **24.** −15.
25. −40. **26.** −6.0. **27.** +9.6. **28.** +6.5.
29. 0. **30.** −0.1. **31.** +10. **32.** −10.
33. 0. **34.** −13. **35.** −2.3. **36.** −18.
37. −29. **38.** +43. **39.** −1.7.
40. +15, +5, +23, −8, 0, +50, −30, +7, −15, +10.
41. 0, −10, +5, +30, −25, −6, +12, +4, −8.2, +1.1, −5.

Page 261

1. +4. **2.** +8. **3.** +17. **4.** −6.
5. +7. **6.** −2. **7.** −10. **8.** −3.
9. +7. **10.** −2.7. **11.** −57. **12.** +95.
3. +12. **14.** +17.4. **15.** $+\frac{5}{6}$. **16.** $-13\frac{5}{8}$.
7. −0.2. **18.** +9.4. **19.** $-3a$. **20.** $-a$.
21. $+3a$. **22.** +5. **23.** +24. **24.** −1.
25. −100. **26.** −3.0. **27.** +2.8. **28.** +9.3.
29. 0.9. **30.** +6.3. **31.** −2°. **32.** −10°.
33. +50°. **34.** Leaves a debt of $4.
35. (a) 100 ft. below sea level; (b) 8° above zero; etc.

Page 264

1. +15. **2.** −15. **3.** −15. **4.** +15.
5. 0. **6.** −48. **7.** −45. **8.** 0.
9. −5. **10.** +100. **11.** −13.5. **12.** +24.
3. −30. **14.** −56. **15.** +12. **16.** +3.
7. −60. **18.** −18. **19.** +70. **20.** −180.

Page 266

1. $+5$. **2.** -5. **3.** -5. **4.** -3.
5. -3. **6.** $+3$. **7.** 3.5. **8.** -0.7.
9. -0.6. **10.** -3. **11.** -7. **12.** -4.
13. -4. **14.** $+20$. **15.** -15. **16.** $+5$.
17. -2. **18.** -3. **19.** 2. **20.** -3.
21. -3.

Pages 268-269

1. $-a$. **2.** $+2x^2$. **3.** $-13xy$. **4.** $-3km$.
5. $-10r$. **6.** $+2a^2b$. **7.** $+3n$. **8.** $+17n$.
9. $-5m^3$. **10.** $+5xyz$. **11.** $-6a^2$. **12.** $-7t$.
13. $-6ab$. **14.** $-20k^2$. **15.** $+49x^3$. **16.** $-4a^3b^3$.
17. $-48a^2bc + 24ab^2c$. **18.** $-15x^2 + 18xy$.
19. $-3x$. **20.** $-2ac$. **21.** $+9k$. **22.** $-0.5mn$.
23. -4. **24.** $+c^2d^2$. **25.** $-5x^2$. **26.** 0.
27. $+6b$. **28.** $-x^2y$.

Page 270

1. $-6x - 6y$. **2.** $-2m + 2n$. **3.** $12a + 12b$.
4. $-6a^2 + 3ab$. **5.** $28x^2 - 7xy$. **6.** $-km - kn$.
7. $-7 + 3b$. **8.** $-a - b$. **9.** $-6y^2 - 7y$.
10. $-3x - 15xy$. **11.** $3a^2 + 9ab$. **12.** $15kn$.
13. $23r^3 - 57r$. **14.** $-xy$. **15.** $4ab - 16ac$.

Review Exercises. Pages 272-275

A.

1. -5. **2.** $-12a$. **3.** $3z^2$.
4. -6. **5.** $+100$. **6.** $-12b^2c$.
7. $-28x^3y^3$. **8.** $+2m^3$. **9.** $-8a^2 + 3ab$.
10. $5z$. **11.** $-2t^2$. **12.** $-3b^2$.
13. $+14x + 6$. **14.** $-2ab + 3a^2 + 12b^2$.
15. $-5b^2$. **16.** 0. **17.** $-29mn$.
18. $+21x^2y^2z^2$. **19.** $-3x^2 + 8xy - 11y^2$.
20. $+3b$. **21.** $-7ab + bc - 5ca$.

22. $m^2 - 3$.
23. $-3b$.
24. 15.
25. $2x^2 - 10x + 13$.
26. $a^2 - 18ab$.
27. $-20x^2 + 16x - 6$.
28. $25pq^2 - 3pr$.

B.

29. 55 B.C.; 75.
30. 77.
31. Withdrawal of $1.
32. Gain of 14 yd.
33. +350 lb.; +100 lb.
34. 25 knots south; +105 knots; −55 knots.
35. 3 floors above.
36. A gain of $100.
37. (a) +5°C; (b) −5°C; (c) +30°C.
38. $370 credit.
39. 4.5 m.p.h. east.
40. Casey wins by 5411 votes.

Recreations. Page 275

1. +1; 28.
2. −50.
3. (a) 0; (b) +1.

Page 303

1. (a) 0; (b) −1; (c) +3.
2. 1.
3. 18.
4. −204.
5. 0.
6. 13.
7. $-1\frac{7}{20}$.
8. $-\frac{1}{4}$.
9. $\frac{1}{16}$.
10. 0.
11. 12.
12. 32.
13. −10°C.
14. −328°F.

Pages 305-306

1. $3(a + 2b - 3c)$.
2. $a(2 - 3b)$.
3. $x(x - y + z)$.
4. $PR(WO + CM)$.
5. $NR^2(UA + CV)$.
6. $3mn(2n + 5m)$.
7. $abc(a - b + c)$.
8. $t(2u + at)$.
9. $2\pi r(2r - h)$.
10. $rah(1 + rah + r^2a^2h^2)$.
11. $6toa(tom + 2cr^2)$.
12. $k^2l^3m(kl - lm - km)$.
13. $2xyz^2(2yz - 3x + 4z)$.
14. $5\pi(r^2 + 2s^2)$.
15. $ab(c + a + b)$.
16. $4\pi r^2(r + h)$.

Page 311-312

1. $3a$; $18a^3$.
2. $2x^2$; $12x^6$.
3. ab; $6a^2bc^2$.
4. xy^2; x^3y^3.
5. $2m$; $120m^2n^3$.
6. CA; RCA^2FOW.
7. $2a/b$.
8. $rh/2$.
9. $7xy$.

10. $rt^2/s.$

11. $\dfrac{x-a}{2}.$

12. $\dfrac{a}{a+2}.$

13. $\dfrac{x+2}{2x}.$

14. $r/s.$

15. $x/15.$

16. $5/6y.$

17. $\dfrac{z-2}{2z^2}.$

18. $1/ab.$

19. $\dfrac{h^2+k^2}{hk}.$

20. $\dfrac{2n-3m}{mn^2}.$

21. $\dfrac{x-2}{x(x+1)}.$

22. $\dfrac{b^2+a^2+2}{ab}.$

23. $\dfrac{tr-us}{r^2s^2}.$

24. $\dfrac{q^2+p^2-2pq}{p^2q^2}.$

Page 314-315

1. $a^2.$

2. $xy.$

3. $6pq/r^2$

4. $3ab^2/2c.$

5. 1.

6. $RSVP.$

7. $\dfrac{a^2}{(a+1)^2}.$

8. $2a.$

9. $2x/yz.$

10. $r^2s^2t^2.$

11. $mn.$

12. $xyz/abc.$

13. (a) a^3/b^3; (b) $x^4/16$; (c) $4r^2/9s^2$.

14. (a) $2/3$; (b) a/b; (c) a/a; (d) bx/ay.

15. (a) $a/3$; (b) $a^2/9$; (c) $1/9$; (d) $2a^2/9b$.

Page 317

1. $x = -3.$

2. $x = 5.$

3. $y = -6.$

4. $x = 24.$

5. $x = \frac{5}{2}.$

6. $r = 60.$

7. $y = 7.$

8. $x = -5.$

9. $z = \frac{2}{3}.$

10. $x = -7.$

11. $-7.$

Review Exercises. Pages 318-321

A.

1. $-9.$

2. 10.

3. $2xy(3x+y-2xy).$

4. $3a^2b^3c^2(c^2-2a^2).$

5. $47/12x.$

6. $\dfrac{c+a-b}{abc}.$

7. $5bc/a.$

8. $m.$

9. $2ax/b.$

10. $2/z$. **11.** -5. **12.** -1.

13. 0. **14.** 1200. **15.** $6k^2m^2n(2kn + 5m^2 - 3mn)$.

16. -15°C. **17.** $\frac{3b + 4ac}{ab}$. **18.** $\frac{y^2 - x^2}{xy}$.

19. $\frac{4 - 5mn}{m^2}$. **20.** 1. **21.** $\frac{3}{2}$.

22. -3. **23.** -2.

B.

24. -10. **25.** -2. **26.** $F = \frac{5}{9}C + 32$; -4°F.

27. $55/3x$. **28.** $1/m$; $3/2m$. **29.** $1000/3x$; $1750/3x$.

30. $47/6r$ hr. **31.** $mnk/2$. **32.** $4xy$ ft.

33. $2 loss.

C.

34. 1. **35.** -4. **36.** -3.

37. -5°F. **38.** 11 a.m.

39. $x = -25$; a submarine.

Recreations. Page 322

1. $20\frac{5}{11}$.
2. 20 m.p.h. (Hint: Consider plane and balloon relative to the air).
3. 20 ft.
4. Hint: Watch Stanley's division.

Page 325

1. 7%. **2.** 12%. **3.** $2\frac{1}{2}$%. **4.** $1\frac{1}{4}$%.

5. $8\frac{3}{5}$%. **6.** $24\frac{1}{2}$%. **7.** 150%. **8.** $62\frac{4}{5}$%.

9. $10\frac{1}{2}$%. **10.** 0.09. **11.** 0.045. **12.** 0.05.

13. 0.0625. **14.** 0.32. **15.** 0.032. **16.** 0.24.

17. 0.024. **18.** 0.108. **19.** 0.2. **20.** 0.33.

21. 0.18. **22.** 1.5. **23.** 0.99. **24.** 0.0025.

25. 0.004. **26.** 30%. **27.** 125%. **28.** $87\frac{1}{2}$%.

29. $66\frac{2}{3}$%. **30.** $83\frac{1}{3}$%. **31.** $56\frac{1}{4}$%. **32.** $9\frac{3}{8}$%.

33. $233\frac{1}{3}$%. **34.** 85%.

Pages 327-329

1. 3.5. **2.** 145. **3.** 192. **4.** 3.72.
5. 12. **6.** 4.3. **7.** $33\frac{1}{3}\%$. **8.** 20%.
9. 95%. **10.** $12\frac{1}{2}\%$. **11.** 170. **12.** 600.
13. 600. **14.** 108. **15.** 1250. **16.** 108.75.
17. 50.4. **18.** 4.32. **19.** 1.5408. **20.** 40.
21. 37.5. **22.** 7.5. **23.** 105. **24.** 740.
25. 1230. **26.** The second but not the first. **27.** 30%.
28. Agriculture 9.5%; mining 1.7%; manufacturing 32.4%.
29. \$8000. **30.** About 24.5 lb. **31.** \$43.15.

Pages 333-335

1. \$24.36. **2.** \$2.25. **3.** \$10.20. **4.** \$3.40.
5. $\$6n$. **6.** $\$0.12k$. **7.** $\$0.02xy$. **8.** 4%.
9. $3\frac{1}{2}\%$. **10.** 5%. **11.** $2\frac{1}{2}\%$. **12.** 10%.
13. $6\frac{1}{4}\%$. **14.** $2x\%$. **15.** \$10,000. **16.** \$2560.
17. \$8000. **18.** \$366. **19.** $\$33\frac{1}{3}k$. **20.** $\$50x/n$.
21. \$691.20. **22.** \$6916. **23.** 5%. **24.** \$1.44.
25. (a) \$68, (b) \$68.40, (c) \$68.40. **26.** \$19; 19%.
27. 14.5%.

Pages 338-339

1. \$36 before; \$63 after. **2.** \$140.25.
3. \$138.84. **4.** \$646.08. **5.** \$120; equal. **6.** \$249,900.
7. \$190.50; \$215.90. **8.** 32 mills. **9.** \$1,845,000.
10. \$375,525; 38 mills; \$16,000,000.

Pages 342-343

1. \$21.60. **2.** \$11.40. **3.** \$11.80.
4. \$27.50; \$16.20; \$49.20. **5.** \$259.20.
6. \$1860.47; \$2790.70; \$3348.84. **7.** 54c. per \$100.
8. \$29.97; \$11.99. **9.** \$3409.09.

Review Exercises. Pages 343-348

1. About 72%. **3.** More than \$24. **4.** \$12.96.
6. \$68.72. **7.** 8%. **8.** \$280.37.

9. \$7.67. **10.** $3\frac{4}{7}\%$.

11. Loss is 1% of cost or $1\frac{1}{99}\%$ of selling price.

12. \$722.65. **13.** \$38.10. **14.** \$61.92; \$38.27; \$10.75.

15. \$150,000. **16.** \$276.60.

17. Increased 4-fold; no change.

18. 1,003,319; 312,962. **19.** $1\frac{1}{3}$ cu. yd.; $16\frac{2}{3}\%$.

20. 80 lb. nitrogen; 160 lb. phosphoric acid; 200 lb. potash.

21. 63 lb. nitrogen; 45 lb. phosphoric acid; 81 lb. potash.

22. $3\frac{19}{27}\%$. **23.** 5 bonds. **24.** \$13.48.

25. Service should receive \$22. **26.** \$59.10.

27. About 6.4%. **28.** About 8.4 years.

29. The first; \$95.50. **30.** 1.9 mills. **31.** 500 lb.

32. \$270.60. **33.** 3.7%. **34.** Grocer gains 40c.

35. (a) $\$xyz/1200$; (b) $\{100(b - a)/na\}\%$; (c) $\$100s/(100 + rt)$.

36. $2 - 12 - 10$.

Page 353

1. 100°. **2.** 90°.

3. $\angle P = 30°$; $\angle Q = 60°$; $\angle R = 90°$. **4.** 60°.

5. 63°. **6.** 80°; 80°; 20°.

7. $\angle D = 90°$; $\angle E = 60°$; $\angle F = 30°$. **8.** (a) $70\frac{1}{2}°$, (b) $70\frac{1}{2}°$.

Pages 355-356

1. (a) ii, v; (b) (i) i, (ii) iv, (iii) iii.

2. (i) Acute-angled, (ii) Acute-angled, (iii) Obtuse-angled, (iv) Right, (v) Obtuse-angled, (vi) Right.

3. One. If there were two obtuse angles, the sum of these two angles would exceed 180°.

4. An obtuse angle. **5.** (a) 130°, (b) 50°, (c) 102°, (d) 90°.

10. (a) A, (b) C, (c) $C + A$, (d) $A + B$.

11. (a) 70°, (b) 20°, (c) 60°, (d) 45°.

13. 90°. **14.** 30°; 60°. **15.** 45°; 135°.

16. 55°; 35°. **17.** 100°; 80°.

Pages 364-366

3. (b) M and N. **4.** (b) A and D.

7. (b) About 170 yards.

Pages 371-372

2. 3.5 in. **3.** 8.1 cm. **5.** (c) 45°.

Page 375

1. $\angle B = 37°$; $\angle C = \angle D = 143°$.
2. 135°; 135°; 45°; 45°.
3. 120°. **4.** 120°.

Page 378

1. (a) $\frac{4}{1}$, (b) $\frac{1}{6}$, (c) $\frac{3}{1}$, (d) $\frac{1}{2}$, (e) $\frac{3}{2}$, (f) $\frac{10}{1}$, (g) $\frac{1}{1}$, (h) $\frac{1}{6}$.
2. (a) The two ratios are equal; (b) equal; (c) equal; (d) equal.
3. (a) 0.71, (b) 8.73,(c) 0.13, (d) 8.38. **4.** The first.

Page 379

1. Similar; all circles are of the same shape.
3. They are not similar because they have different shapes. Consider a square of side 2 inches and a rectangle of length 11 inches and width 1 inch.
5. They are not similar because they have different shapes.

Page 380

1. No. The second condition is not fulfilled.
2. No. The second condition is not fulfilled.

Pages 382-383

2. Yes. The triangles have two angles of the one respectively equal to two angles of the other.
3. $x = 4$, $y = 2$. **4.** (a) $\frac{FE}{ED}$, (b) $\frac{AE}{ED}$, (c) $\frac{DB}{BA}$.
5. 6 units.

Page 385

2. 72 ft.

Pages 390-391

1. (a) 60 yd. (b) 24 yd. **2.** 24 ft.
3. No. Find the length of the diagonal of the bottom.
4. $CA = 13$; $BC = 8$; $AB = 9$; $AB = 24$; $BC = 11$; $CA = 37$.
6. 8 miles.
7. (a) $\frac{\sqrt{2}}{1}$; (b) Yes, because all right isosceles triangles are similar.
8. $18\frac{1}{2}$ ft.
9. $DE = 7.1$ ft. $DF = 7.1$ ft. $AD = 10$ ft.
$AE = AF (= BE = ED = DF = FC) = 7.1$ ft.

Page 393

1. The following groups have the property possessed by 8, 15 and 17:— (i), (ii), (iv), (v), (vi), (viii), (ix), (x), (xi) and (xii).

Review Exercises. Pages 396-402

2. (a) No, (b) Yes, (c) Two, (d) Yes; for example, a triangle with the angles 100°, 40°, 40°.
3. (a) (i) 60°, (ii) 1°, (iii) 90°, (iv) 89°, (v) $(90 - n)°$, (vi) $n°$, (vii) $90° - (m + n)°$ or $(90 - m - n)°$;
(b) Yes, (c) $\angle A = \angle B$, (d) $x = 18$.
4. (a) (i) 150°, (ii) 91°, (iii) 180°, (iv) 179°, (v) 90°, (vi) 1°, (vii) $(180 - n)°$, (viii) $n°$, (ix) $180° - (A + B)$ or $180° - A - B$.
(b) Yes, (c) $\angle A = \angle B$, (d) 120°, (e) 30°, 150°.
5. 68°. **6.** 45°. **7.** 72°, 35°, 73°.
8. 75°, 15°. **9.** 75°, 75°, 30°. **10.** 90°.
11. (a) 30°, (b) 90°, (c) 90°.
15. (a) $\frac{5}{2}$. (b) $5\frac{5}{16}$ in. (c) $3\frac{3}{4}$ in. (d) 13 in. (e) $\frac{25}{4}$.
16. (a) Construct a right triangle in which the two sides containing the right angle are equal respectively to a side of each of the two squares.

(b) Construct a right triangle in which the hypotenuse is equal to the side of the larger square, and in which another side is equal to the side of the smaller square.

17. 3960 ft. **18.** About 2 hr. 40 min.

19. 15, 20, 25; 18, 24, 30.

21. The triangles would not be congruent. Hence, the distance *CA* would be either greater or less than the distance *AB*.

23. The instrument described on page 42 can be used for this purpose. To find the difference between $\angle BAC$ and its supplement, produce *BA*, place the top pin of the instrument at *A*, and open it so that the middle strip lies along *AC* and so that one of the outer strips lies along *AB*.

24. 17 ft. 6 in.

25. $\angle DCE$ = vertically opposite $\angle ACB$; $\angle D = \angle B$ (each being a right angle); $CD = CB$ (by construction). Hence, two angles and a side in ΔCDE are respectively equal to two angles and a side in ΔCBA. Therefore, the triangles are congruent. Therefore, corresponding sides, such as *DE* and *BA* are equal.

26. Margaret walked along the line *BC* until she came to the trees. There she marked a point *D* in line with *C* and *B*. At *D*, she turned to the right and walked along a line *DX* perpendicular to *BD*. She stopped at *E*, a point in line with *C* and *A*. She measured *BC*, *CD* and *DE*. Then, knowing that the triangles *ABC* and *EDC* were similar, she calculated *AB* from the equation $\frac{AB}{ED} = \frac{BC}{CD}$. (How did she know that the triangles *ABC* and *EDC* were similar?)

27. If the area of a circle of radius *R* units is equal to the sum of the areas of two circles the radii of which are *a* and *b* units, then $\pi R^2 = \pi a^2 + \pi b^2$. That is to say, $R^2 = a^2 + b^2$. Hence, it is necessary to find a line the square on which is equal to the sum of the squares on two given lines. (See Ex. 16(a) page 399.)

29. 80°. **30.** 28, 195, 197.

31. Since $\angle B = \angle D$ (each being 90°) and $\angle ACD = \angle ECD$, the two triangles are similar. Therefore, $\frac{AB}{ED} = \frac{BC}{CD}$. If the observer measures BC, CD and ED, he can find AB from the equation.

Recreations. Pages 403-404

1. Hint: make a net of the cube. (See p. 75.) Does Spiderella's position on the face of the cube affect the length of her route?
2. A number of nets of the room may be made. There are three possible routes. By using the Pythagorean theorem, verify that the length of the shortest route is 40 feet.
3. (a) 63 ft. (b) 63 ft.
4. White. (The hunter could not have killed this bear by shooting due south if he were not standing at the North Pole.)

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